Astrophysical Radiative Processes

* D R A F T *

Alessandro Patruno

April 23, 2022

ISBN

Contents

2 Blackbody and Thermal Radiation

Blackbody radiation is a type of thermal radiation that is emitted by an ideal body whose property is to emit (and absorb) every frequency of radiation with perfect efficiency. In simpler terms, a blackbody is the ideal perfect emitter/absorber. As we will see, this type of radiation is fundamental in order to understand any other type of radiation. A common myth that can be found in different textbooks and that is repeated over and over again is that blackbody radiation spectrum emerges directly from basic principles of quantum mechanics and it cannot be explained classically. Although the blackbody radiation was explained by M. Planck by quantizing the energy of photons emitted by atoms, the release of an arbitrarily high amount of energy, sometimes called the ultraviolet catastrophe, was something well understood at the time. Indeed the limitations of Rayleigh's formula and the equipartition theorem were very well known and understood by the physicists of the time and it was clearly stated in Rayleigh's work that the formula (leading to the ultra-violet catastrophe) was valid only in the low-frequency regime and could not be applied to the entire energy spectrum.

Beside this historical note, the great merit of M. Planck was to understand that atoms need not to emit a continuous energy range, even if the classical physics of the time had no clear justification for this assumption.

To understand blackbody and thermal radiation we will proceed by first outlining the laws of thermodynamics. From these, we will define the concept of thermal equilibrium which is essential to understand what is meant by thermal radiation (and blackbody radiation) and to derive its main properties. Finally, we will use the derived properties of thermal radiation to solve the equation of radiative transport and to do so we will use the so-called radiative diffusion approximation which has a broad applicability in astrophysical settings. For example, this type of approach is useful to describe radiation emitted by stars, accretion disks, nebulae, stellar atmospheres and many other astrophysical environments.

2.1 Laws of Thermodynamics

The *first law of thermodynamics* (conservation of energy) states that energy can be transformed from one form to another, but it cannot be created or destroyed:

$$dU = dQ - dW \tag{2.1}$$

where dU is the infinitesimal change in internal energy of the system, dQ is the heat **added to** the system and dW is the work **done by** the system.

The second law of thermodynamics relates the heat transfer between two bodies with their temperature and entropy. The second law refer to the fact that heat cannot be transferred from a colder to a hotter body without some other change occurring at the same time:

$$dS = \frac{dQ}{T} \tag{2.2}$$

These two laws are logically preceded by the Zeroth law of thermodynamics, which defines the concept of temperature of a body and the meaning of *thermal equilibrium*. It can be proved experimentally that a high temperature object in contact with a low temperature object transfers heat to the lower temperature object. The temperature of the two bodies will approach the same value and remain constant over time (in absence of losses or other external influences). When this constant temperature is reached, the two bodies are said to be in thermal equilibrium.

There is a great deal of confusion when discussing thermal equilibrium since it is often used as a synonym for *thermodynamic equilibrium*. However, these two concepts are different although related. A body in thermodynamic equilibrium has reached thermal equilibrium as well as mechanical, chemical and radiative equilibrium. This means that two bodies in thermodynamic equilibrium need to have achieved the same temperature, pressure, chemical potentials and there must be a zero net flux of radiation. The radiative equilibrium condition can be expressed as:

$$h = \int_0^\infty h_\nu d\nu = 0 \tag{2.3}$$

where h_ν is equal to the negative monochromatic gradient of the flux:

$$h_\nu = -\nabla F_\nu \tag{2.4}$$

2.2 Equipartition Theorem

A powerful result of statistical mechanics is that when a system is in thermal equilibrium, its energy is shared equally amongst all its accessible degrees of freedom. Remember that the degree of freedom correspond to the dimension of the phase space, the latter being the space representing the set of all possible accessible states of a system. When the degree of freedom is quadratic in the energy, then the average energy of each degree of freedom is equal to $\frac{1}{2}k_{\mathrm{B}}T$. For an ensemble of particles moving in three dimensions and in thermal equilibrium, the total average translational kinetic energy per particle is therefore:

$$E_k = \frac{3}{2}k_{\mathrm{B}}T. \tag{2.5}$$

This value is useful to try to understand the effect of matter and radiation interacting under the conditions of thermal equilibrium. Indeed a photon has an energy of $E_{\mathrm{ph}} = h\nu$ that can be compared with the expression of the average kinetic energy above. If for example a monochromatic photon field with average energy E_{ph} interacts with matter with average translational kinetic energy E_k and $E_{\mathrm{ph}} \ll E_k$, then when the radiation is absorbed by matter, the net contribution of the photon field to the total energy of the system will be small and the energy of the photon field will be quickly redistributed across the degrees of freedom of the absorbing material. In this case one can say that the radiation has been *thermalized* by matter.

2.3 Thermal Equilibrium

As we have seen in the previous sections, the definition of thermal equilibrium is given by the Zeroth law of thermodynamics. A practical interpretation of thermal equilibrium is that a body that has reached this state will absorb as much energy as it emits. If it is unable to absorb enough energy to compensate its emission, then it will cool down until it reaches the same temperature of the surroundings. If it absorbs more energy that it is able to emit then it will heat up until reaching the same temperature of its surroundings.

Thermal equilibrium applies to both matter and radiation. Matter in thermal equilibrium obeys the Zeroth law of thermodynamics and the velocities of the particles are not the same but follow the Maxwell-

Boltzmann distribution:

$$F(v)dv = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT} dv \tag{2.6}$$

where v is the velocity of the particle and m its mass. The Maxwell-Boltzmann distribution written in this way is valid for non-relativistic particles, but many astrophysical systems involve speeds which are a significant fraction of the speed of light.

In that case the same physical principles apply but the distribution need to be adapted to account for the relativistic effects thus arising. It is possible to generalize the Maxwell-Boltzmann distribution such that it can correctly describe both relativistic and non-relativistic systems. Instead of using the velocity of the particle, it is more convenient to use the momentum p whose relativistic expression is:

$$p = \gamma\beta mc \tag{2.7}$$

where $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$. The general expression for the Maxwell Boltzmann equation then becomes:

$$F(p)dp = \frac{p^2 e^{-\gamma\Theta}}{\Theta m^3 c^3 K_2(1/\Theta)} dp \tag{2.8}$$

where $\Theta = kT/mc^2$ and K_2 is the modified Bessel function of the second kind. This relativistic expression is sometimes also called the Maxwell-Jüttner equation.

For radiation to be in thermal equilibrium a *necessary* condition is that it is *emitted by matter in thermal equilibrium*. However, this condition is not sufficient since there is also another important condition that needs to be met in order to reach thermal equilibrium. Indeed if the photons are emitted by matter in thermal equilibrium and they are immediately free to escape the system without further interactions, then the radiation emerging will not be in thermal equilibrium, even if the radiation is thermal radiation. Photons need to be absorbed and emitted many times before escaping the system in order to be considered in thermal equilibrium with matter. This will become more clear in the following sections, when we will discuss the principle of detailed balance.

2.4 Blackbody Radiation

Blackbody radiation is a type of idealized radiation emitted by a body that can absorb radiation at all wavelengths with maximum efficiency (no reflection). When a blackbody is in thermal equilibrium, it must emit radiation to obey the Zeroth law of thermodynamics and maintain a constant temperature. The emission occurs also with maximum efficiency, in the sense that no other body can emit thermal radiation with an efficiency higher than a blackbody. Therefore the blackbody is the perfect absorber and the perfect emitter of thermal radiation.

To create a real version of a body with properties as close as possible to that of the ideal blackbody emitter, it is useful to consider a closed enclosure of arbitrary shape with thick walls so that any radiation inside it cannot escape. If one waits a sufficiently long time, the enclosure will reach thermal equilibrium at temperature T. This means that both radiation and matter have reached a thermal equilibrium since matter will have reached a constant temperature and the radiation it emits is also in thermal equilibrium by definition, since all radiation emitted will also be absorbed and re-emitted many times.

If one opens a small hole in the enclosure, small enough so that any radiation escaping is only a tiny fraction of the total, the thermal equilibrium will be maintained. The radiation escaping from the small hole is a very close real life version of the idealized blackbody emission. This happens regardless of the material composing the enclosure and regardless of the shape and objects present within the enclosure. To demonstrate this property, imagine to attach to the first enclosure a second one at the same temperature T with similar characteristics (arbitrary shape, thermal equilibrium and a small hole on its walls). The two enclosures should be positioned so that the two small holes can face each others, so that radiation from one hole can in principle enter in the other hole. To simplify the problem, one can place a narrow pass-band filter between the two holes in order to allow only a specific frequency ν to pass through and block any other radiation frequency. Since we are considering only one frequency we can consider the specific brightness of the first hole at the filter frequency ν and call it I_ν and the specific brightness of the second hole as I'_ν. If radiation flows from the first to the second enclosure, then let´s call the amount of heat associated with this radiation exchange as δQ. The entropy of the first enclosure must now decrease and the entropy of the second enclosure must increase according to the second law of thermodynamics. The total change in entropy of the whole system is the sum of the two:

$$dS = \frac{\delta Q}{T_1} - \frac{\delta Q}{T_2} \tag{2.9}$$

where the subscripts 1 and 2 refer tot the first and second enclosures. But since we have said that the two enclosures are in thermal equilibrium and have the same temperature, the change in entropy of the system is exactly zero. Furthermore, no heat would have flowed in the first place because of thermal equilibrium. Therefore there cannot be any net flow of energy between the two enclosures or the system is violating the second law of thermodynamics. Therefore the two values of the specific brightness must be the same and so $I_\nu = I'_\nu$. Since the frequency ν has been chosen arbitrarily, this property of the specific brightness has to be true for every spectral component.

Another important property of blackbody radiation is that it is *isotropic* and *homogeneous*, meaning that the flux of radiation must be the same in every direction ed in every location. To demonstrate isotropy we can use a counterargument to show that it leads to unphysical conditions inside one cavity. For example, let's suppose that the radiation propagates from left to right with an intensity higher than the radiation propagating in the bottom-top direction. We can now introduce two identical bodies (same mass, shape, material etc.) within the enclosure and wait that they both reach thermal equilibrium. Let's also suppose that these two bodies are small enough so that they do not introduce a significant perturbation to the radiation energy density within the enclosure (in other words we can use them as test bodies for our experiment). We can position the two bodies in such a way that one intercepts more radiation going from let to right and the other more of the radiation from bottom to top. For example we can select a rectangular shape for our bodies and place the first of them such that the longer side is parallel to the left-right direction and the other body with the longer side aligned to the bottom-top direction. Since the flux of radiation is more intense in the bottom-top direction, the first body will become hotter than the second body in a shorter amount of time. This is a violation of the second law of thermodynamics since one can use the temperature difference between the two bodies to create a Carnot cycle between the two bodies and convert heat into work without any other thermodynamic variation of the system. In other words one can absorb energy from one source (the radiation within the cavity) to continuously produce work. The same would be true if we filter certain radiation frequencies. Therefore, since this is a non-physical result, the radiation cannot have a preferred direction of propagation and so the cavity must be filled with isotropic radiation.

To demonstrate that blackbody radiation is also homogeneous we can use a similar reasoning with the same two bodies introduced before.

This time let's check the system after it has achieved thermal equilibrium. When this happens, both bodies have to absorb and emit the same amount of energy. The amount of energy absorbed by each body depends on the absorption coefficient of each body and by their surface extension. Since both bodies absorb and emit the same quantity of energy, the energy incident on each of them must be identical. This quantity depends on the energy density of radiation and on geometrical factors:

$$dE_{inc} = u_\nu(T)\, c\, dS\, dt\, d\nu \tag{2.10}$$

where dS is the infinitesimal surface area element of the body. Since $dE_{\mathrm{inc},1} = dE_{\mathrm{inc},2}$ for body one and two, and since the geometrical factors are identical for the two bodies, it follows that the energy density must be the same, $u_{\nu,1}(T) = u_{\nu,2}(T)$, regardless of the position of the two bodies. Thus the energy density of blackbody radiation must be identical in each point of the cavity which means it is homogeneous. The property of isotropy and homogeneity applies to any cavity of any shape and material, so that if the temperature T is the same for two cavities of different shape and material then the energy density of radiation has to be the same. Once again if this had not been the case then we would violate the second law of thermodynamics once more. This is true for each single spectral component of blackbody radiation. This tells us an important fact: if you measure the specific brightness of a blackbody emitter at temperature $T_1 > T_2$ then it must be higher than the value you obtain for the colder body at *any* frequency. In other words, plotting the blackbody specific brightness curve for a body hotter than another, you would see the curve of the hotter body always above the curve of the colder body, with the two curves crossing only at the origin and asymptotically at at infinity.

A final property of blackbody radiation is that it is unpolarized. This is also a consequence of the laws of thermodynamics, since a blackbody can absorb all type of radiation and thus has no preferred polarization state. Another way to demonstrate this principle is by using the fact that photons obey the Bose-Einstein statistics. The mean occupation number of each mode of the electromagnetic field is:

$$n_i(E_i) = \frac{1}{\exp(E_i/k_B T) - 1} \tag{2.11}$$

where E_i refers to the energy of the $i-th$ mode and k_B is the Botzmann constant. Since photons of different polarization have the same energy (in vacuum) then the occupation number of each mode is the same since it depends only on the energy of the photon and on the temperature of the cavity. Therefore the occupation number of each polarization state must be the same and therefore blackbody radiation emits a statistical mixture of all polarization states, which is equivalent to unpolarized radiation.

Summarizing, if we have thermal equilibrium between two enclosures of arbitrary shape and material, the specific brightness is independent on the properties of the enclosure itself. The only parameter that has an effect on the specific brightness is just the temperature T of the enclosures. The radiation inside the cavity must be isotropic, homogeneous and unpolarized. The specific brightness I_ν must therefore be a universal function of the frequency and temperature alone that we call the Planck function $B_\nu(T)$.

2.5 Properties of the Planck Spectrum

The Planck function describes the specific brightness of a blackbody emitter:

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/k_\mathrm{B}T) - 1} \tag{2.12}$$

This expression can be derived by ...

Let's now check the meaning of this expression in detail. The number '2' appears in Eq.2.12 because light has two independent polarization states (left and right circular polarization). The Planck constant 'h' appears because of course photons are quantized. The term $1/c^2$ appears because photons propagate at the speed of light 'c' and the square appears because to calculate a specific brightness one has to multiply the spectral energy density $u_\nu(T)$ by the speed of light:

$$B_\nu(T) = \frac{u_\nu(T)c}{4\pi} \tag{2.13}$$

The numerator of Eq.2.12 is the density of states of photons in a box $2\nu^2/c^3$ multiplied by the average energy per state:

$$E_\mathrm{avg} = \frac{h\nu}{\exp(h\nu/k_\mathrm{B}T) - 1} \tag{2.14}$$

Finally, remember that "-1" in the denominator is there because photons are bosons, so there can be multiple photons with the same quantum number and the use of the Bose-Einstein statistics reflects this fact. It is also worth noticing that the chemical potential of photons is zero, otherwise we would have seen an expression like $(h\nu - \mu)/k_\mathrm{B}T$, where μ is the chemical potential.

2.6 Kirchhoff's Law of Therman Emission

When we introduced a body of arbitrary shape and material inside the enclosure we demonstrated that the radiation escaping from a tiny hole on a wall is still blackbody radiation. The question now is about the body inside the cavity, what will be its emission properties? Let's imagine to put two bodies of small size and identical shape and surface area inside the cavity. Let's call them body 1 and 2 and let's also suppose that the two bodies are made by different material. If the first body absorbs more radiation than the second one, then it has to emit more as well. The amount of energy absorbed will depend on the energy density u around the bodies (that we have previously shown to be identical because of homogeneity of blackbody radiation), on the geometrical factors (which are also identical by construction) and on the absorption coefficient of each of the two bodies. Therefore the energy absorbed has to be:

$$dE_{\mathrm{abs},1} = \xi u \alpha_{\nu,1} \quad (2.15)$$

$$dE_{\mathrm{abs},2} = \xi u \alpha_{\nu,2} \quad (2.16)$$

where ξ is a geometrical factor, identical for the two bodies. The energy emitted will instead depend on the specific emission coefficient j_ν and by the geometrical factors, beside the temperature T of the enclosure:

$$dE_{\mathrm{em},1} = \eta j_{\nu,1} \quad (2.17)$$

$$dE_{\mathrm{em},2} = \eta j_{\nu,2} \quad (2.18)$$

Since at thermal equilibrium the energy absorbed has to be the same as the energy emitted, it follows that, for a certain temperature T we have:

$$\mathrm{K}_\nu(T) = \frac{\xi u}{\eta} = \frac{j_{\nu,1}}{\alpha_{\nu,1}} = \frac{j_{\nu,2}}{\alpha_{\nu,2}} \quad (2.19)$$

What is this function $K_\nu(T)$? If the two small bodies have absorption coefficient equal to one at all wavelengths (perfect absorber) then both of them will emit as blackbodies once they reach thermal equilibrium. Therefore it is clear that this universal function $K_\nu(T)$ that depends only on temperature and frequency has to be the Planck function $B\nu(T)$. Furthermore, the ratio between the specific emission coefficient and the absorption coefficient is the definition of the source function S_ν. Therefore, for bodies in thermal equilibrium of arbitrary shape and material we have the Kirchhoff's law of thermal radiation:

$$S_\nu = \frac{j_\nu}{\alpha_\nu} = B_\nu(T) \quad (2.20)$$

This law states that a good absorber is also a good emitter (and vice versa). It also states that it is impossible for any body in thermal equilibrium to emit more radiation than a blackbody. Summarizing:

$$I_\nu = B_\nu(T) \rightarrow \text{blackbody radiation} \tag{2.21}$$

$$S_\nu = B_\nu(T) \rightarrow \text{thermal radiation} \tag{2.22}$$

It is important to stress that blackbody radiation is thermal radiation but vice versa is generally not true: thermal radiation is not necessarily blackbody radiation. In astrophysical situations the thermal emission becomes blackbody radiation when the photons are absorbed and emitted several times before leaving an object in thermal equilibrium. This occurs when the optical depth of the system is large and the original emission spectrum of matter in thermal equilibrium is continuous. In that case one can expect to observe (approximately) blackbody radiation. If the thermal object is instead optically thin, then the radiation will escape the system with few or no absorption events occurring and the radiation will therefore be thermal and follow Kirchhoff's law, but will not be blackbody radiation. The same is true if the emitting body is unable to absorb/emit radiation at certain specific frequencies: the emitted radiation can then never reach thermal equilibrium and the emitted spectrum will not be that of a blackbody).

2.7 Principle of Detailed Balance

As we have seen, when a body is in radiative equilibrium with its environment then the total thermal radiation leaving the object is equal to the total thermal radiation it absorbs. In other words when two bodies are at the same temperature, they will stay in mutual thermal equilibrium. A body at temperature T immersed in a bath of radiation emitted by another body at the same temperature will therefore emit on average as much light as it absorbs, following radiative equilibrium. This happens because of the principle of detailed balance, stating that in thermodynamic equilibrium every elementary process is balanced by its opposite. In thermodynamic equilibrium the amount of thermal radiation emitted at every frequency and in all directions by a body at temperature T is equal to the corresponding amount that the body absorbs regardless of whether the body in question is black or not.

For example, if the body is a perfect blackbody then the intuition about this statement is simple, since all radiation hitting the surface of the body will be absorbed and then re-emitted. The amount of radiation absorbed and emitted can be calculated with the blackbody curve and it depends solely on the temperature of the body. When the body is not a perfect blackbody, for example because the body is small compared to the certain wavelengths of light, or because its emissivity is smaller than 1 in certain frequencies, then the principle of detailed balance still applies and so we can expect that a good absorber is also a good emitter.

2.8 Intuition about Kirchhoff's Law

To develop an intuition for Kirchhoff's law, imagine having a body with a certain temperature T in thermal equilibrium. For example, a piece of plastic material has reached thermal equilibrium with the surrounding environment. Let's say that the material has a certain color, for example, suppose it looks red to your eyes during the day. Where does the color red come from? This radiation is reflected from the Sun, indeed the mixture of color frequencies that correspond to the color you see are the ones that the rock cannot absorb. From Kirchhoff's law and the principle of detailed balance, we know that a bad absorber is also a bad emitter. Suppose now you take an energy spectrum of the piece of plastic, still in thermal equilibrium, in a dark room so that the sunlight does not interfere with your measurement. What will you see? You expect to see thermal radiation emitted by the body at temperature T. This radiation will be emitted at all frequencies that the body can absorb, therefore you expect to see a deep "absorption" band around the frequencies that correspond to the red color since the plastic body is unable to emit them. At the other frequencies, you will measure a specific emission coefficient that is given by the Plack function times the absorption coefficient $B_\nu(T)\alpha_n u$. The specific intensity/brightness of the body is then given by the equation of transport. A confusing aspect of this is that we said in the previous chapter that when the optical depth is large, then we have $I_\nu \approx S_\nu \approx B_\nu(T)$. And yet we are now saying that the specific brightness that we will see is not $B_\nu(T)$. The reason is that we don't have to forget that the optical depth τ_ν is frequency-dependent. When the frequency ν corresponds to the red color in question, then $\tau_{\nu_red} \approx 0$. So in this case we *cannot* expect $I_{\nu_red} \approx S_{\nu_red} \approx B_{\nu_red}(T)$. A related question is then the following: if $\tau_{\nu_red} \approx 0$ then why light is not crossing the medium? This happens because reflection occurs at the microscopic level. Reflection is a phenomenon that occurs at the atomic level when light waves induce a polarization on the individual atom of the material (or on the free electrons in a metal) that causes a dipole emission. This dipole emission is composed of secondary waves that interact according to the laws of Huygens and Fresnel to give rise to the known reflection and transmission phenomena. This physics is not accounted for in the equation of transport –remember we are using the radiative transfer approximation and not Maxwell equations– and therefore this conclusion might seem counterintuitive.

It is important to notice that, in our example of red plastic, we have made the assumption that only the red frequencies are those that cannot be absorbed/emitted. Therefore the spectrum of the red plastic will look like a blackbody everywhere except at those frequencies. However, it is important to stress that real materials have a certain ''inefficiency" in absorbing/emitting at basically all frequencies. Therefore the spectrum of a real body will look like an indented curve at basically all frequencies (like in the Figure 2.1).

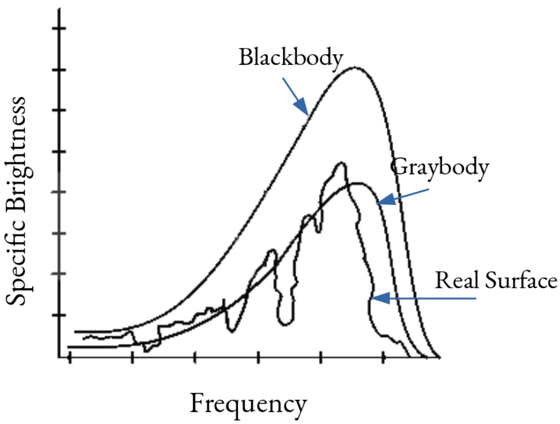

Figure 2.1: Specific brightness of a real surface vs. blackbody and graybody emission. A graybody is an imperfect blackbody, i.e., an object that can emit/absorb at all frequencies but not with perfect efficiency, $\alpha_\nu = k$ with $k \in (0,1)$.

It is important to notice that, in our example of red plastic, we have made the assumption that only the red frequencies are those that cannot be absorbed/emitted. Therefore we have said that the spectrum of the red plastic will look like a blackbody everywhere except at those frequencies. However, it is important to stress that real materials have a certain ''inefficiency" in absorbing/emitting at basically all frequencies. Therefore the spectrum of a real body will look like an indented curve at basically all frequencies (like in the Figure **??**).

2.9 Application of Kirchhoff's Law

Let's now apply Kirchhoff's law of thermal equilibrium to the equation of transport in order to describe three simple physical cases (Figure 2.2). This application shows how the principle of detailed balance works as we can see how a good absorber is also a good emitter and vice-versa.

Case 1 First let's take an optically thick source, for example, a thick hot gas cloud made of hydrogen plasma and let's assume that the source function S_ν is constant. This type of plasma, as we will see, can emit a continuum spectrum because electrons interact with the free protons and the acceleration generates photons covering the whole frequency range. The cloud has to be optically thick so that radiation can reach thermal equilibrium along with the plasma itself. Besides the plasma cloud, we assume there is no other foreground or background object emitting or absorbing light. Therefore we can set $I_{\nu,0} = 0$ in the equation of transport. Therefore $I_\nu = S_\nu(1 - e^{-\tau_n u})$. Since the plasma is very dense, the optical depth will be $\tau_\nu \gg 1$ and the equation of transport is:

$$I_\nu = S_\nu = B_\nu(T). \tag{2.23}$$

This means that you will see a blackbody spectrum (Planck spectrum).

Case 2 The same hot thick plasma cloud is now surrounded by a cloud of cold and thin gas. With "cold'' we mean that the thermal emission from this cloud can be considered negligible when compared to the radiation coming from the hot plasma and therefore $S_\nu \approx 0$. The hot plasma acts as a background source of radiation, therefore this time $I_{\nu,0} = B_\nu(T)$. If the cold gas is composed of some atomic/-molecular species, encoded by the cloud color green and red, then it will be optically thin at all frequencies except those that correspond to the atomic/molecular energy transitions where τ_{green}

and τ_{red} are both much larger than 1. The equation of transport is now:

$$I_\nu = B_\nu(T)e^{-\tau_\nu} \tag{2.24}$$

and therefore the spectrum observed will be a continuum with narrow absorption bands corresponding to the color green and red where the intensity falls exponentially.

Case 3 We now keep the same gas cloud and increase its temperature T, while at the same time we remove the hot plasma cloud. There is no background blackbody source now and the equation of transport is:

$$I_\nu = S_\nu(1 - e^{-\tau_\nu}). \tag{2.25}$$

Here again, τ_ν is small everywhere except for the red and green color frequencies. We thus see the source function only at the green/red frequencies (since $e^{-\tau_\nu} \approx 0$) whereas everywhere else it will be mostly dark due to the small optical depth ($S_\nu(1 - e^{-\tau_\nu}) \approx S_\nu \tau_\nu \approx 0$).

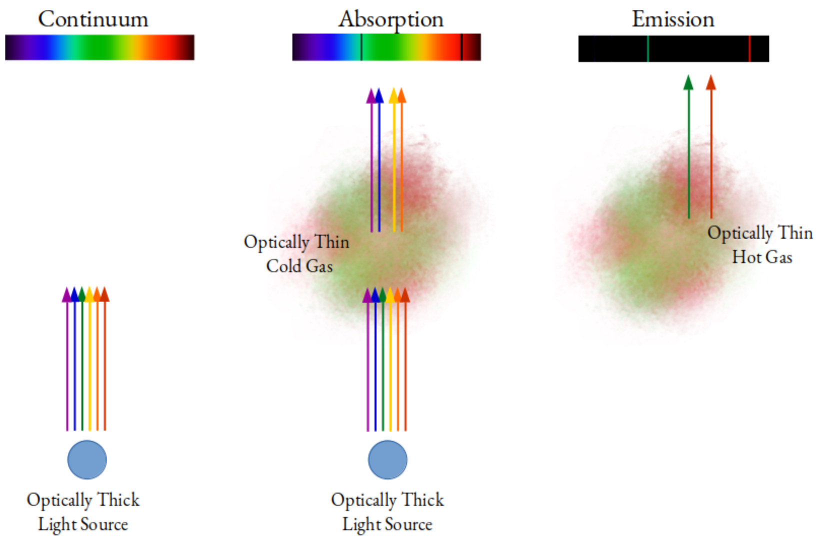

Figure 2.2: The principle of detailed balance. On the left an optically thick hot source emits a continuum spectrum. In the center, the radiation crosses a cold thin cloud of gas that absorbs some specific frequencies corresponding to line transitions of the elements composing the gas. On the right the same gas is heated such that it emits radiation with no background radiation.