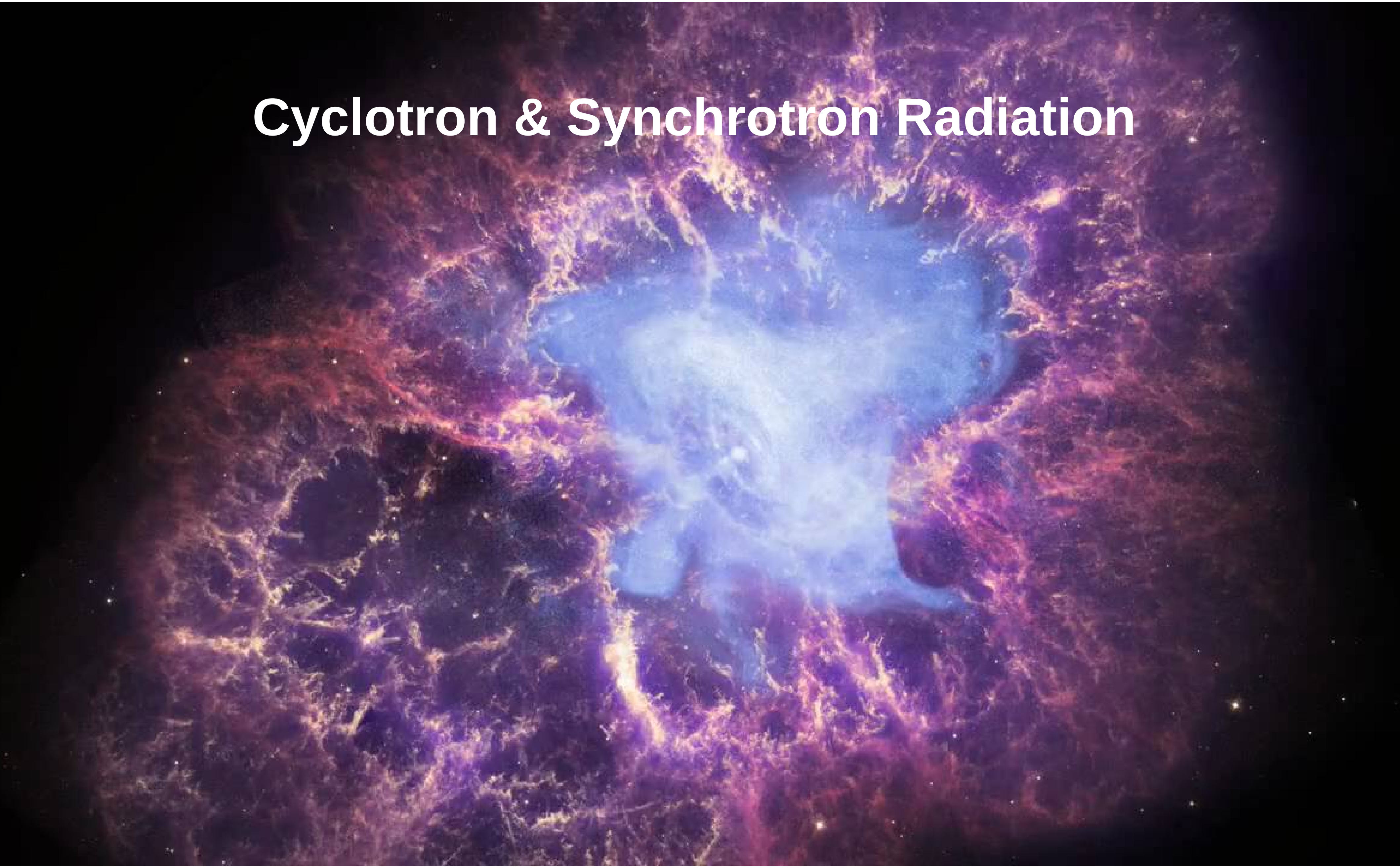
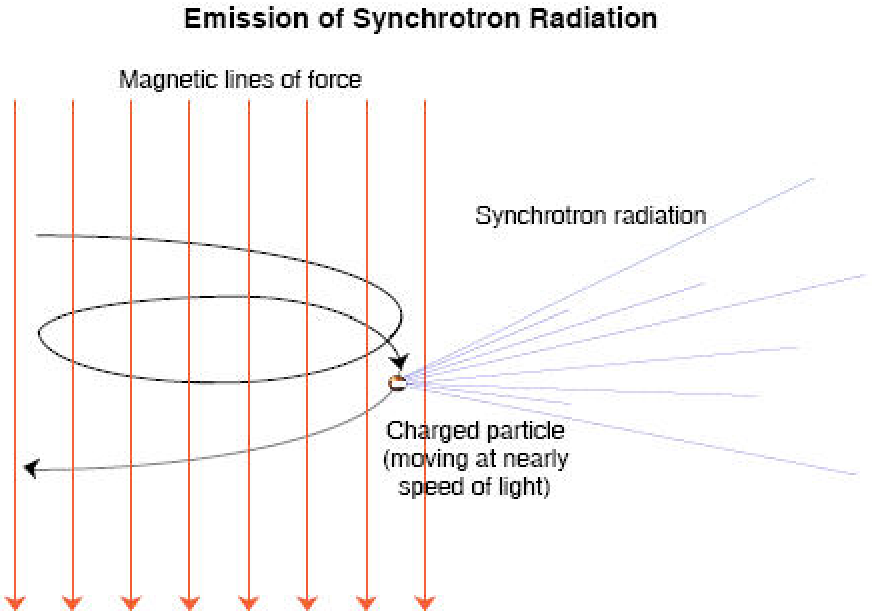
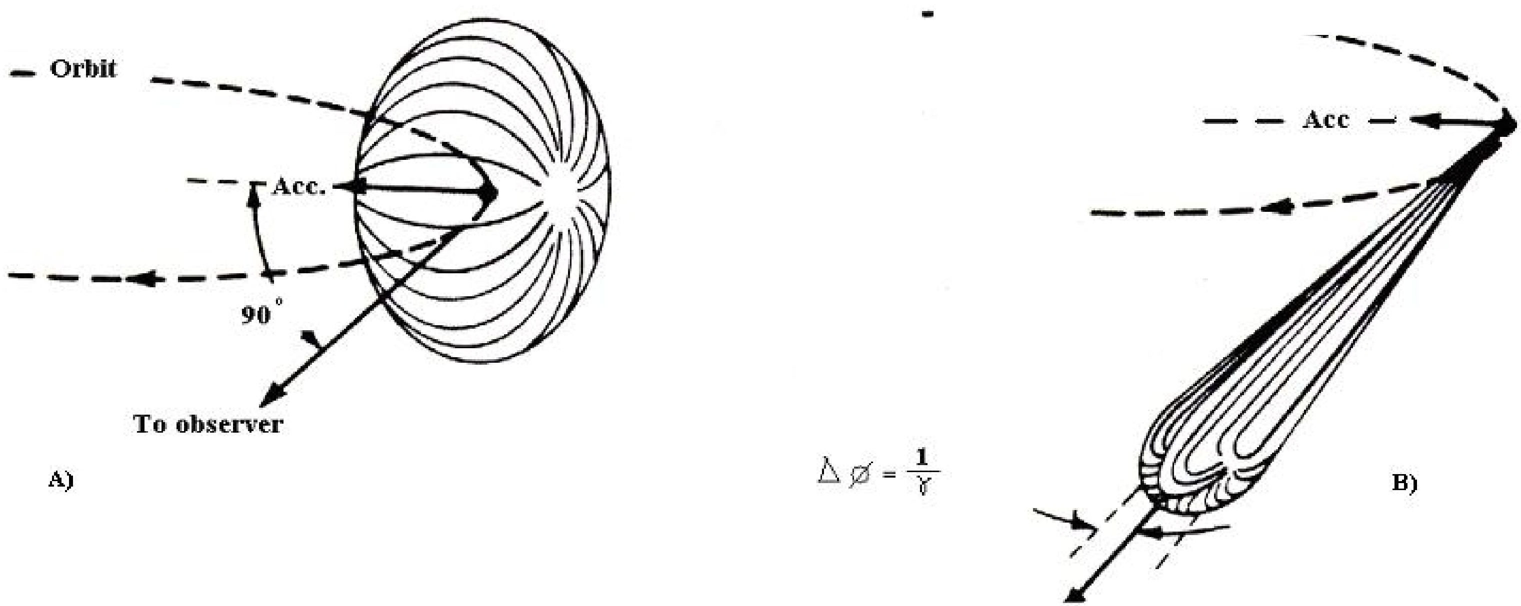
Cyclotron & Synchrotron Radiation



Synchrotron Radiation is radiation emerging from a charge *moving relativistically* that is accelerated *by a magnetic field*.

The relativistic motion induces a change in the radiation pattern which is very collimated (beaming, see Lecture 3).

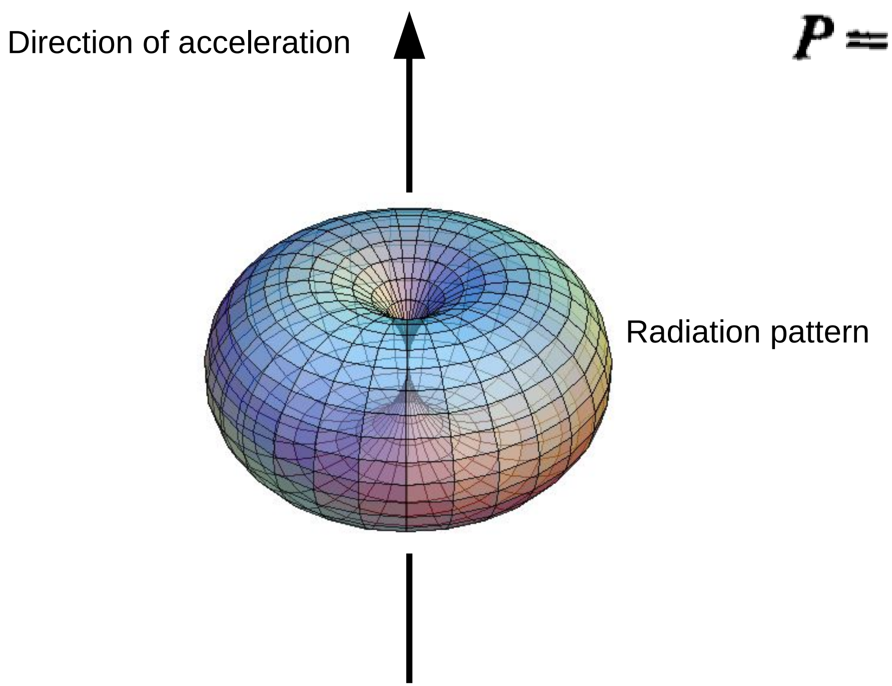




Cyclotron Radiation: power & radiation pattern

To understand synchrotron radiation let’s first begin with the non-relativistic motion of a charge accelerated by a magnetic field.

That the acceleration is given by an electric field, gravity or a magnetic field does not matter for the charge, which will radiate according to the Larmor’s formula (see Lecture 3)



$$P = \frac{2q^2\dot{u}^2}{3c^3}$$

Remember that the radiation pattern is a torus with a sin^2 dependence on the angle of emission:

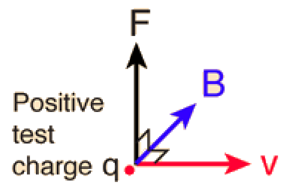
$$\frac{dW}{dt\,d\Omega} = \frac{q^2\dot{u}^2}{4\pi c^3}\sin^2\Theta$$

Cyclotron Radiation: gyroradius

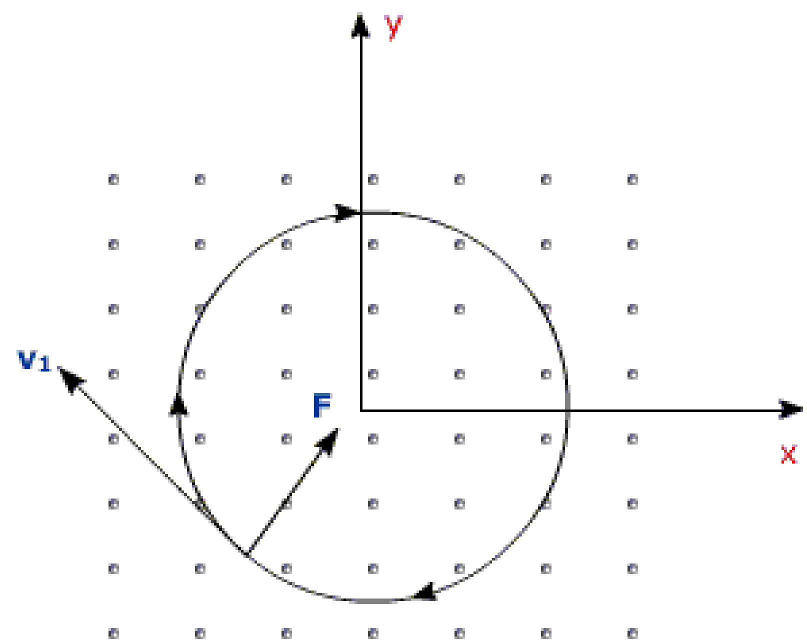
So let's take a charge, say an electron, and let's put it in a uniform **B** field. What will happen?

The acceleration is given by the Lorentz force.

$$\vec{F} = q\vec{v} \ x \ \vec{B}$$



If the B field is orthogonal to v then: $F = qvB$



Equating this to the centripetal force gives the "Larmor radius":

$$F = \frac{m v^2}{r_L} = qvB \rightarrow r_L = \frac{mv}{qB}$$

We can also find the cyclotron angular frequency:

$$F = \frac{m v^2}{R} = m \omega_L^2 R \rightarrow \omega_L = \frac{qB}{m}$$

Cyclotron Radiation: cyclotron frequency

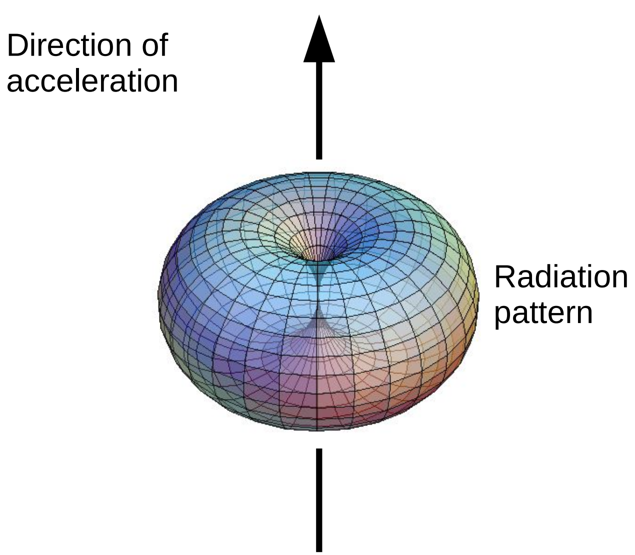
From the angular frequency we can find the period of rotation of the charge:

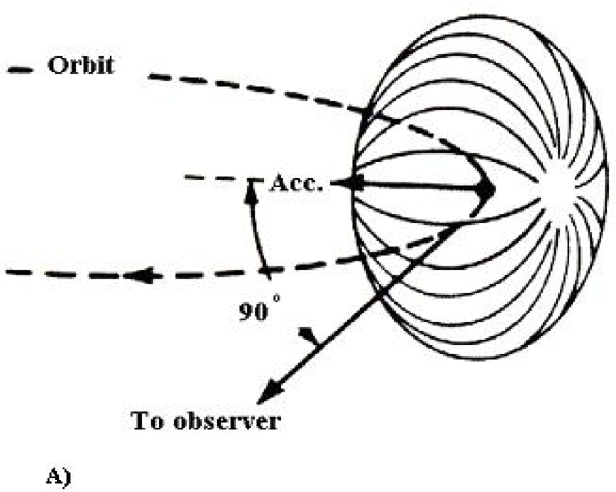
$$T = \frac{2\pi}{\omega_L} = \frac{2\pi m}{qB}$$

Note that the period of the particle *does not depend on the size of the orbit and is constant if B is constant.*

The charge that is rotating will emit radiation at a single specific frequency:

$$\nu_L = \frac{\omega_L}{2\pi} = \frac{qB}{2\pi m}$$



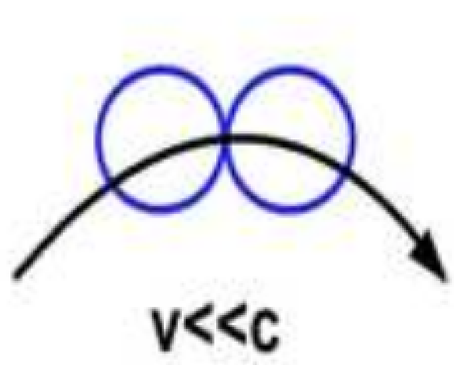


Cyclotron Radiation: power spectrum

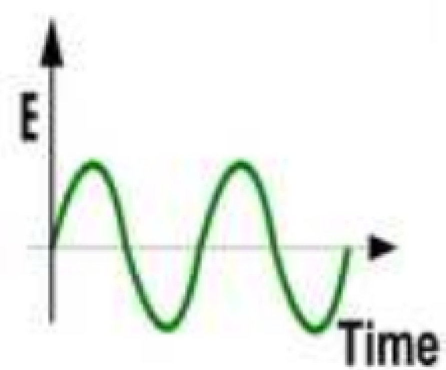
Since the emission appears at a single frequency $\nu_L = \frac{\omega_L}{2\pi} = \frac{qB}{2\pi m}$

and the dipolar emission pattern is moving along the circle with constant velocity, the electric field measured will vary sinusoidally and the power spectrum will show a single frequency (the Larmor or cyclotron frequency).

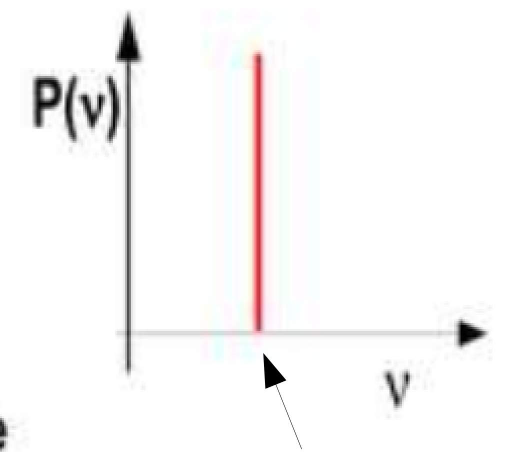
Radiation Pattern



Electric Field E(t)



Power Spectrum P(nu)



Cyclotron Radiation: kinetic energy

Suppose now you have a charged particle, say a proton, with a kinetic energy of 1 MeV.
The definition of electronvolt eV is the following:
the amount of energy an charge gains after being accelerated in an electric potential of 1 Volt (1 Volt ~300 statVolt in cgs units).

Both an electron and a proton gain the same energy by definition, but of course the mass of the proton is ~2,000 times larger than the electron's, thus its velocity is way smaller. (An implication for this is that protons need to be *much more* energetic than electrons to become relativistic).

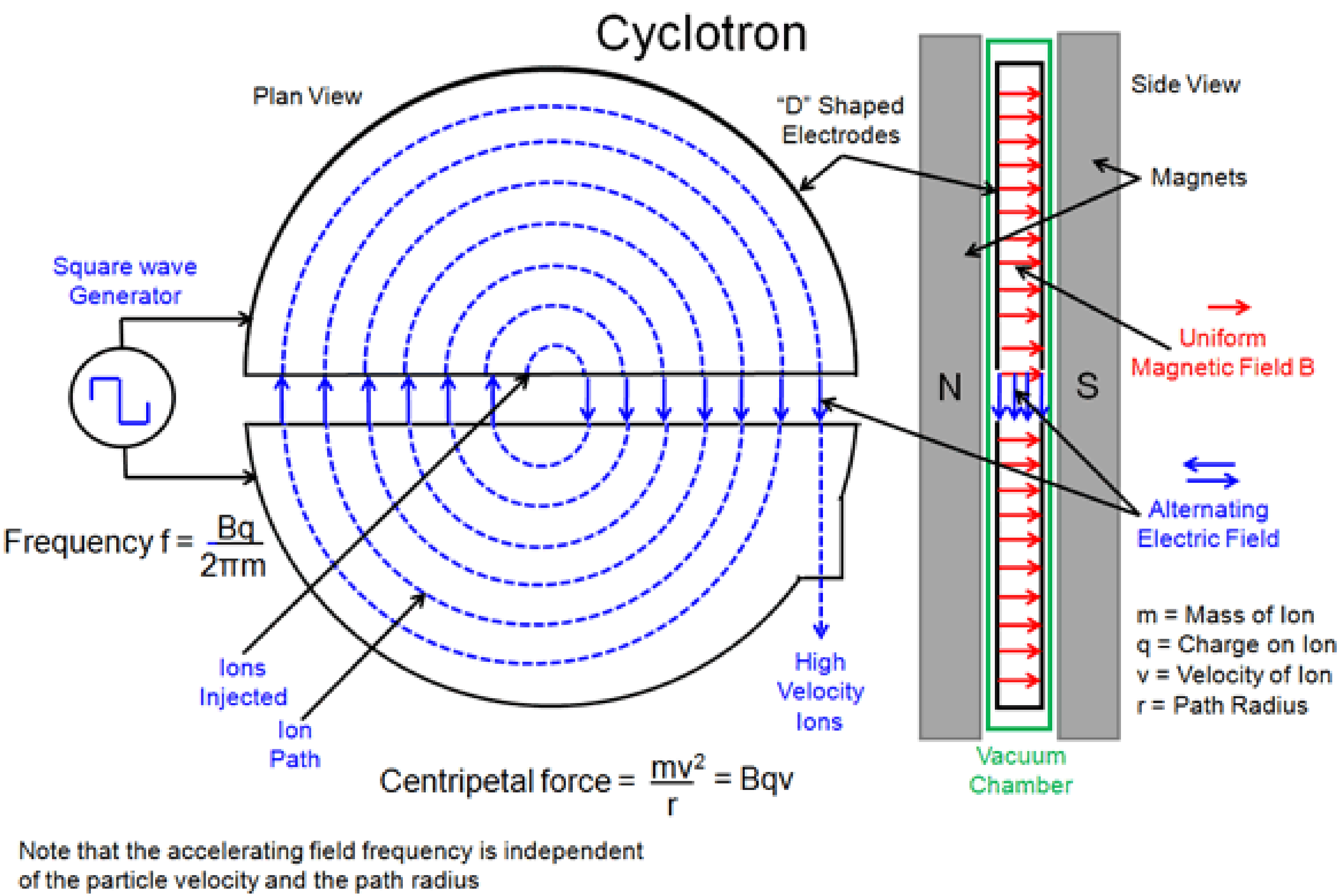
$$E_k=\frac{1}{2}mv^2=qV \rightarrow v=\sqrt{\frac{2qV}{m}}$$

Let's put this velocity v back into the Larmor radius' formula: $r_L=\frac{mv}{qB} \rightarrow r_L=\sqrt{\frac{2mV}{qB^2}}$

(q = 1.6e-19 C; m = 1.67e-27 kg; V = 1,000,000 V; B = 1 T)

So if I have a 1 MeV proton, and a B field of say, 1 T (1 Tesla = 10^4 G) then my Larmor radius is: 15 cm.

What will happen if you have a 1 MeV electron in the same field?



$$r_L = \frac{mv}{qB} \rightarrow r_L = \sqrt{\frac{2mV}{qB^2}}$$

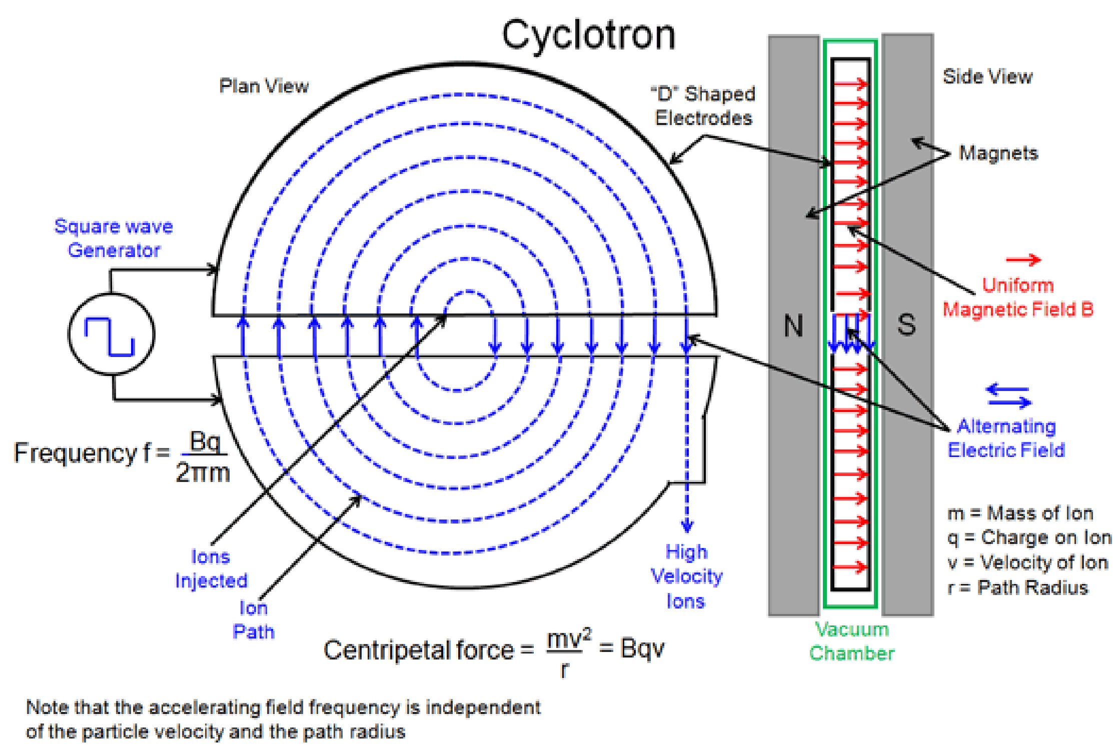
$$E_k = \frac{1}{2} m v^2 = q V \rightarrow v = \sqrt{\frac{2qV}{m}}$$

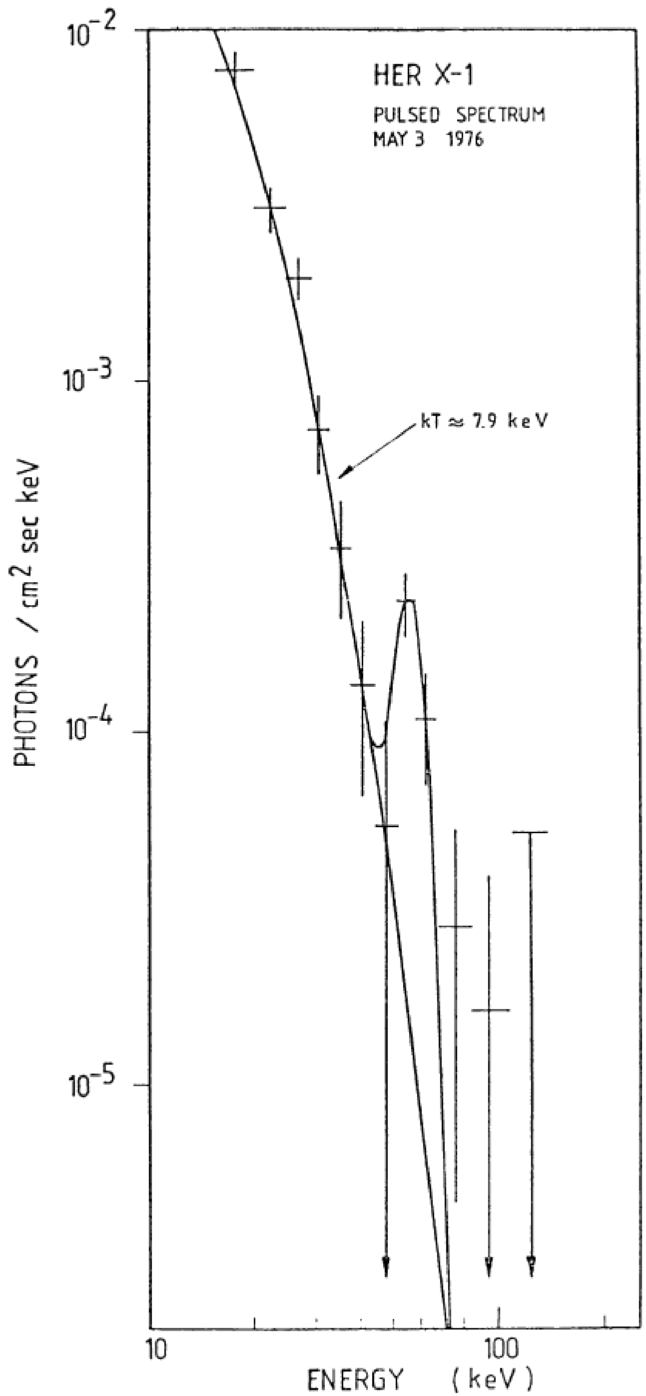
The “square wave” in the plot in the slide before means that you can apply a varying electric field so that it changes only sign (that’s the meaning of the square wave).

How often do you need to change the sign of the electric field to accelerate the proton?

You need to change the field twice every period: $T = \frac{2\pi}{\omega_L} = \frac{2\pi m}{qB} \approx 67\, ns$

Therefore you need to put an alternating electric field with a square wave at a frequency of 30 MHz

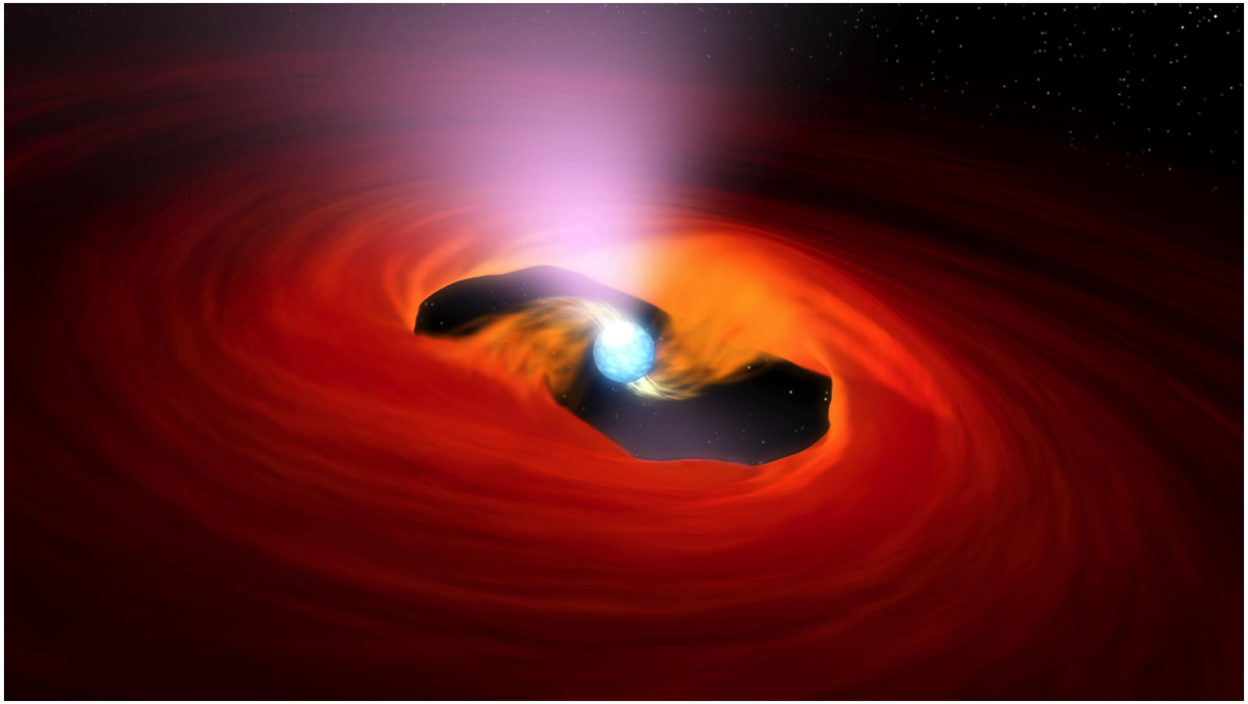




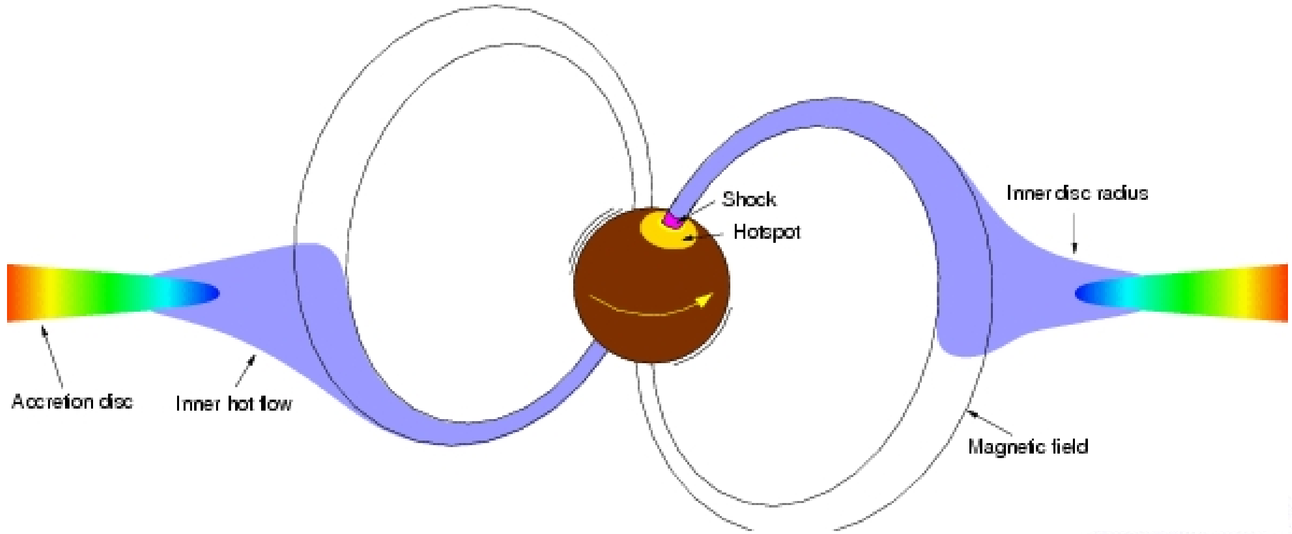
In 1977 German astronomer J. Trumper identified a cyclotron emission line in the accreting pulsar Hercules X-1.

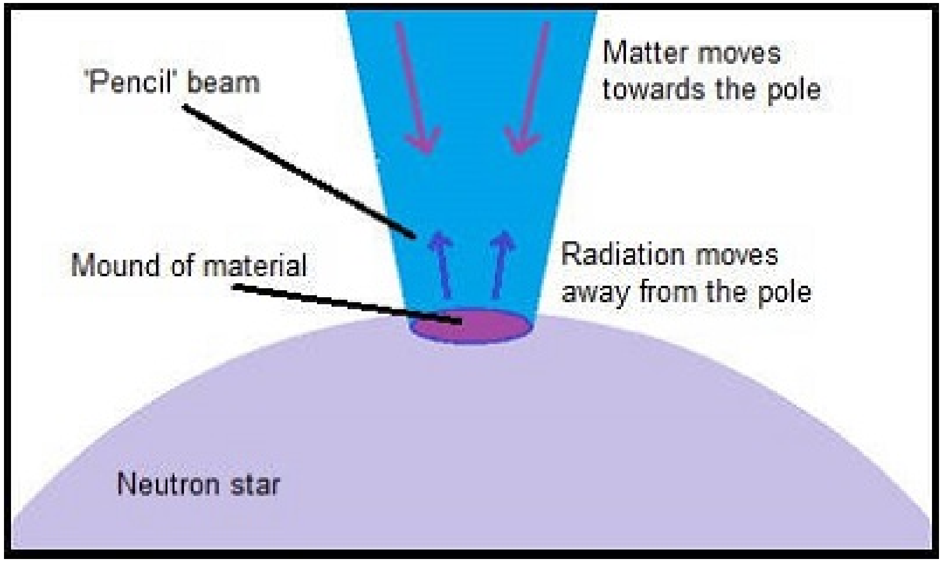
The X-ray spectrum shows an emission line at around 55 keV.

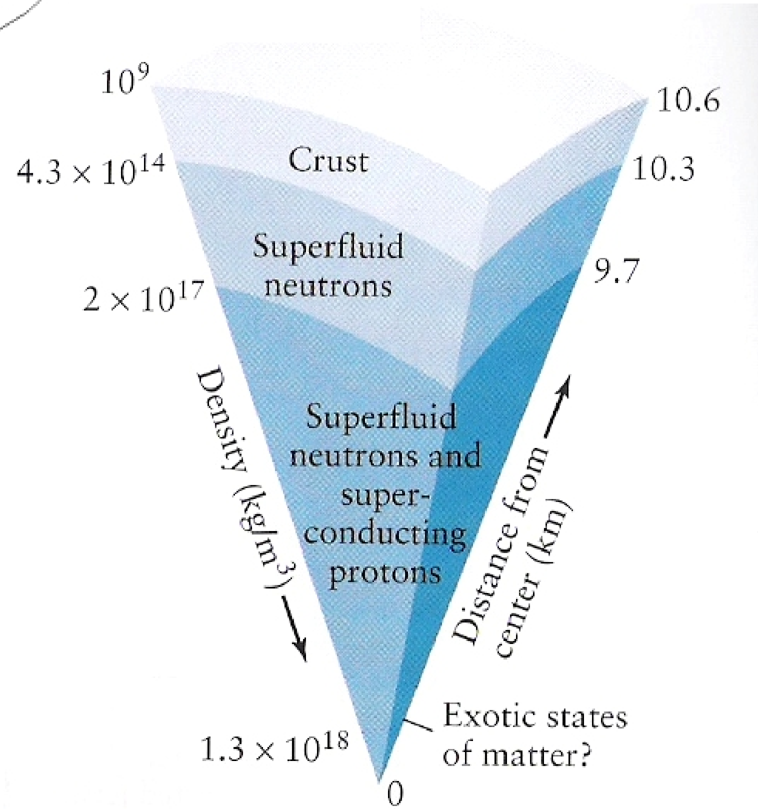
Trumper proposed that the hot electrons around the neutron star magnetic poles are rotating around a strong B field of 5e12 Gauss, giving rise to an emission line at around 55 keV.



Hercules X-1: Geometry







Relativistic Case: From Cyclotron to Synchrotron

We now drop our assumption that v<<c (*non relativistic particles)* and we describe what happens to the radiation of a charge accelerated in a B field when the speeds approach c.

For this is **important** that you review Lecture 2-3 on relativistic effects.
Let's start by remembering the Lorentz transformations of time:

$$\Delta t = \Delta t' \gamma$$

Then remember that the frequency is 1/time, so a frequency transforms as:

$$\nu = \nu' / \gamma$$

Therefore what we have called the Larmor frequency and radius become now the so-called **gyration frequency and gyration radius**:

$$\nu_b = \frac{\omega_b}{2\pi} = \frac{qB}{2\pi m \gamma} = \frac{\nu_L}{\gamma} \qquad r_L = \sqrt{\frac{(\gamma+1) mV}{qB^2}} = \frac{\gamma m v}{q B}$$

The period of rotation therefore is: $T_b = \dfrac{2\pi}{\omega_b} = \dfrac{2\pi m \gamma}{qB}$

The period now *does depend on the particle velocity* (Lorentz factor gamma) and as the velocity approaches c, the period *increases*.

The term *synchrotron* might now become clear: in synchrotron machines, the strength of the B field is not kept constant, but it is increased with time so that as the Lorentz factor gamma increases, the *frequency and the radius* of gyration are constant.

Synchrotron facilities are widespread and have very different uses. The most famous one is perhaps the *Large Hadron Collider,* which is a synchrotron machine used to generate relativistic protons up to 7 TeV in energy (per beam). In this case the synchrotron radiation represents an “annoying” energy loss.

All the magnets on the LHC are electromagnets. The main dipoles generate powerful 8.3 T magnetic fields (Earth’s magnetic field is ~0.00005 T). The electromagnets use a current of 11,080 Amp to produce the field, and superconductive material allows the high currents to flow without losing any energy to electrical resistance.

Example: Large Hadron Collider

LHC 8 T magnetic fields require a current of 11,000 Amp!!!
To sustain such huge currents, the resistance of the conductors need to be zero → superconductors.
Comparison: an airplane jet engine requires 1,000 Amp
Your house is not using more than 100 Amp.

LHC accelerates protons in 5 stages.

Stage 1: *Linear Accelerator*
Energy: 50 MeV
Speed: 1/3 c

Stage 2: *Proton Synchrotron Booster*
Circumference: 160 m
Energy: 1.4 GeV (mc^2> p rest mass)
Speed: 0.92 c

Stage 3: *Proton-Synchrotron*
Circumference: 620 m
Energy: 25 GeV
Speed: 0.99935 c

Stage 4: *Superproton Synchrotron*
Circumference: 7 km
Energy: 450 GeV
Speed: 0.999993 c

Stage 5: *LHC*
Circumference: 27 km
Energy: 7 TeV
Speed: 0.99999999 c

Synchrotron Radiation in Astrophysics

Magnetic fields and relativistic particles are plentiful in astropysics. Synchrotron emission is seen in a wide variety of environments.

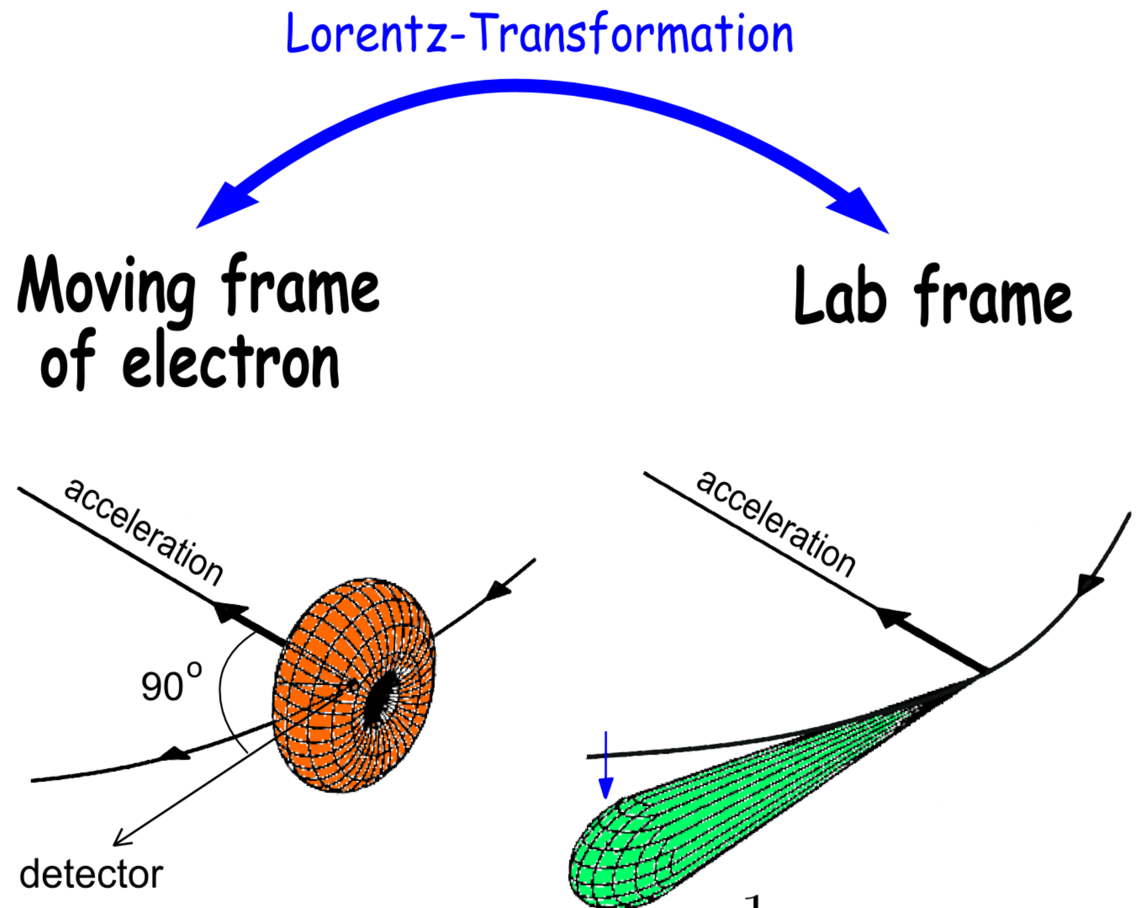
Location	Field strength (Gauss)
Interstellar medium	10^{-6}
Stellar atmosphere	1
Supermassive Black Hole	10^{4}
White Dwarf	10^{8}
Neutron star	10^{12}
This room	0.3
Crab Nebula	10^{-3}

Note: From here onward I switch back to cgs units

Synchrotron Radiation: Emission Pattern

Take a relativistic electron moving around a B field. To understand how the radiation pattern changes from cyclotron to synchrotron we can do the following:

- we start with the radiation pattern in the electron rest frame (where we know the radiation pattern)
- then we do a Lorentz transformation from the rest frame to the lab frame.



$$\pm\frac{1}{\gamma} \text{ opening angle}$$

$$\frac{1}{\gamma} = \frac{m_o c^2}{E} = \sqrt{1-(\frac{v}{c})^2}$$

Synchrotron Radiation: Emission Pattern

Take a relativistic electron moving around a B field. To understand how the radiation pattern changes from cyclotron to synchrotron we can do the following:

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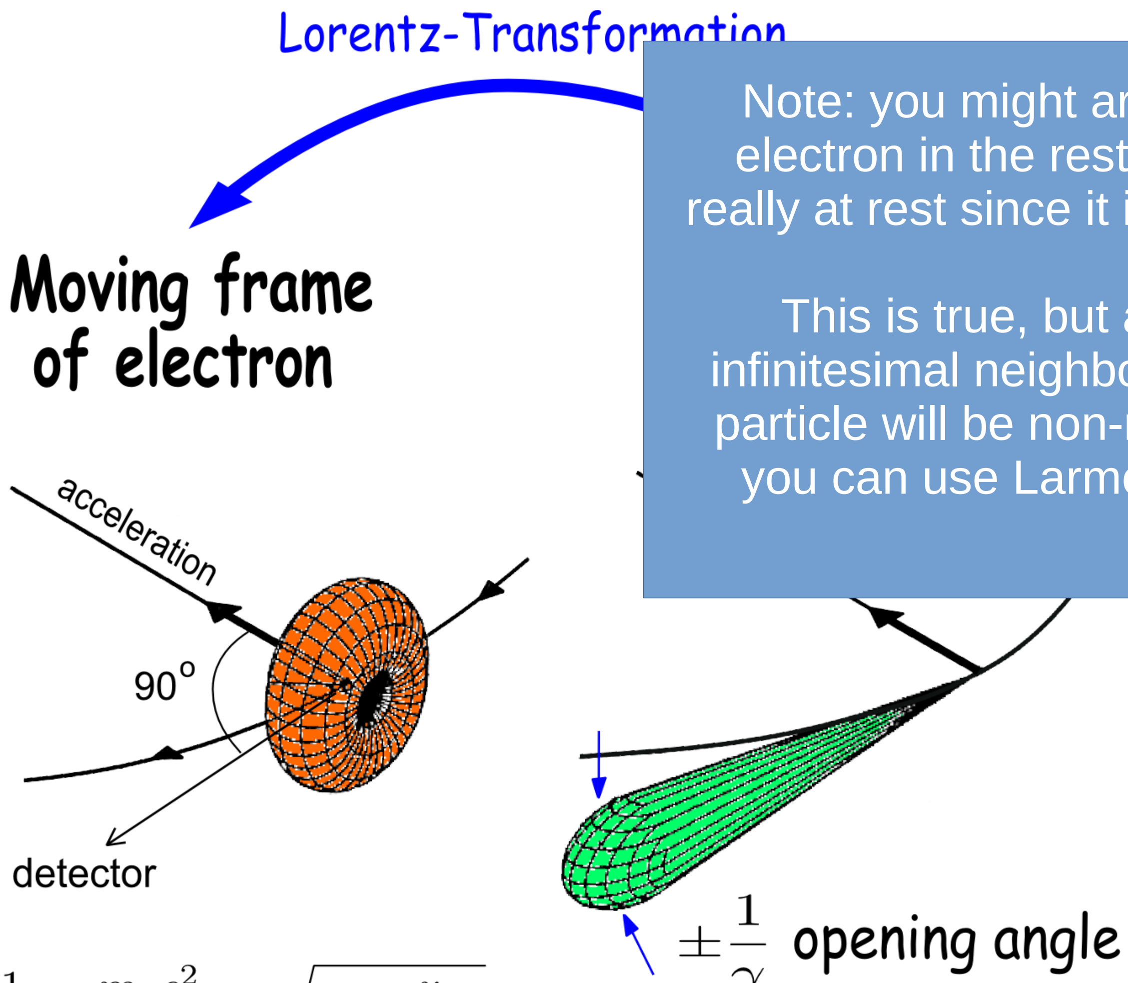


Note: you might argue that the electron in the rest frame is not really at rest since it is accelerating.

This is true, but at least for infinitesimal neighboring times the particle will be non-relativistic and you can use Larmor's formula.

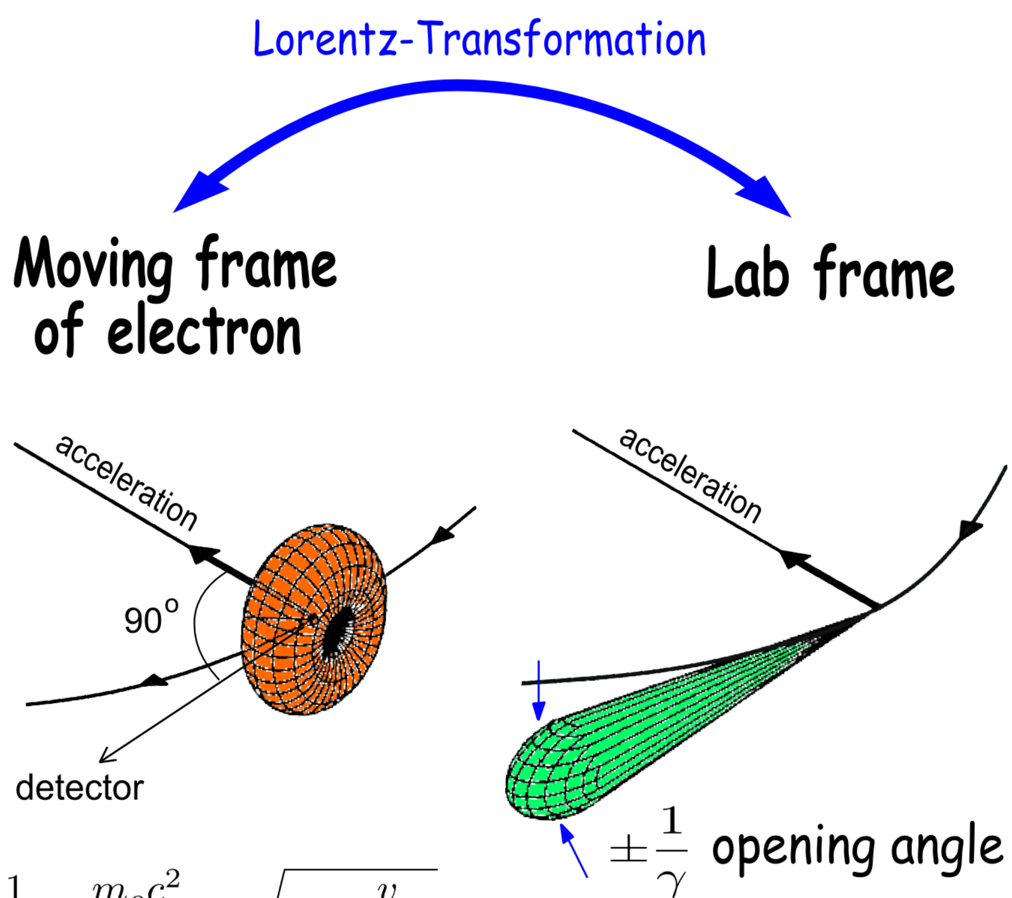
$$\frac{1}{\gamma} = \frac{m_o c^2}{E} = \sqrt{1-(\frac{v}{c})^2}$$

Synchrotron Radiation: Emission Pattern

To transform the radiation pattern what we really need is to transform the emission angle theta and see how it changes under Lorentz transformations.

But we have already seen this in Lecture 2: aberration of light

$$\tan\theta = \frac{\sin\theta'}{\gamma(\cos\theta' + v/c)}$$



$$\frac{1}{\gamma} = \frac{m_o c^2}{E} = \sqrt{1 - (\frac{v}{c})^2}$$

Synchrotron Radiation: Power

In the case of cyclotron, as well as (non-rel) Bremsstrahlung we saw that we can use the Larmor's formula (Lecture 4) to calculate the power emitted by an accelerated charge:

First, let's take the usual rest frame K' with velocity **v** and the lab frame K.

Power is energy over time. Let's Lorentz transform energy and time.

Energy: $dW = \gamma \, dW'$

Time: $dt = \gamma \, dt'$

Therefore: $P = P'$

So the total power emitted will be a **Lorentz invariant** (but this does not mean that the angular dependence of radiation will be the same of course).

Synchrotron Radiation: Lorentz Transformation of Accelerations

The Larmor's formula in a non-relativistic reference frame is: $P = \dfrac{2q^2a^2}{3c^3}$

It is now convenient to split the acceleration in its parallel and perpendicular components to the direction of the rest frame **v**.

In Lecture 3 we saw how velocities transform. Now we see how accelerations transform.

As usual start with the Lorentz transformations of coordinates and velocities (Lecture 3):

$$dt = \gamma\left(dt' + \frac{v}{c^2}\,dx'\right) = \gamma\sigma\,dt',$$

$$du_x = \gamma^{-2}\sigma^{-2}\,du'_x,$$

$$du_y = \gamma^{-1}\sigma^{-2}\left(\sigma\,du'_y - \frac{vu'_y}{c^2}\,du'_x\right).$$

where here we have defined for convenience: $\sigma \equiv 1 + \dfrac{vu'_x}{c^2}$

Therefore:

$$a_x = \frac{du_x}{dt} = \gamma^{-3}\sigma^{-3}\frac{du'_x}{dt'} = \gamma^{-3}\sigma^{-3}a'_x,$$

$$a_y = \frac{du_y}{dt} = \gamma^{-2}\sigma^{-3}\left(\sigma\frac{du'_y}{dt'} - \frac{vu'_y}{c^2}\frac{du'_x}{dt'}\right),$$

$$= \gamma^{-2}\sigma^{-3}\left(\sigma a'_y - \frac{vu'_y}{c^2}a'_x\right)$$

And a similar result holds for the z-component.

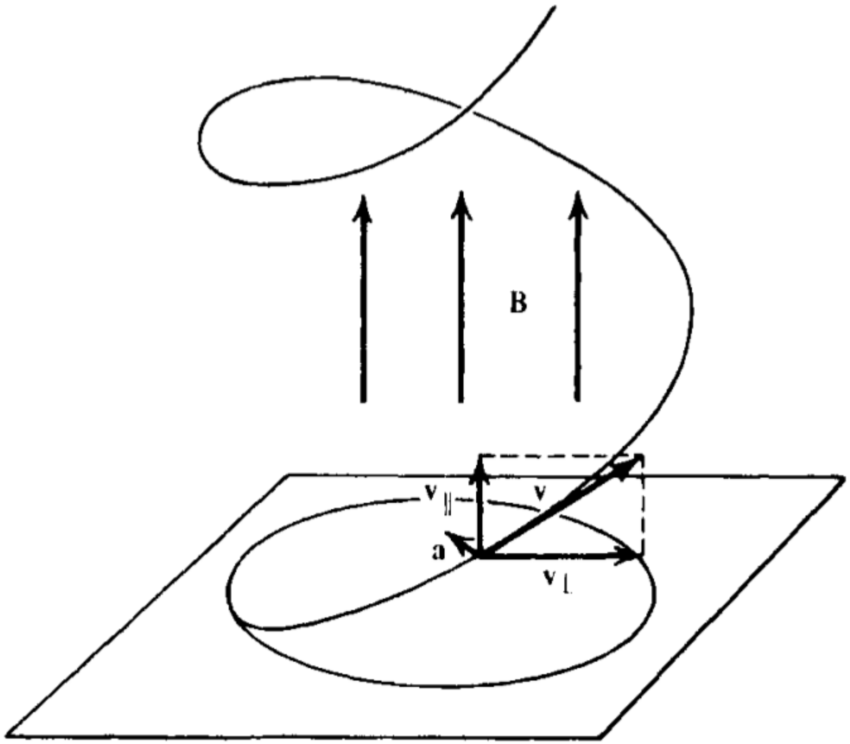
Now, if the particle is at rest instantaneously in K', then the velocity $u'_x = u'_y = u'_z = 0.$

Also, sigma = 1 in this case.

Therefore, take the velocity **v** along the x-axis and from the equations above:

$$a'_{\|} = \gamma^3 a_{\|},$$

$$a'_{\perp} = \gamma^2 a_{\perp}.$$



So take now a helical path of a particle in a uniform **B** field. Let's decompose the velocity and accelerations into their parallel and perpendicular components and write the total emitted power:

$$
\begin{aligned}
P = P' &= \frac{2q^2}{3c^3}\left[(a'_{\|})^2 + (a'_{\perp})^2\right] \\
&= \frac{2q^2\gamma^4}{3c^3}(\gamma^2 a_{\|}^2 + a_{\perp}^2)\ .
\end{aligned}
$$

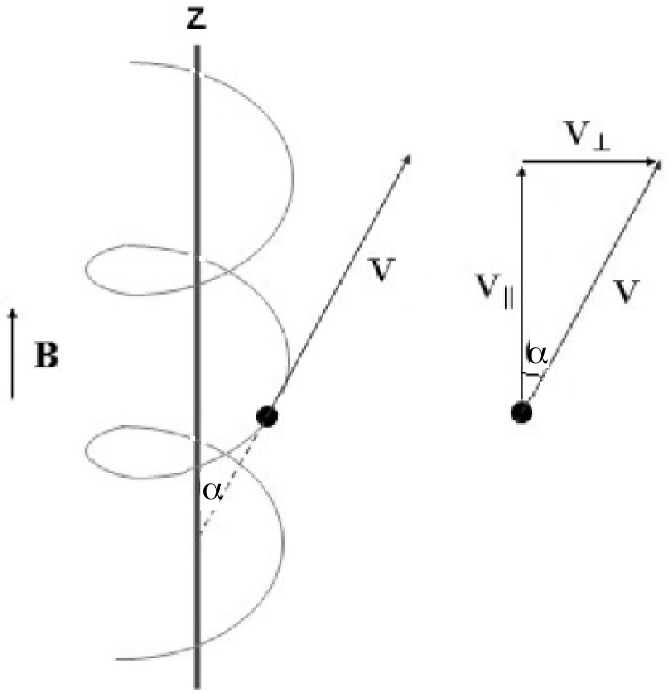
Now from this formula one might suppose that the parallel component is more important than the perpendicular one. This is true when the two accelerations are comparable.
But think for a moment about what happens in a ultra-relativistic beam of particles.
If v~c, you cannot really accelerate the particle in the direction of **v** (the parallel component) whereas with a modest B field you can strongly bend the direction of motion and have a large perpendicular acceleration. So for ultra-relativistic particles the parallel component gives almost zero contribution!!!

Synchrotron Radiation: Power and Pitch Angle

Since we know the angular frequency of radiation (gyrofrequency): $a_\perp = \omega_B v_\perp$

Substituting this back in the emitted power equation we get:

$$\boxed{P = \frac{2q^2}{3c^3}\gamma^4 \frac{q^2B^2}{\gamma^2 m^2 c^2} v_\perp^2}$$



However, if we have many particles,
each of them will have a different **pitch angle alpha**.
So the perpendicular velocity needs to be averaged over all pitch angles. If we do that, we can write the formula:

$$P = \left(\tfrac{2}{3}\right)^2 r_0^2 c \beta^2 \gamma^2 B^2,$$

and rewrite it as:

$$\boxed{P = \frac{4}{3}\sigma_T c \beta^2 \gamma^2 U_B.}$$

Where $\sigma_T = 8\pi r_0^2/3$ is the Thomson cross section
and $U_B = B^2/8\pi$ is the magnetic energy density.

IMPORTANT: The formula written before is valid *only* for electrons emitting synchrotron radiation. The reason why we write this formula only for electrons is because in basically all astrophysical cases you have electron synchrotron. This is because electrons become relativistic much more quickly than protons as they are easier to accelerate.
However using the preceding expression (with the red contour in the previous slide) is fine for any particle of mass m and charge q.

Example: Suppose the protons of the LHC are accelerated up to an energy of 7 TeV and then they are left to cool down due to synchrotron emission. On which timescale do they cool down?

Proton energy: 7 TeV → velocity v~ 29979245800*0.99999999 cm/s
B = 80,000 G
q = 4.8e-10 statC
gamma ~ 7,000
m = 1.67e-24 g

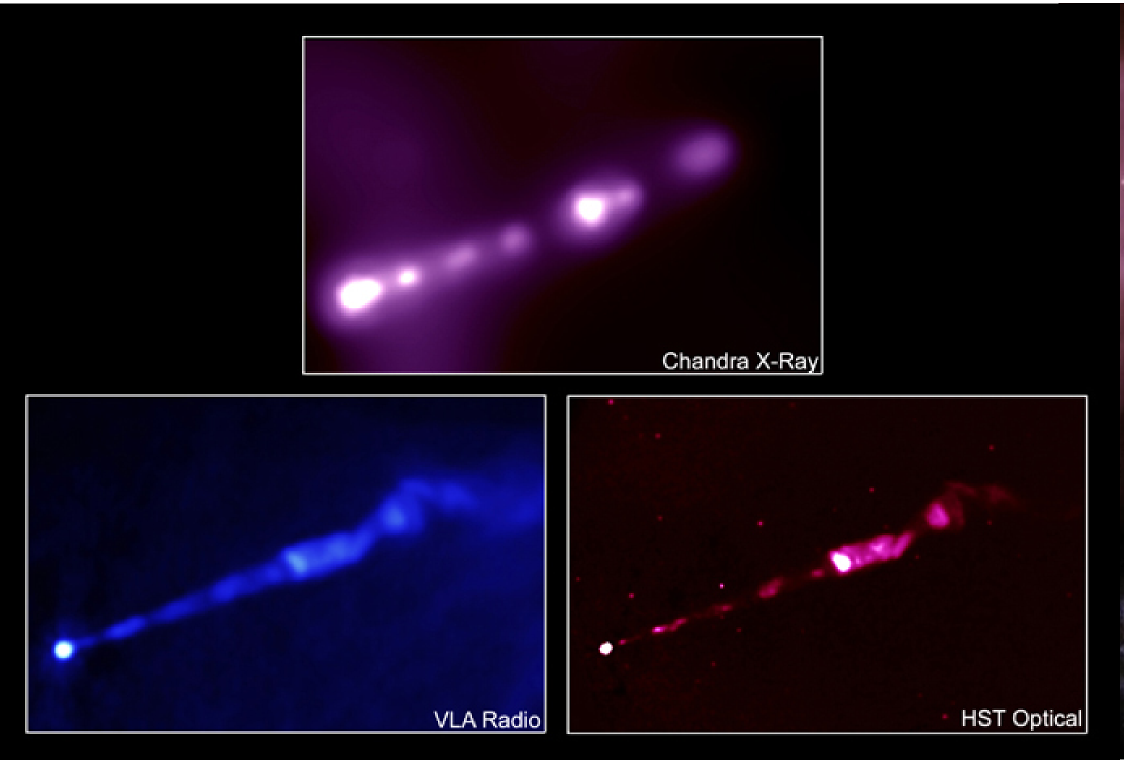
Timescale = (Proton energy) / (synchrotron power) = 10 erg/ 4.9e-5 erg/s = 2.5 days

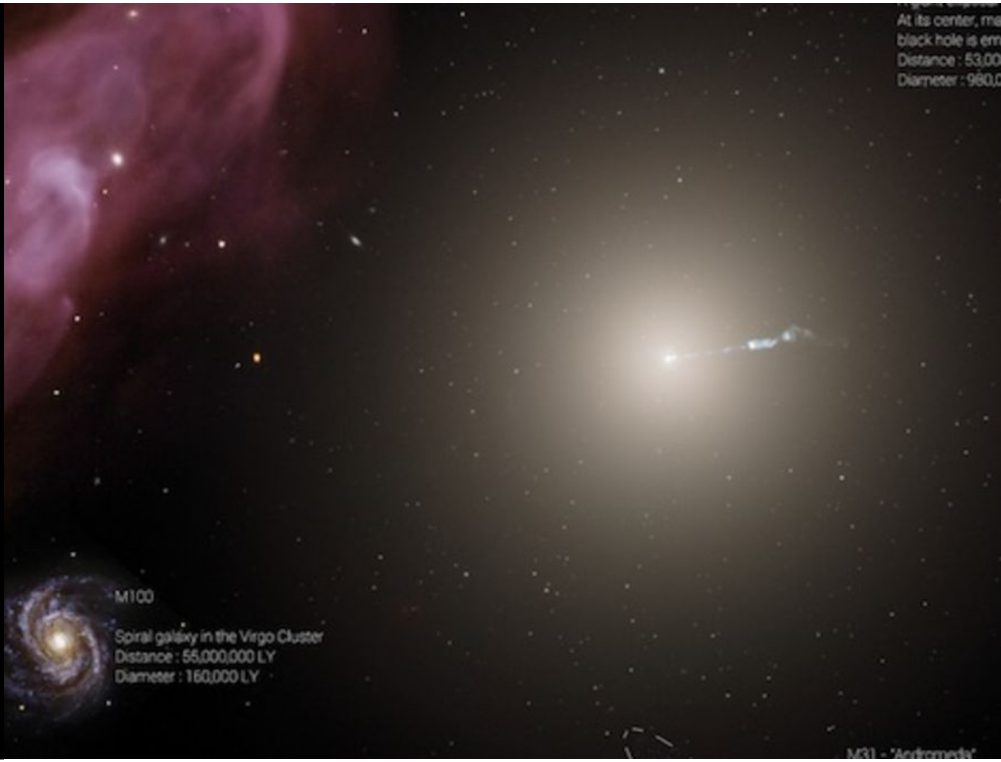
Now let's do the same calculations for a 7 TeV electron (gamma ~ 13,000,000):

Timescale = (Electron energy) / (synchrotron power) = 10 erg/ 4.9e-5 erg/s = 9 ns

Electrons would cool down a factor 10^13 faster than protons!! → LHC INFEASIBLE at 7 TeV

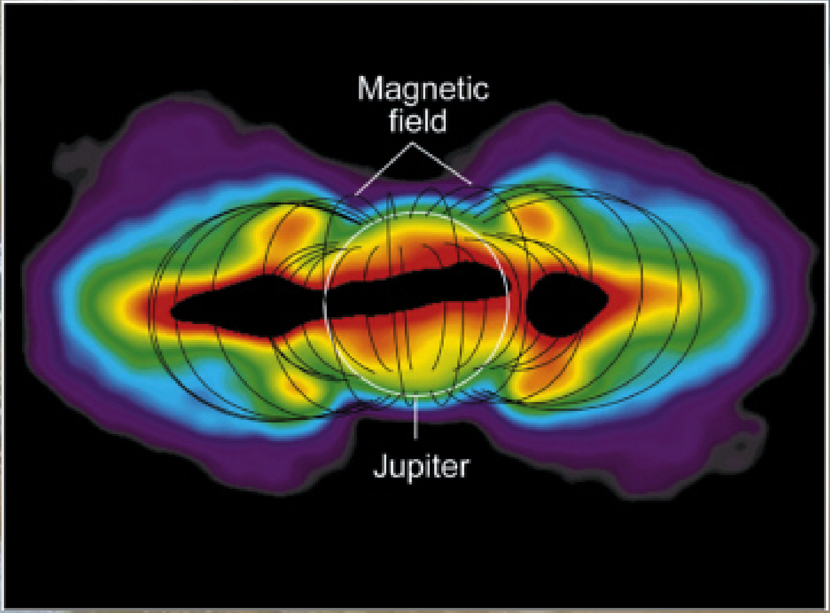
Synchrotron in Astrophysics

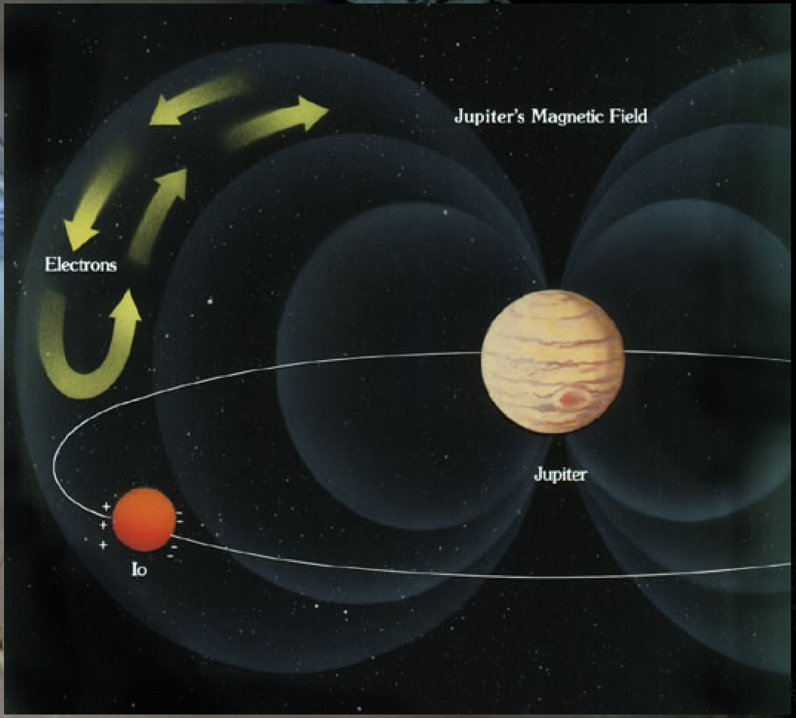




Astrophysical jets are most likely generated by relativistic particles being launched close to a black hole (or even a neutron star when in a binary). Such particles are thought to be electron/positron pairs which then spiral along **B** field lines and generate synchrotron radiation. However, we also know that cosmic rays most likely come from Active Galactic Nucei, where strong **B** fields around supermassive black holes launch streams of ultra-relativistic particles which include protons. So it’s still unclear whether jet emission is due to leptons or hadrons.

Synchrotron Radiation: Jupiter's Belt





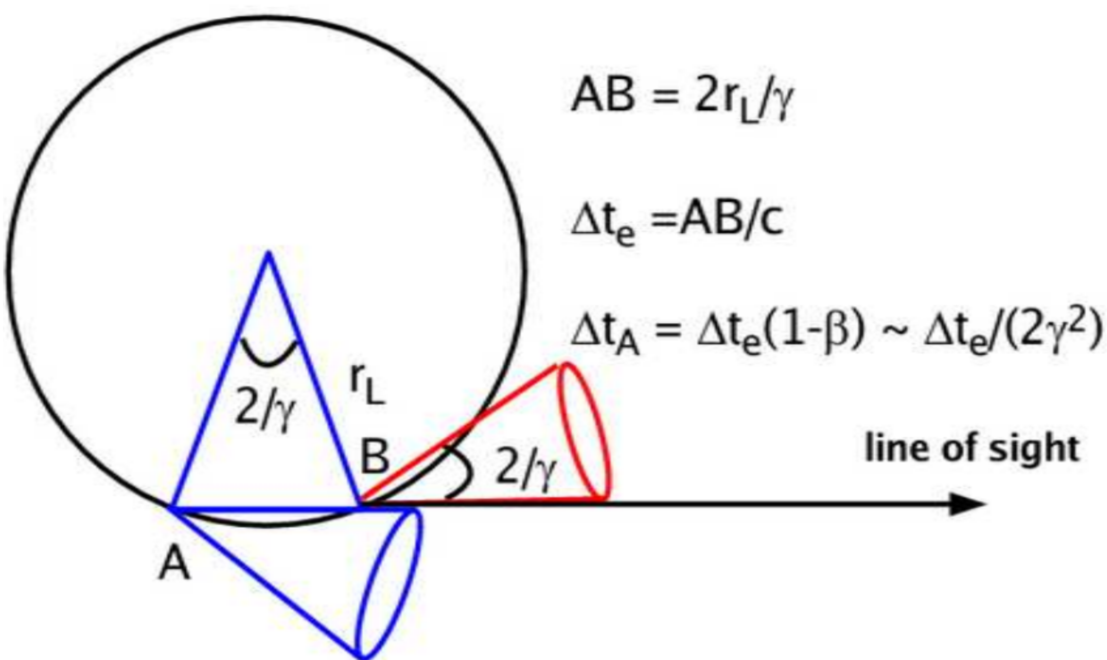
Synchrotron Radiation: Single Particle Spectrum

It's important now to make a distinction between the *emitted* radiation and the *received* radiation. Indeed the received radiation will be such that the observer can see it only when the narrow beam points towards the observer.

This will be a fraction much smaller than the gyration period! The electro(magnetic) field received by the observer will thus be pulsed and therefore it must be composed by many frequencies (think about the Fourier power spectrum of a narrow pulse…).
How small is this period of time? First of all, suppose that we have one electron and the pitch angle is 90 degrees.

The electron will radiate towards the observer only for a brief amount of time:

$$\Delta t_e \approx \frac{AB}{v} = \frac{2 r_L}{\gamma v} = \frac{1}{\pi \gamma \nu_B}$$

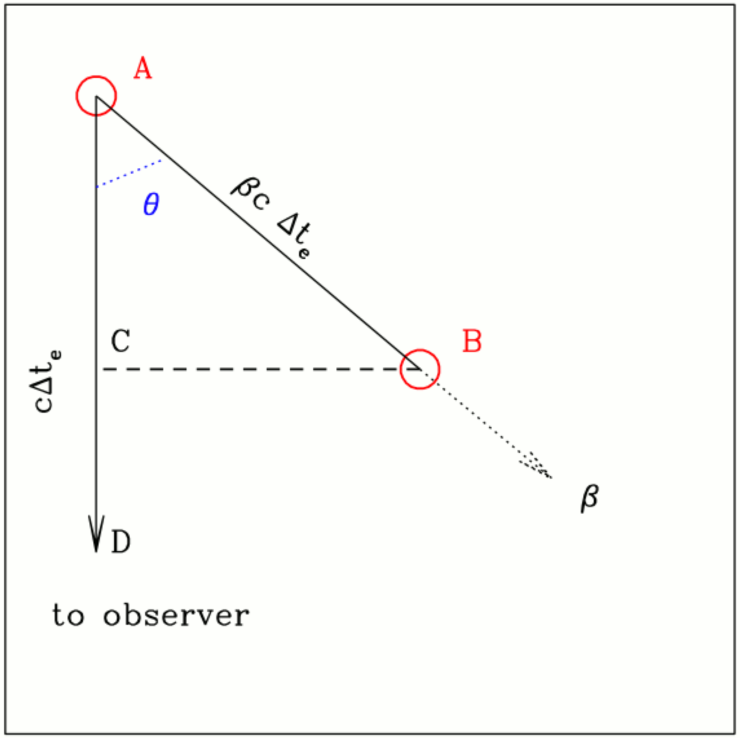


Now a question: here we are seeing photons. The expression above is the emission time. What happens when we measure time intervals with photons? Do we have the same result as when we measure time intervals with clocks? (Lecture 5)

Synchrotron Radiation: Single Particle Spectrum

In Lecture 3 we saw that:

$$\Delta t_A = \Delta t_e (1 - \beta \cos \theta)$$



Here therefore it must be the same with the difference that theta here is ~0 because we are emitting photons in the same direction of motion:

$$\Delta t_A \approx \Delta t_e (1 - \beta)$$

We can write:

$$\Delta t_{\mathrm{A}} = \Delta t_{\mathrm{e}} (1 - \beta) = \Delta t_{\mathrm{e}} \frac{(1 - \beta^2)}{1 + \beta} \sim \frac{\Delta t_{\mathrm{e}}}{2\gamma^2} = \frac{1}{2\pi\gamma^3 \nu_{\mathrm{B}}}$$

The inverse of this is an angular frequency. We can thus build a characteristic synchrotron frequency in this way:

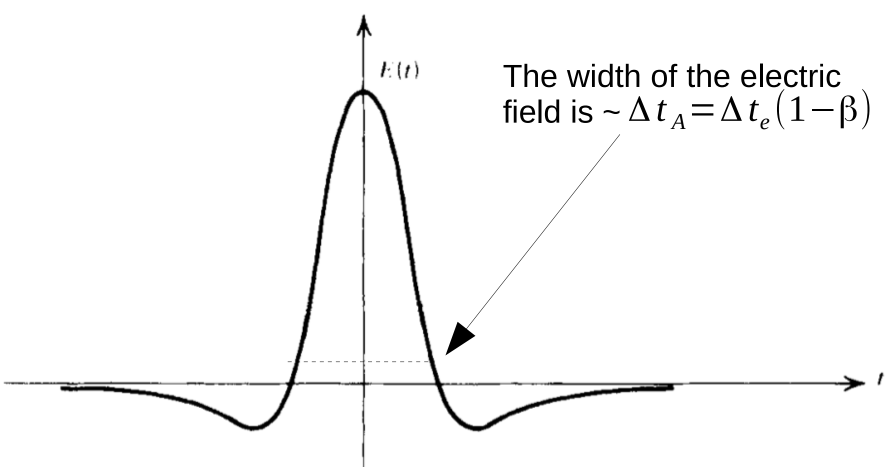
$$\nu_{\mathrm{s}} = \frac{1}{2\pi \Delta t_{\mathrm{A}}} = \gamma^3 \nu_{\mathrm{B}} = \gamma^2 \nu_{\mathrm{L}} = \gamma^2 \frac{eB}{2\pi m_e c}$$

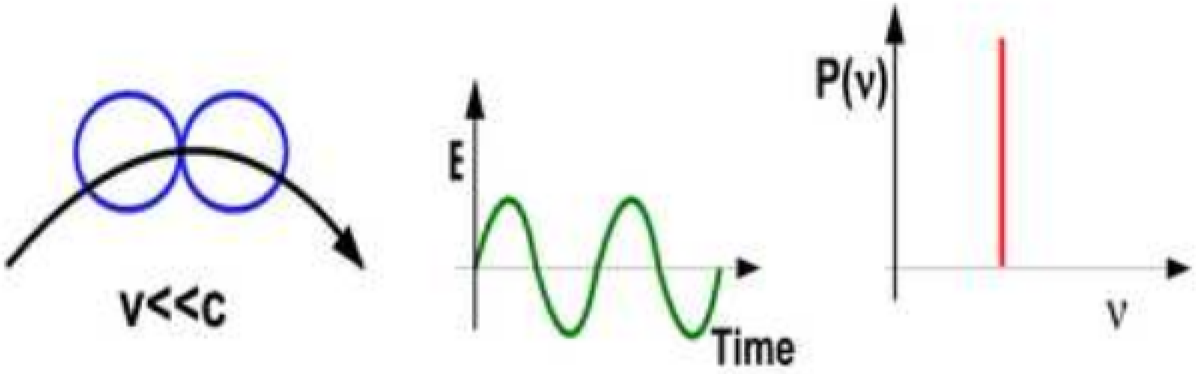
Synchrotron Radiation: Single Particle Spectrum

$$\nu_s = \frac{1}{2\pi\Delta t_A} = \gamma^3\nu_B = \gamma^2\nu_L = \gamma^2\frac{eB}{2\pi m_e c}$$

First thing to note: this frequency is gamma^3 larger than the gyration frequency. This makes sense since we see the emission only for a tiny fraction of the orbit, whereas the gyrofrequency reflects the whole circular motion of the electron.

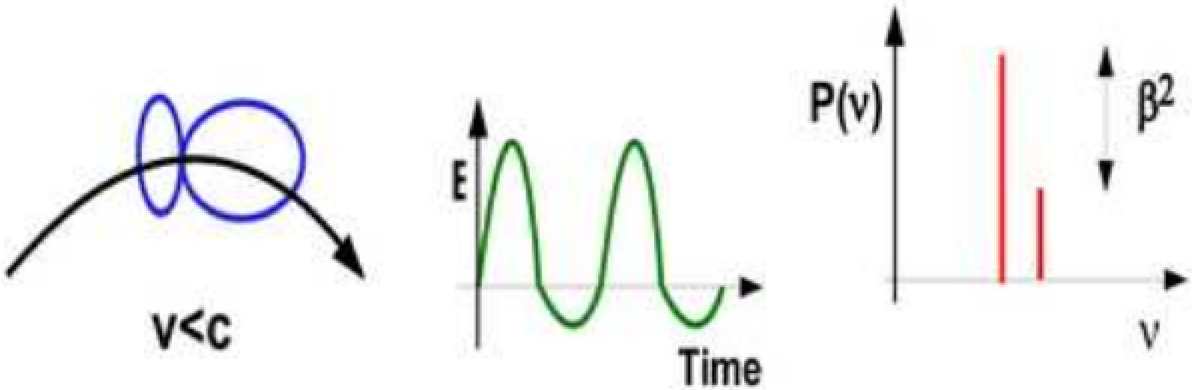
Second: the *observed* electric field of the rotating charge will be narrow and thus the radiation will spread across many frequencies. The characteristic frequency where the power will drop is of the order of the synchrotron frequency above.





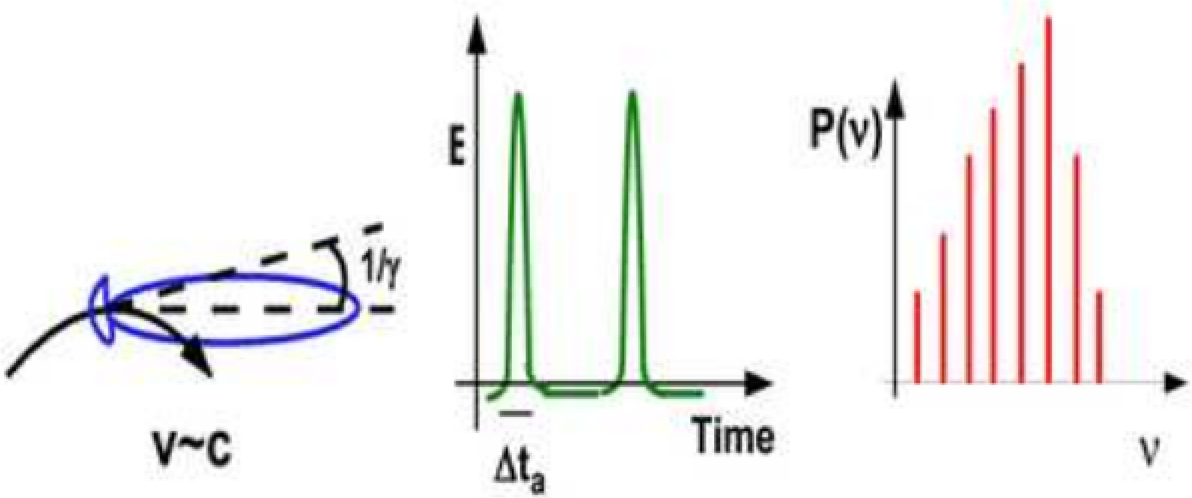
Cyclotron radiation

The charge is moving in a circle, so the electric field variation is sinusoidal



Cyclotron-synchrotron radiation

The charge is moving in a circle, but some aberration of light starts to become visible



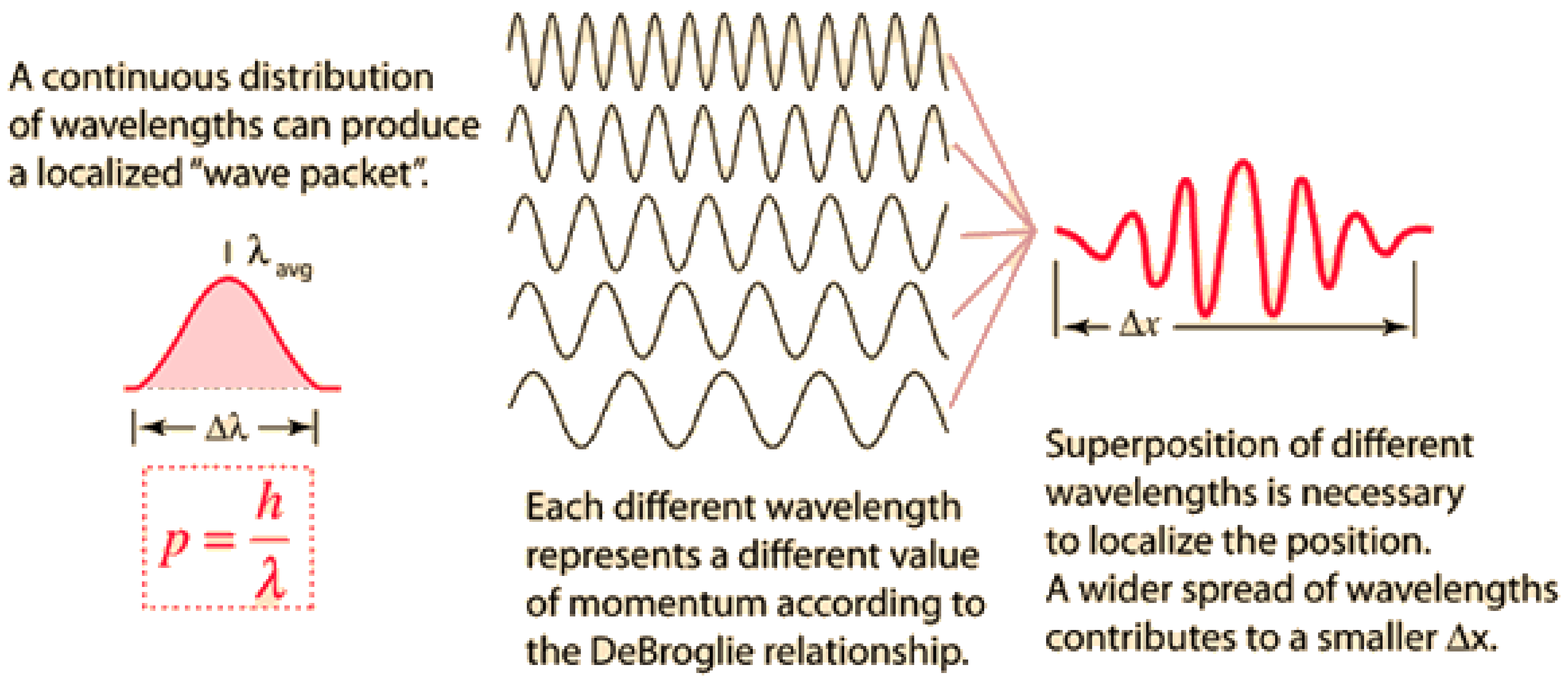
Synchrotron radiation

The charge is moving in a circle, aberration of light is extreme and radiation is seen only for a tiny amount of time when the cone 1/gamma points towards the observer

Synchrotron Radiation: Single Particle Spectrum

The fact that the *single* particle spectrum is not a single frequency can be easily understood by using Fourier transforms. This is related to the so-called time-energy uncertainty relation.

Another way is to think about this is a wave packet localized in time.



Waves and Fourier Transforms
(Section 2.8 R&L)

$$\Delta t \Delta \omega > 1$$

The spectrum of radiation depends on the time variation of the electromagnetic field.
If you observe the variation of, say, the electric field E(t) for a time Dt then you can define the spectrum with a frequency resolution Domega = 1/Dt.

$$\hat{E}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(t) e^{i\omega t} dt.$$

Fourier Transform

$$E(t) = \int_{-\infty}^{\infty} \hat{E}(\omega) e^{-i\omega t} d\omega.$$

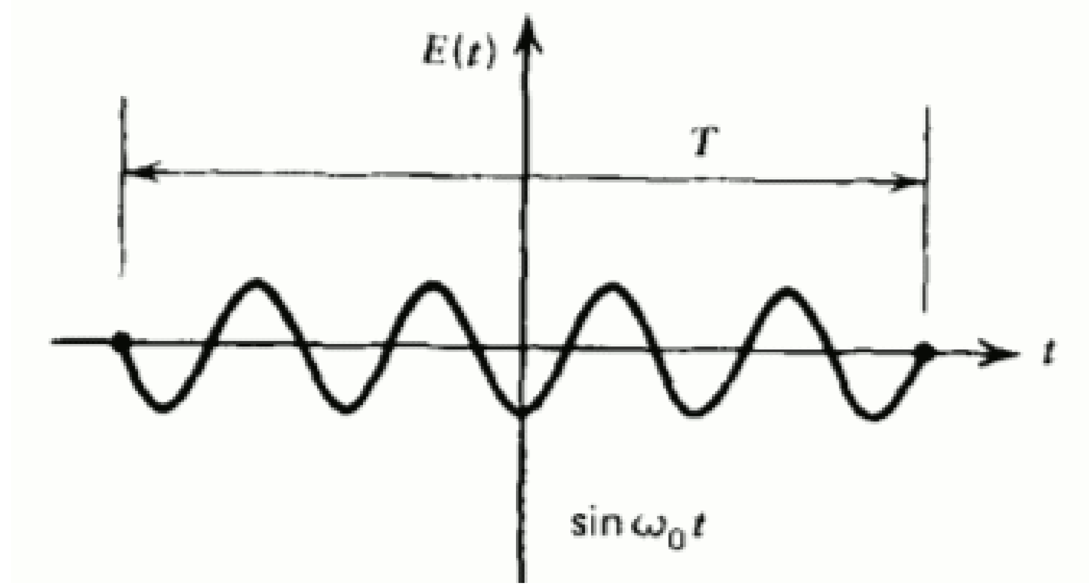
Inverse Fourier Transform

Relation between Power and Fourier Transforms

Suppose you have a sinusoidal electro-magnetic field.
We know that the average magnetic and electric fields are B = E/c
Therefore the Poynting vector is:

$$\frac{dW}{dt\,dA} = \frac{c}{4\pi} E^2(t).$$

The total energy per unit area in the pulse is:



Relation between Power and Fourier Transforms

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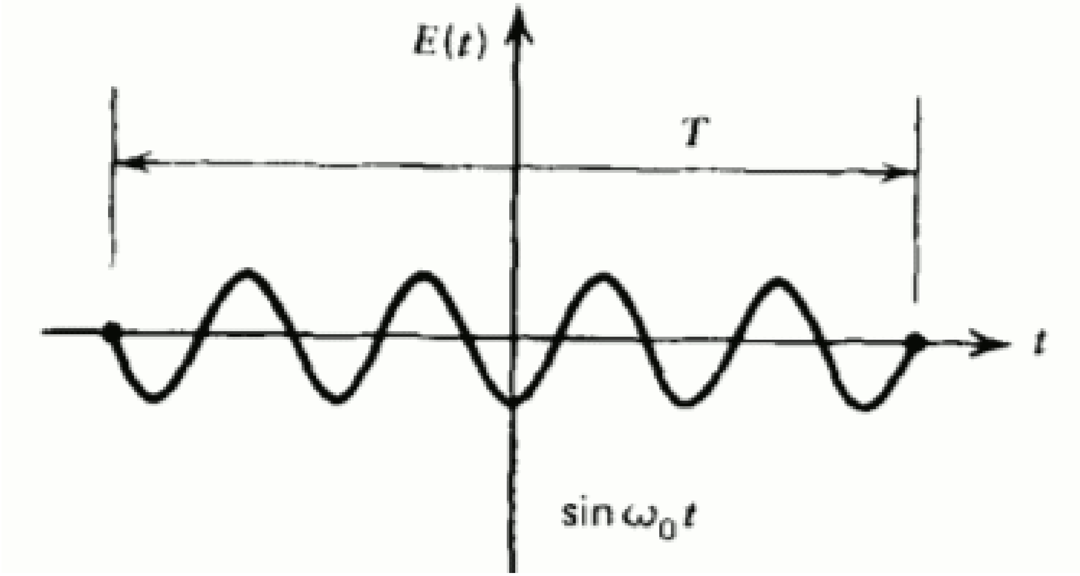
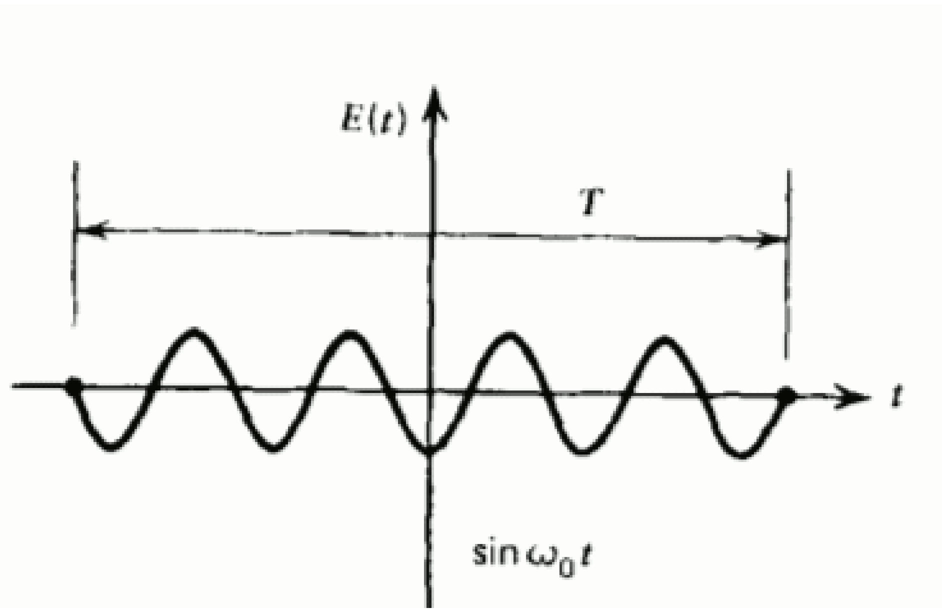


The total energy per unit area in the pulse is:

$$\frac{dW}{dA} = \frac{c}{4\pi} \int_{-\infty}^{\infty} E^2(t)\,dt.$$

Fourier Transforms



Now we take the Fourier Transform:

$$E(t) = \int_{-\infty}^{\infty} \hat{E}(\omega) e^{-i\omega t} d\omega .$$

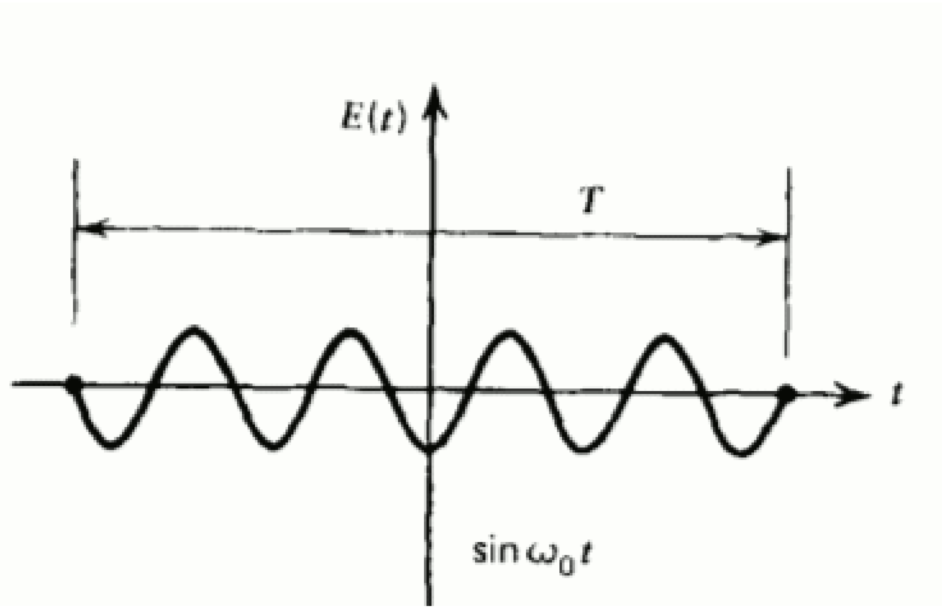
$$\hat{E}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(t) e^{i\omega t} dt .$$

$$\frac{dW}{dA} = \frac{c}{4\pi} \int_{-\infty}^{\infty} E^2(t) dt .$$

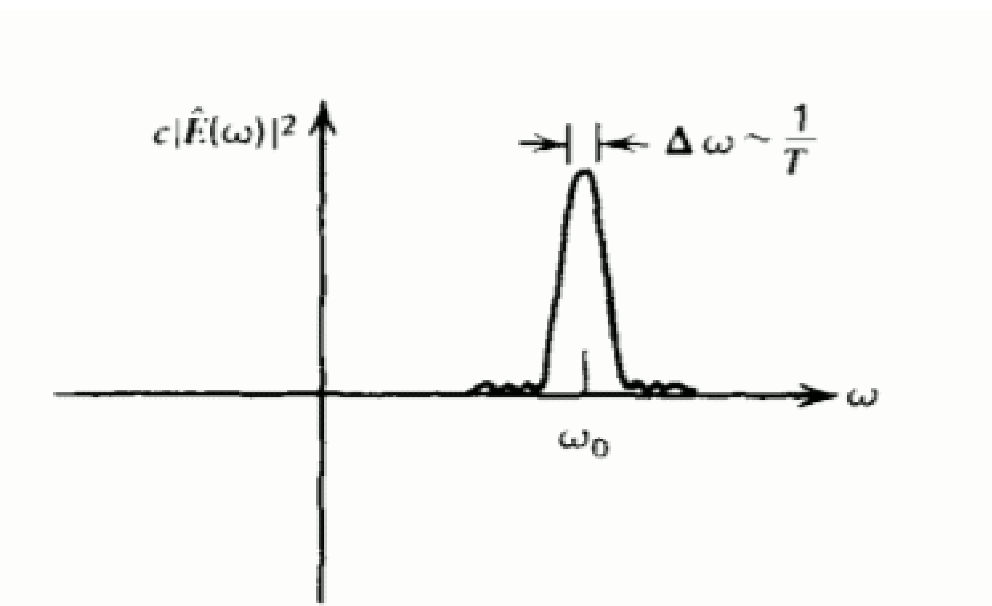
$$\frac{dW}{dA} = c \int_0^{\infty} |\hat{E}(\omega)|^2 d\omega ,$$

$$\frac{dW}{dA\, d\omega} = c |\hat{E}(\omega)|^2$$

Fourier Transforms



$$E(t) = \int_{-\infty}^{\infty} \hat{E}(\omega) e^{-i\omega t} d\omega .$$



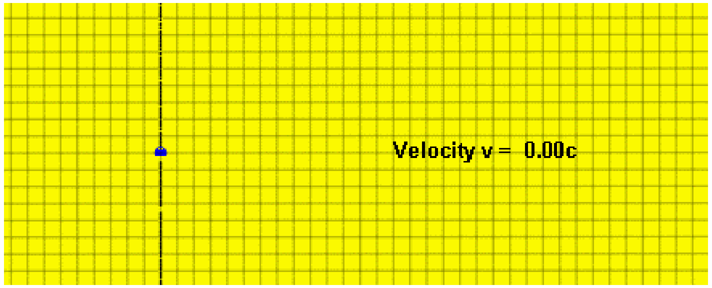
$$\hat{E}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(t) e^{i\omega t} dt .$$

Synchrotron Radiation: Single Particle Spectrum

In summary: We haven't derived in a rigorous way the single particle spectrum yet, but we can expect the following.

1. The power spectrum will show a broad range of frequencies
2. The width of the power in the power spectrum will be of the order of $\nu_s = \frac{1}{\Delta t_A}$
3. We can expect a quick (exponential) cutoff above these frequencies.
4. The peak of the power must be somewhere around ν_s too, since most of the power is emitted in the interval Δt_A
5. We expect *much more than half* of the power to be emitted within 1/gamma.

For point 5. please check Lecture 3 where we showed that $I = \delta^4 I'$

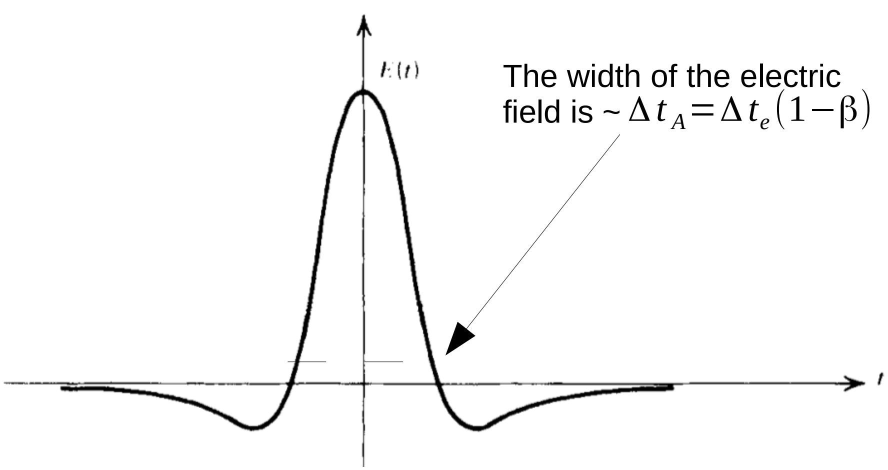


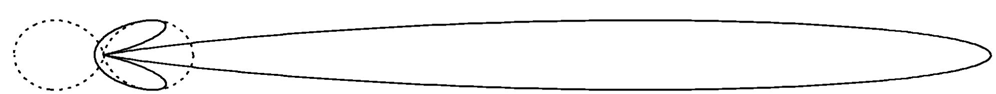
Synchrotron Radiation: Single Particle Spectrum

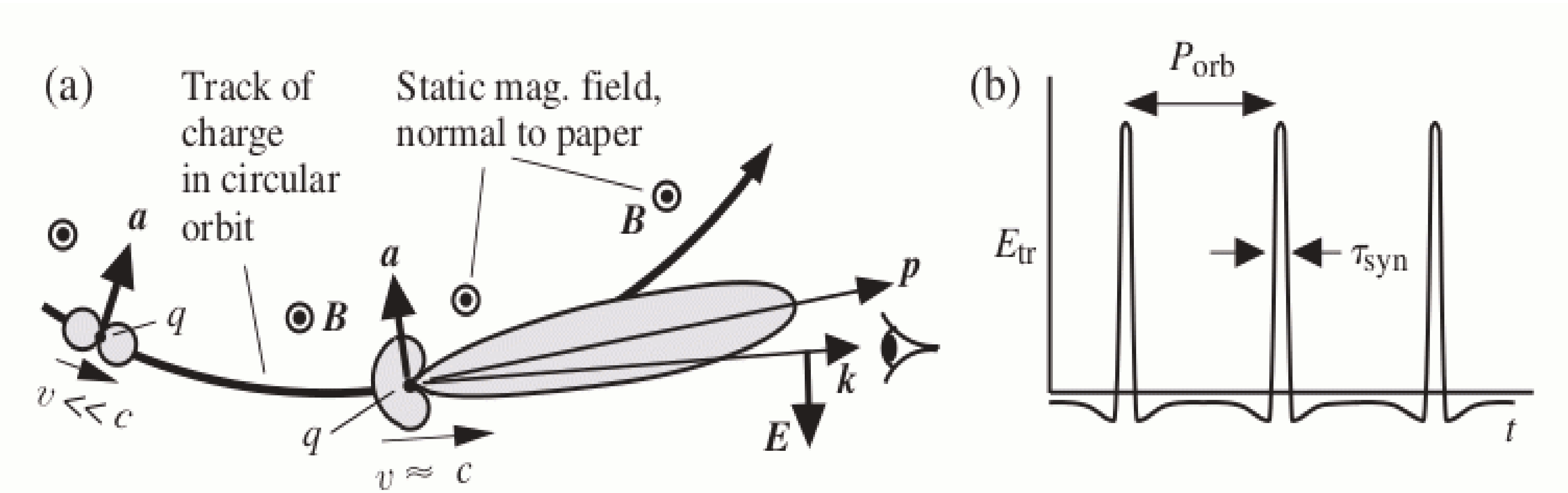
$$\nu_{\rm s} = \frac{1}{2\pi\Delta t_{\rm A}} = \gamma^3\nu_{\rm B} = \gamma^2\nu_{\rm L} = \gamma^2\frac{eB}{2\pi m_e c}$$

First thing to note: this frequency is gamma^3 larger than the gyration frequency. This makes sense since we see the emission only for a tiny fraction of the orbit, whereas the gyrofrequency reflects the whole circular motion of the electron.

Second: the *observed* electric field of the rotating charge will be narrow and thus the radiation will spread across many frequencies. The characteristic frequency where the power will drop is of the order of the synchrotron frequency above.







$$P = \frac{4}{3}\sigma_T c \beta^2 \gamma^2 U_B$$

$$\omega_b = \frac{qB}{mc\gamma}$$

$$\omega_c = \frac{3}{2}\gamma^3 \omega_b \sin\alpha$$

Synchrotron critical frequency (definition)

$$\nu_c = \frac{3}{2}\nu_s \sin\alpha$$

Synchrotron Cooling Time

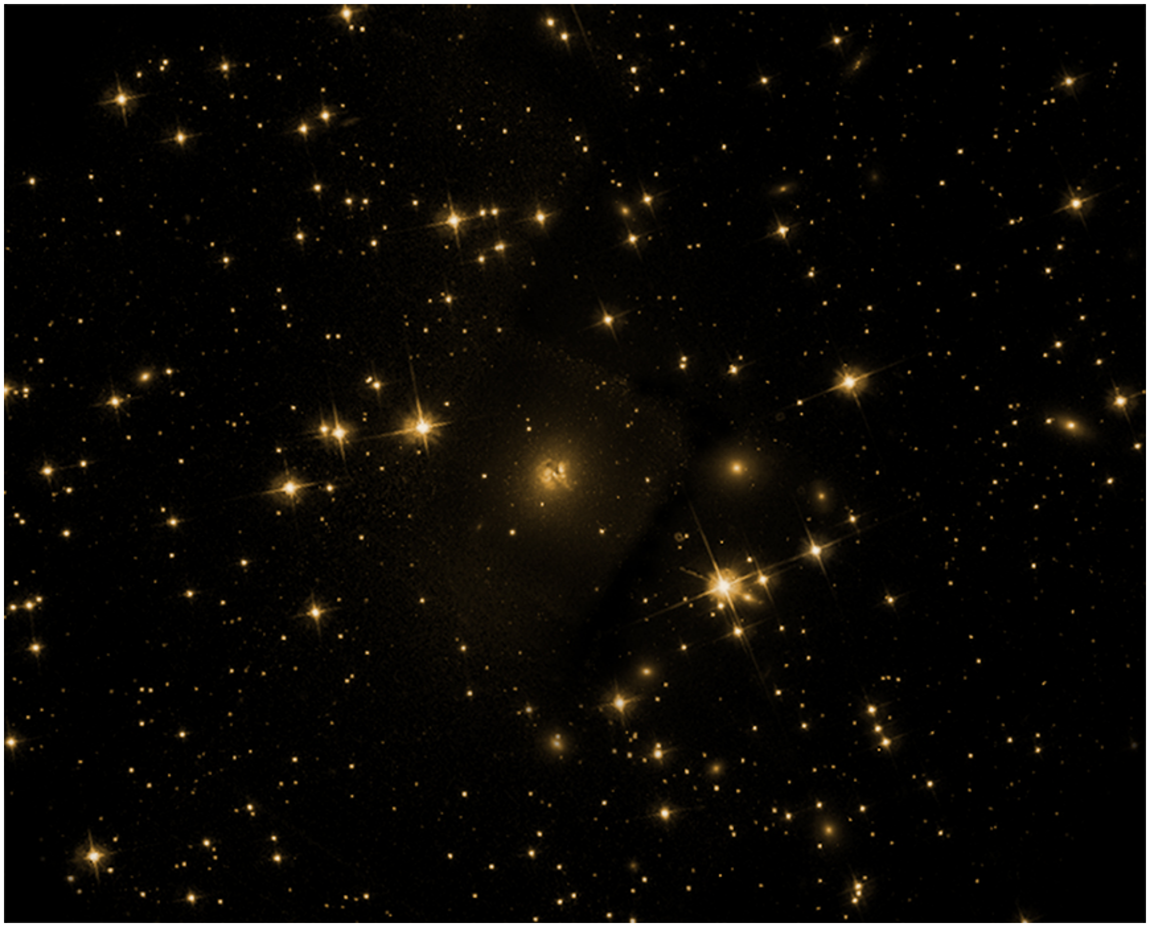
Once we have the total emitted power we can easily calculate the cooling time of an ensemble of electrons emitting synchrotron.

$$t_{\rm syn} = \frac{E}{P} = \frac{\gamma m_{\rm e} c^2}{(4/3)\sigma_{\rm T} c U_B \gamma^2 \beta^2} \sim \frac{7.75 \times 10^8}{B^2 \gamma} \ {\rm s} = \frac{24.57}{B^2 \gamma} \ {\rm yr}$$

As an example, take a supermassive black hole in an Active Galactic Nucleus.
The magnetic field around the black hole is typically of the order of 1,000 G
The Lorentz factor is also of the order of 1,000, so the electrons cool down on a timescale of just 0.77 seconds.

If instead you calculate the same cooling time very far away from the black hole, where gamma is still 1,000 but the B filed is much smaller (e.g., B~1e-5 G), the cooling time is then of the order of 250 Myr.

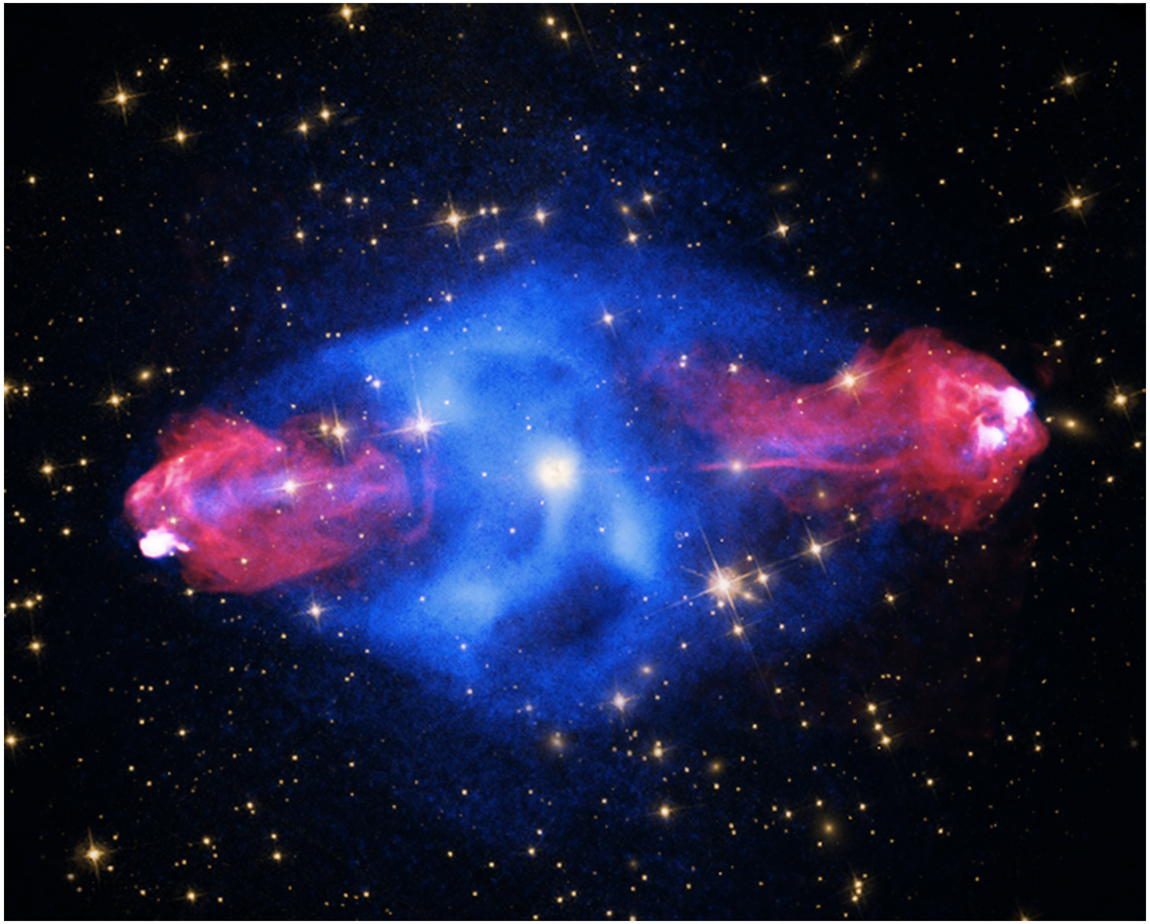
Synchrotron Cooling Time



The galaxy in the center is Cygnus A, at a distance of 260 Mpc. This image is taken in optical where the source is not particularly remarkable.

It is a very famous source especially for its low-energy emission.

Synchrotron Cooling Time



The red light is radio emission from synchrotron radiation in the so-called radio lobes.

These are the end regions of powerful jets generated in the center of the galaxy around a supermassive black hole of mass of the order of 1,000,000,000 Msun.

Synchrotron Single Particle Spectrum

For reasons of time we do not derive the expression of the power emitted by a single particle.

$$P(\omega) = \frac{\sqrt{3}}{2\pi} \frac{q^3 B \sin\alpha}{mc^2} F\left(\frac{\omega}{\omega_c}\right)$$

First let's define the quantity $x = \frac{\omega}{\omega_c}$. Then the function F(x) is: $F(x) \equiv x \int_x^\infty K_{\frac{5}{3}}(\xi)\, d\xi$

This function F(x) contains the frequency dependence of the synchrotron power of the single particle. $K_{5/3}(\xi)$ is the Bessel function of oder 5/3.

The function F(x) admits two asymptotic limits, one at low frequencies x<<1 and one at high frequencies x>>1.

$$F(x) \sim \frac{4\pi}{\sqrt{3}\,\Gamma\left(\frac{1}{3}\right)} \left(\frac{x}{2}\right)^{1/3}, \qquad x \ll 1,$$

$$F(x) \sim \left(\frac{\pi}{2}\right)^{1/2} e^{-x} x^{1/2}, \qquad x \gg 1.$$

Asymptotic limits

$$F(x) \sim \frac{4\pi}{\sqrt{3}\,\Gamma\left(\frac{1}{3}\right)}\left(\frac{x}{2}\right)^{1/3}, \qquad x \ll 1,$$

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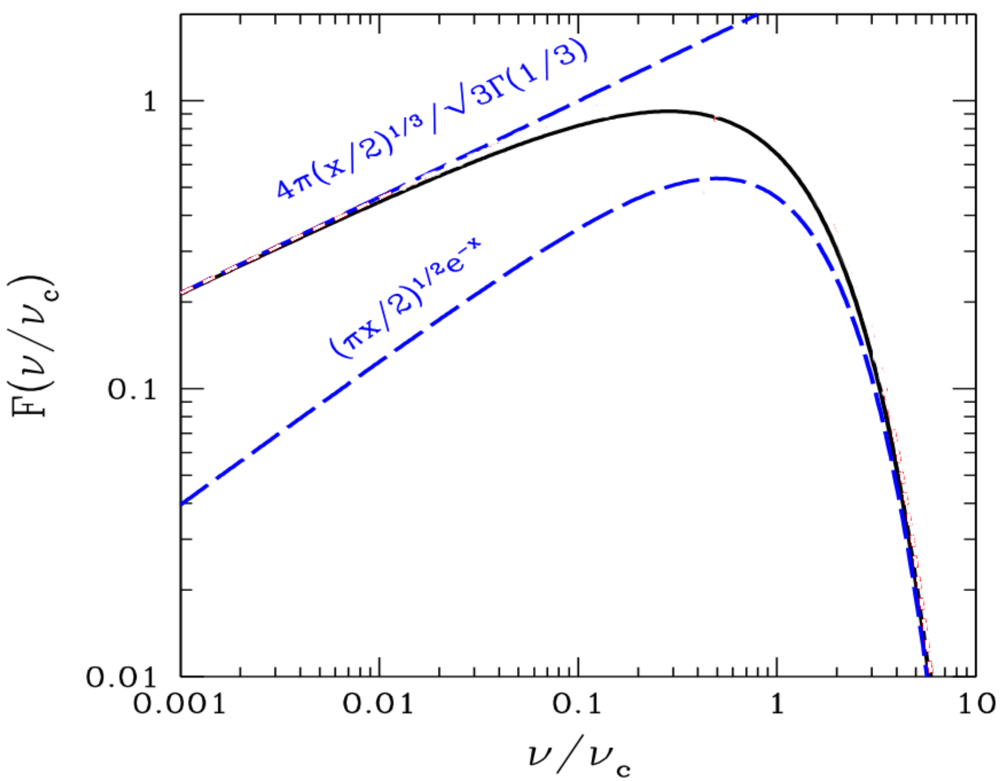
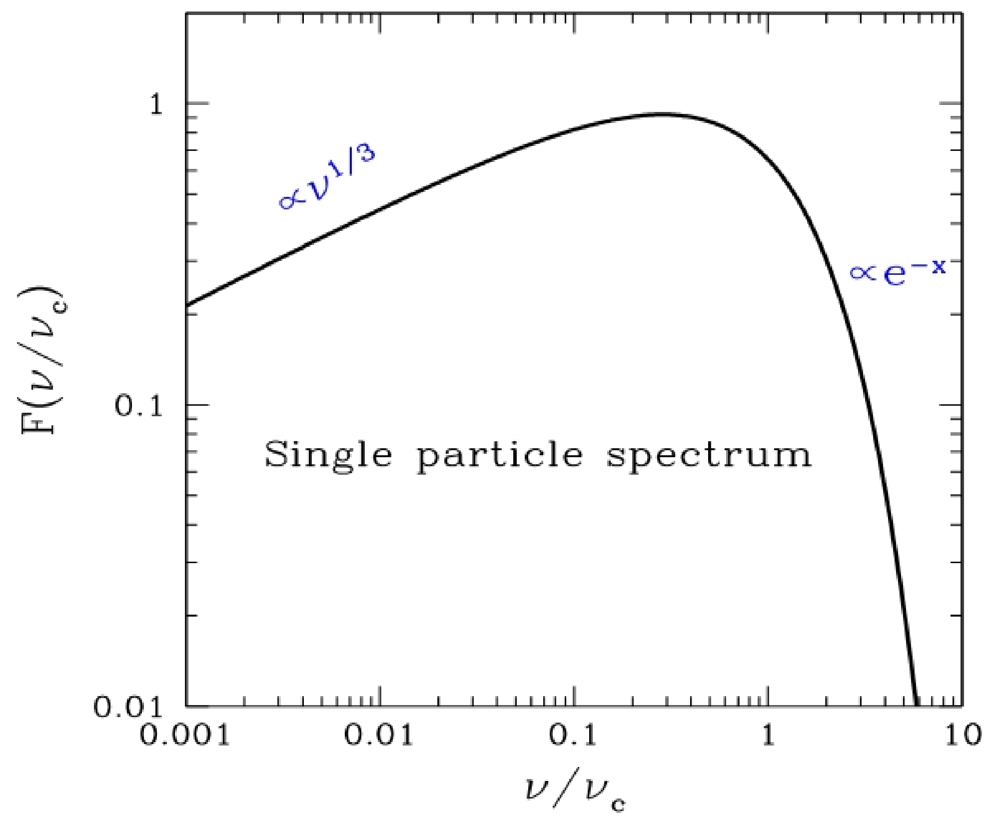
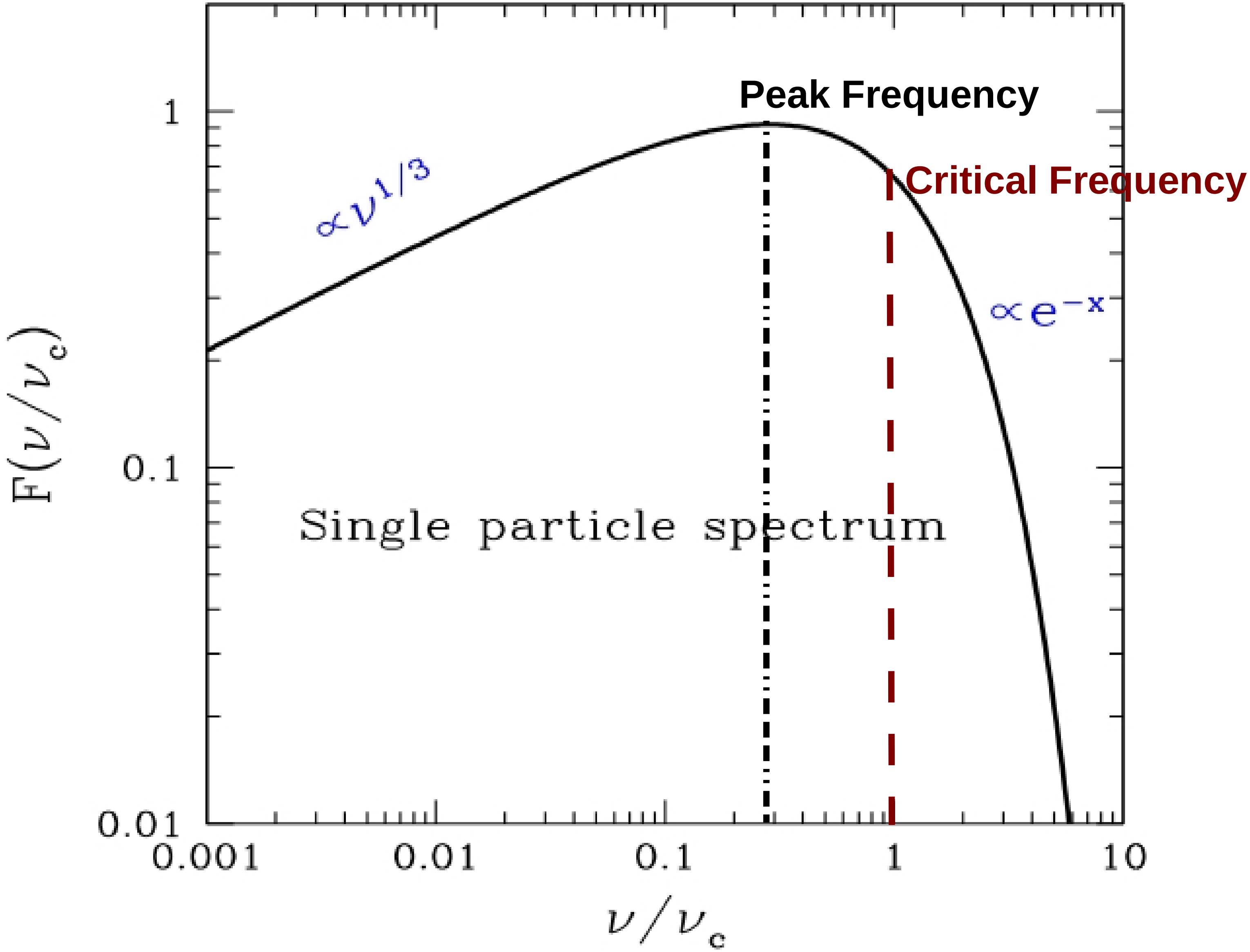
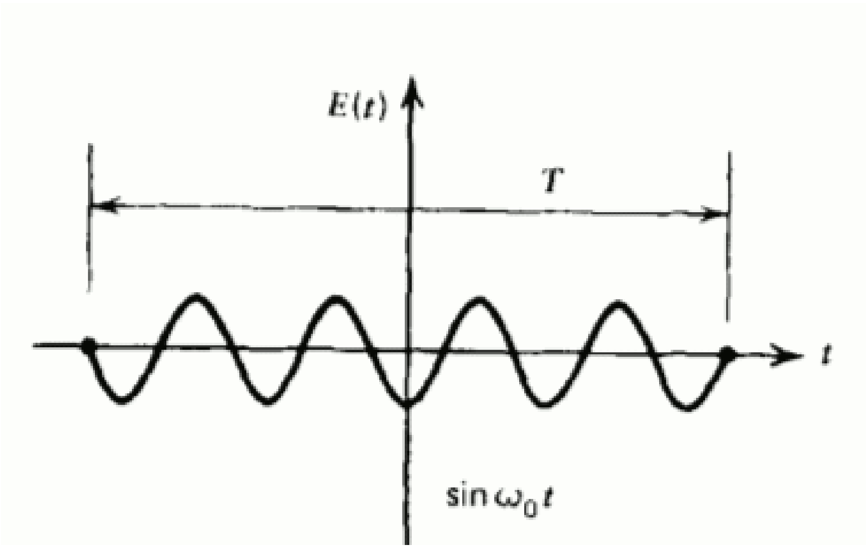




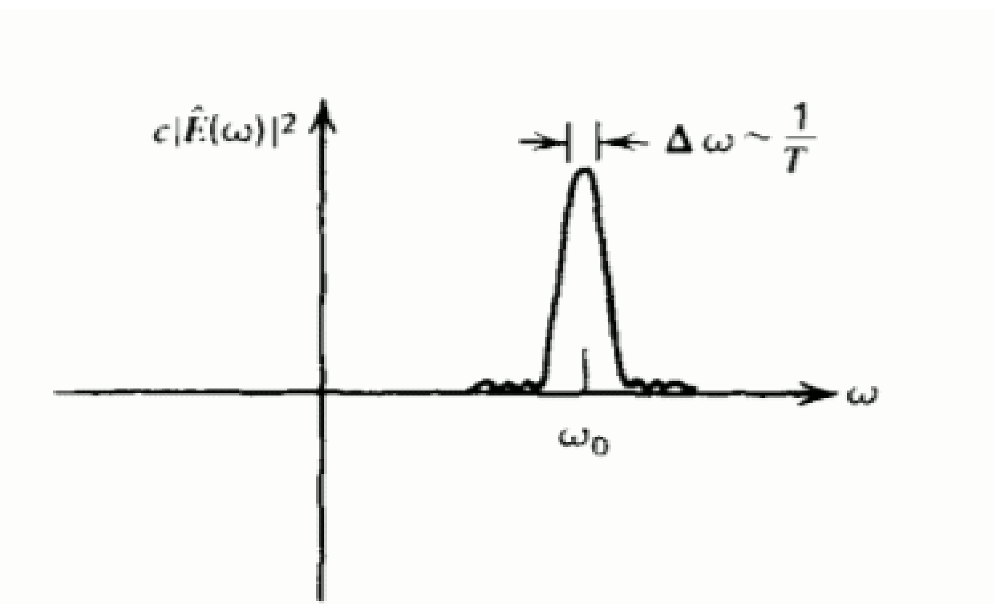


Why this synchrotron emission (SINGLE electron) is **not a single frequency?**

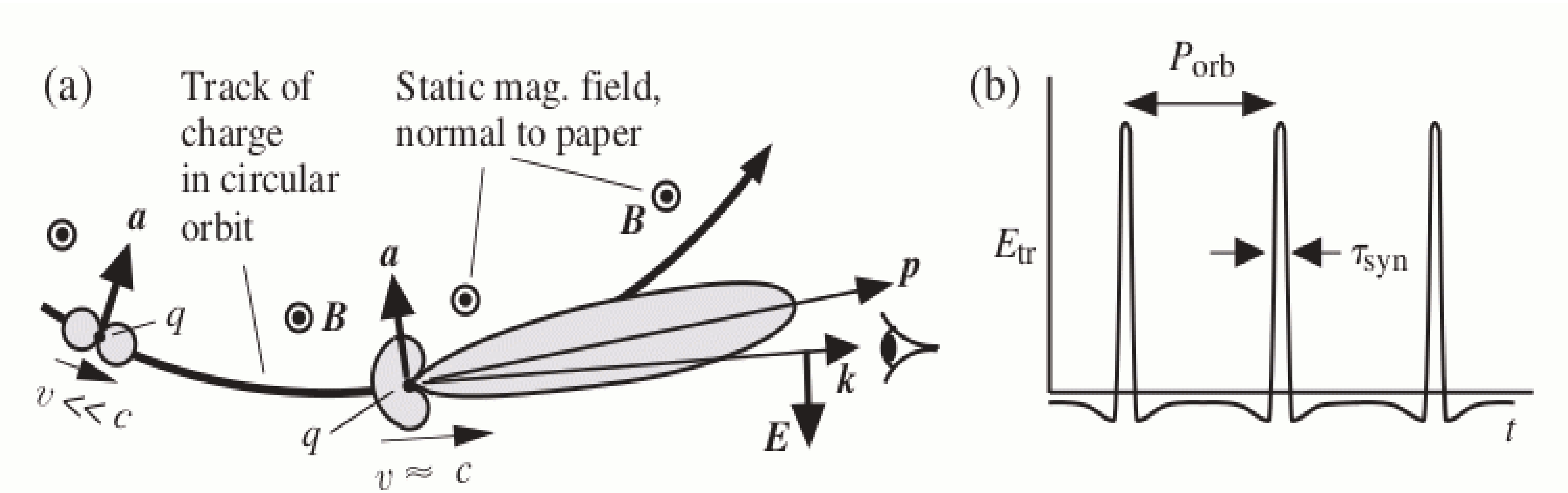
Fourier Transforms

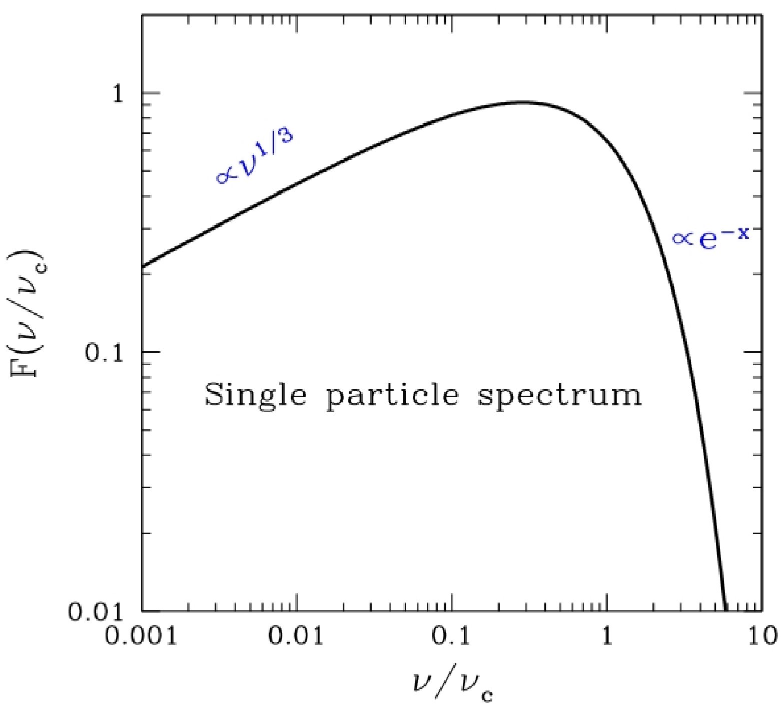


$$E(t) = \int_{-\infty}^{\infty} \hat{E}(\omega) e^{-i\omega t} d\omega .$$



$$\hat{E}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(t) e^{i\omega t} dt .$$

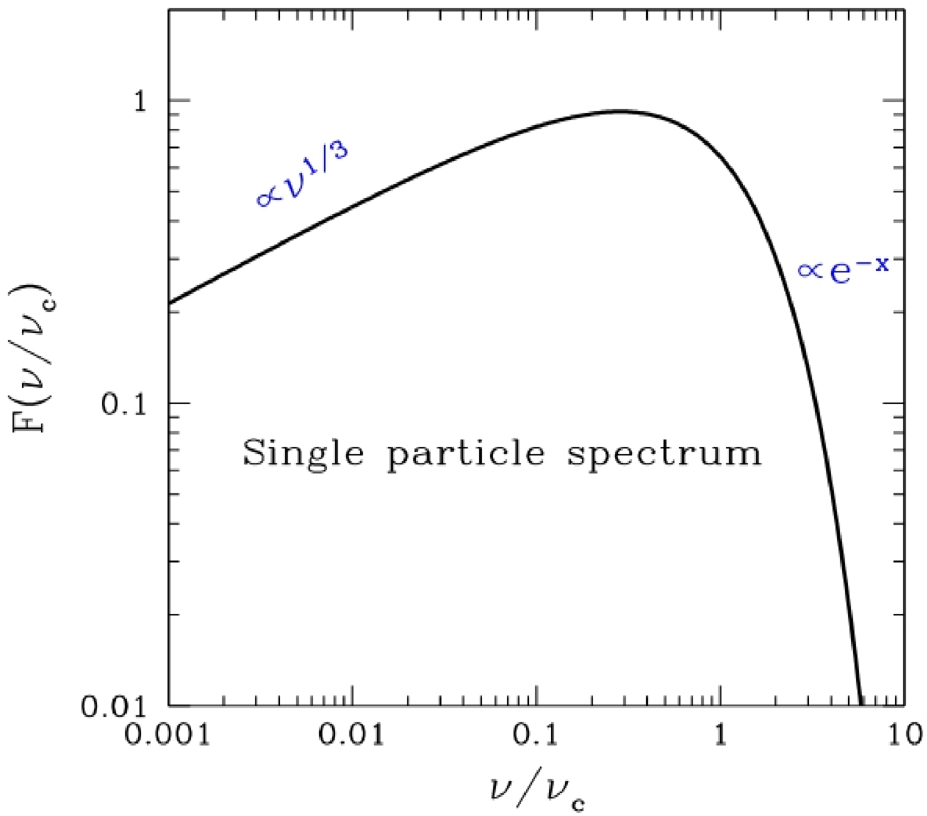


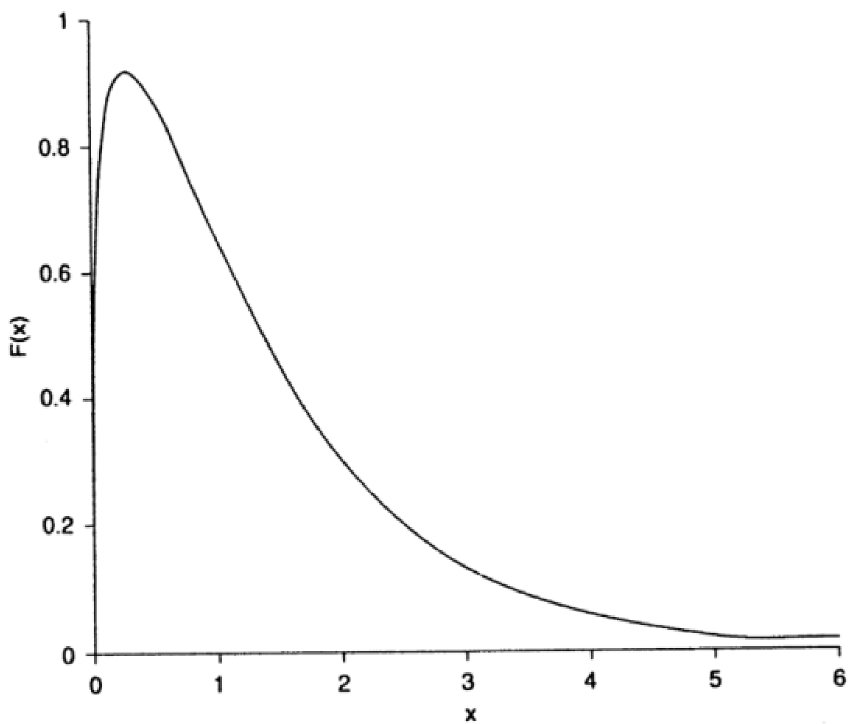


The spectrum of a single particle has its broad shape for two reasons:

1. The electro-magnetic field is not sinusoidal
2. The duration of the "pulse" is finite in time

*This is a general property of **any** signal.*





1. The single-particle spectrum extends up to something of the order of the critical frequency before decreasing exponentially.

2. The peak of the single-particle spectrum occurs at 0.29 x critical frequency

3. The width of the peak is of the order of the critical frequency

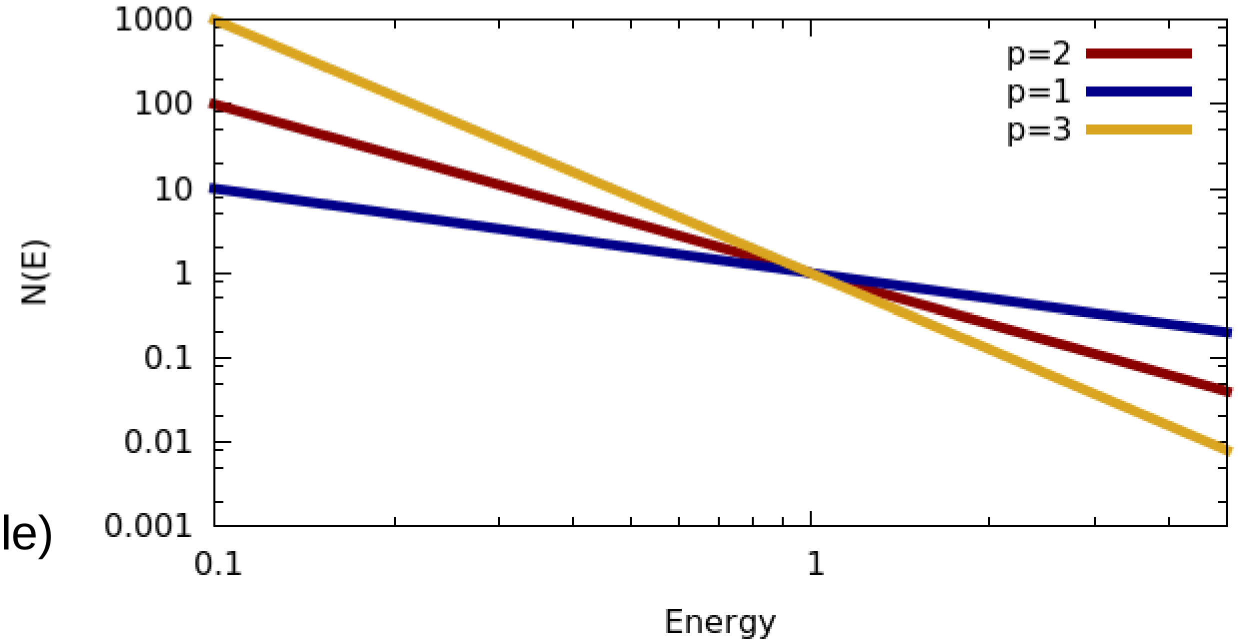
4. The meaning of the low frequency power-law index “1/3” is not immediately obvious and we will skip its detailed derivation here (see Section 6.4 on R&L).

We know how the spectrum of one particle emitting synchrotron will look like.
What if we have an ensemble of particles (with energies E between E1 and E2)?

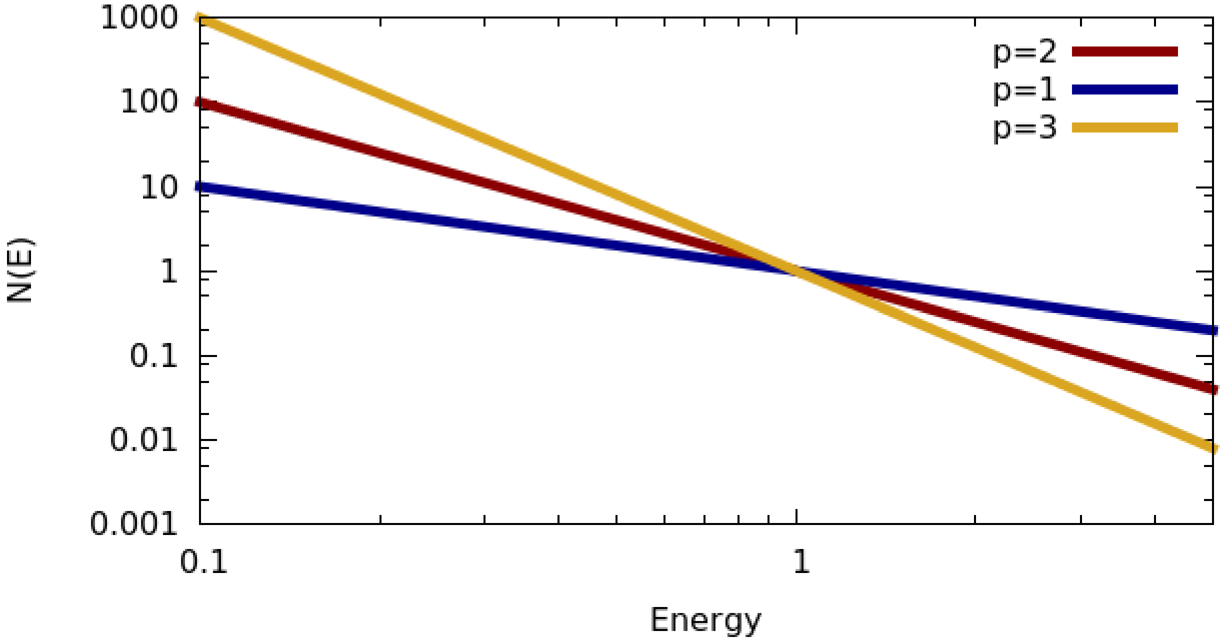
$$N(E)\,dE = C\,E^{-p}\,dE$$

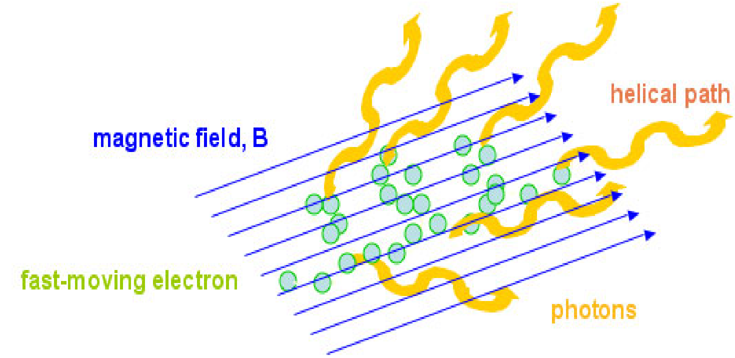
$$N(\gamma)\,d\gamma = C\,\gamma^{-p}\,d\gamma$$

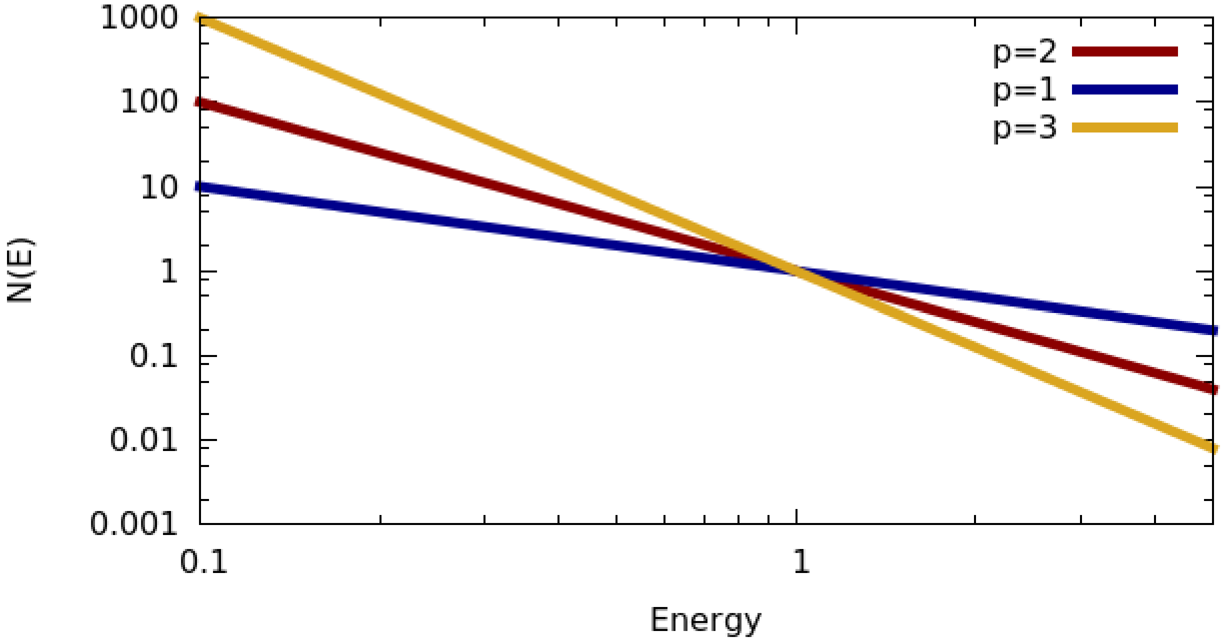
(**C** is a constant that depends on the pitch angle)

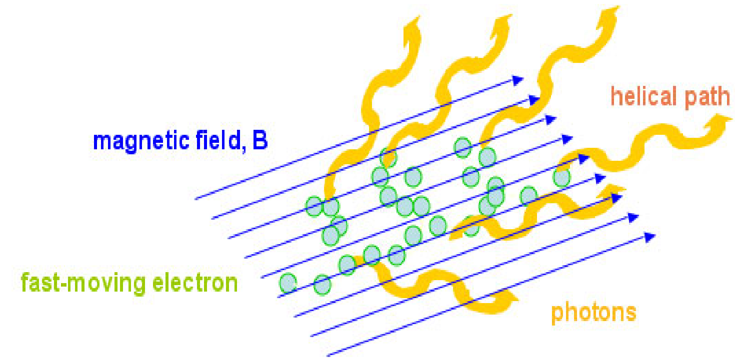


We choose a power-law distribution because this is a *very common* case in many different physical, biological and other natural (and man-made) phenomena.

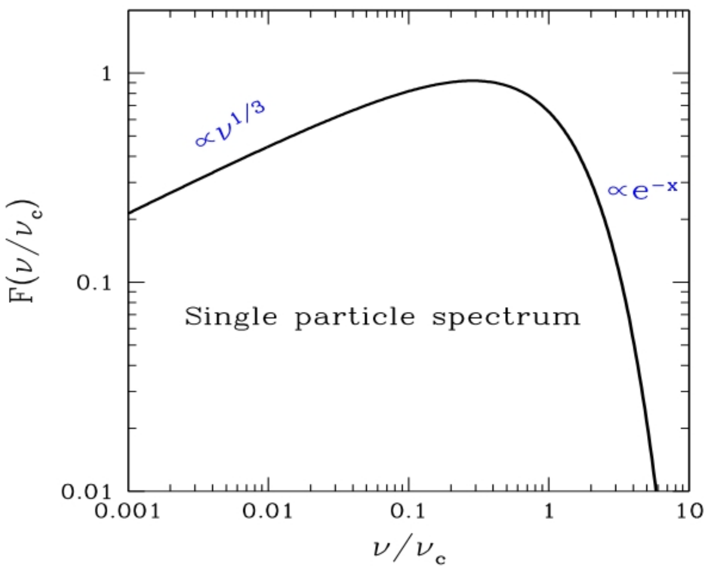

1000
100
10
1
0.1
0.01
0.001
N(E)
0.1
1
Energy
p=2
p=1
p=3

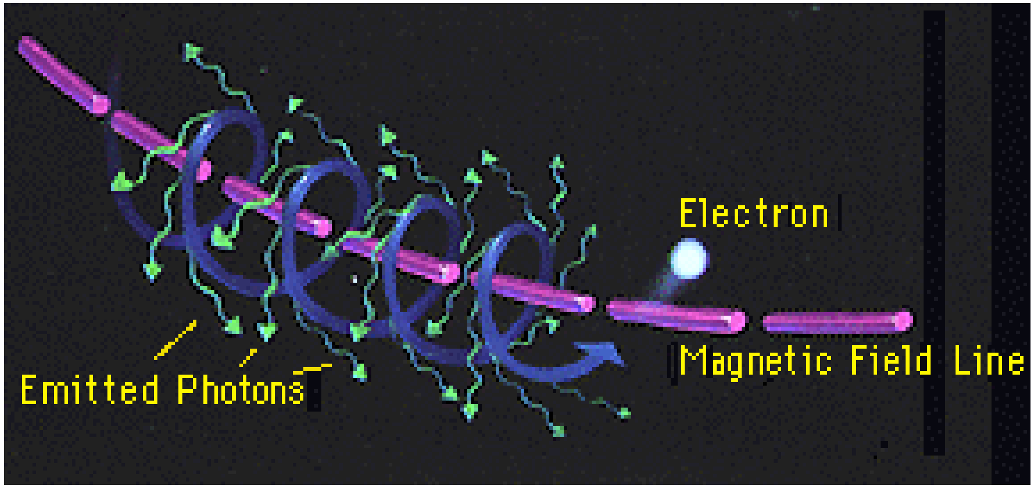

magnetic field, B
helical path
fast-moving electron
photons





X (Convolution)





The total power emitted at a specific frequency by our ensemble of (power-law distributed) particles is:

$$P_{tot}(\omega) = C \int_{\gamma_1}^{\gamma_2} P(\omega)\,\gamma^{-p}\,d\omega$$

We substitute the frequency omega with the variable **x**:

$$x = \frac{\omega}{\omega_c}$$

Remember also that the critical frequency is proportional to gamma^2

$$P_{tot}(\omega) \propto \omega^{-(p-1)/2} \int_{x_1}^{x_2} F(x)\, x^{(p-3)/2}\,dx$$

It's important to note that the function F(x) contains the dependence of the magnetic field (since the critical frequency depends on the magnetic field)

Therefore we expect that the total power emitted depends on the frequency but also on the magnetic field.

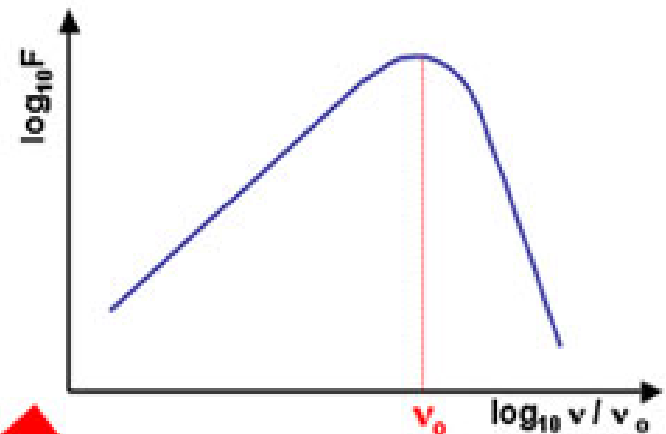
After some algebra (read section 6.4 if interested) one finds:

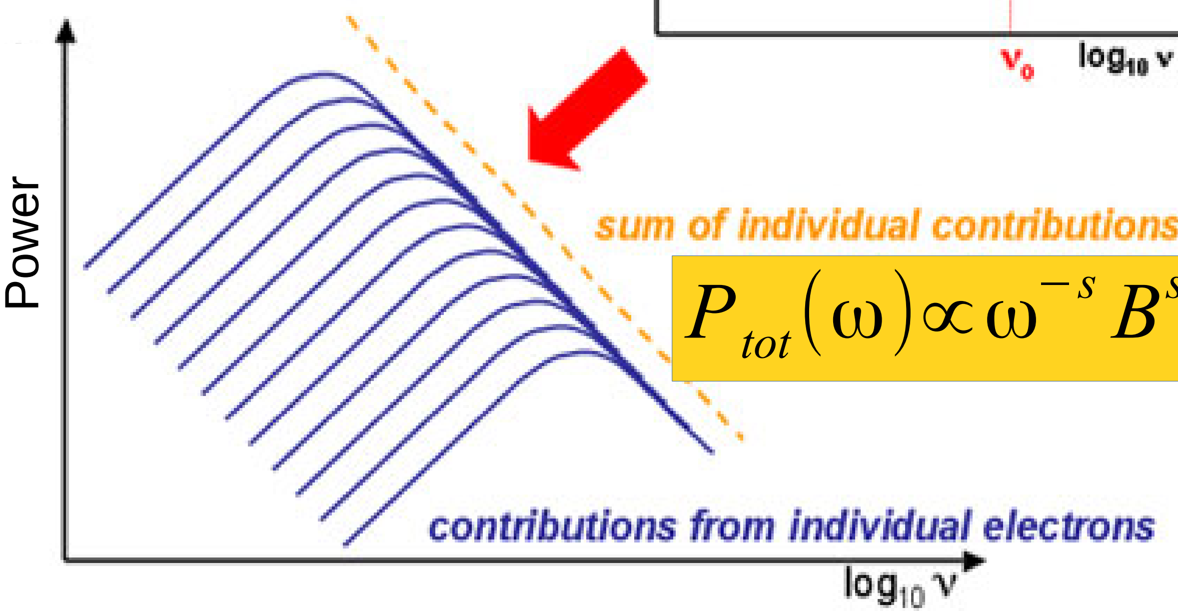
$$P_{tot}(\omega) \propto \omega^{-s} B^{s+1}$$

where we have defined the *spectral index* **s = (p-1)/2**

$$P_{tot}(\omega) = C \int_{\gamma_1}^{\gamma_2} P(\omega)\, \gamma^{-p}\, d\omega$$

$$P_{tot}(\omega) \propto \omega^{-s} B^{s+1}$$





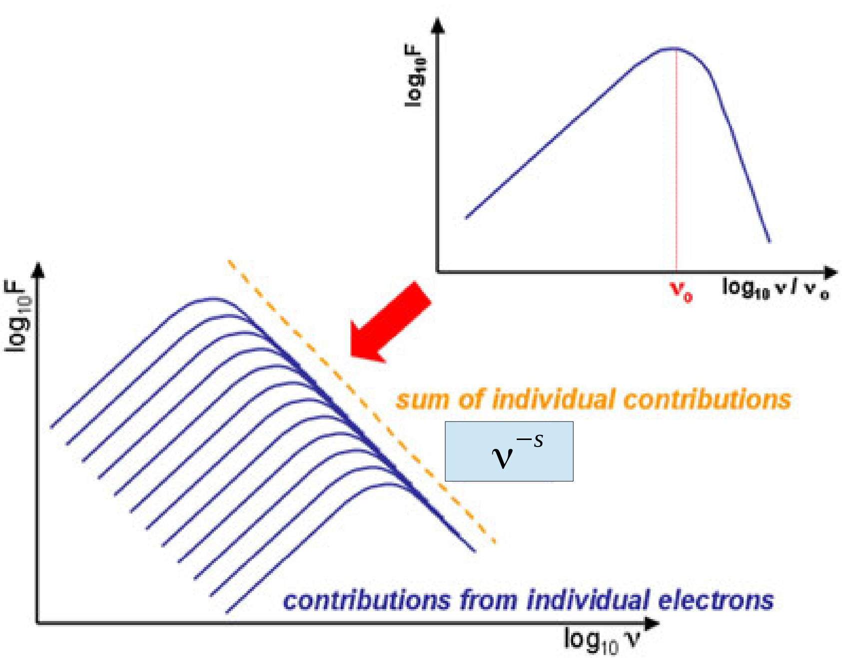
$$P_{tot}(\omega) \propto \omega^{-s} B^{s+1}$$

Synchrotron:
A power law electron energy distribution produces a power law spectrum

Self-Absorption

If you noticed, we superimposed different single particle spectra and we said that the initial power-law distribution of electrons (spectral index “p”) gives a power law distribution of emitted radiation (spectral index “s”).

However, according to the principle of detailed balance (see Lecture 1 and 2), to every emission process there is a corresponding absorption process – in the case of synchrotron radiation, this is known as synchrotron self-absorption.



Therefore this power-law with spectral index **“s”** cannot be the whole story...

When does (self)-absorption occur?

Synchrotron Self-Absorption

After some algebra, that we will skip here, one finds:

$$\alpha_\nu = \frac{\sqrt{3}\, q^3}{8\pi m}\left(\frac{3q}{2\pi m^3 c^5}\right)^{p/2} C(B\sin\alpha)^{(p+2)/2}\Gamma\left(\frac{3p+2}{12}\right)\Gamma\left(\frac{3p+22}{12}\right)\nu^{-(p+4)/2}.$$

We can write the source function as usual:

$$S_\nu = \frac{j_\nu}{\alpha_\nu} = \frac{P(\nu)}{4\pi\alpha_\nu} \propto \nu^{5/2}$$

Why in this case the slope of the self-absorbed part is 5/2, whereas for thermal bremsstrahlung it was 2?

Synchrotron Self-Absorption

After some algebra, that we will skip here, one finds:

$$\alpha_\nu = \frac{\sqrt{3}\, q^3}{8\pi m}\left(\frac{3q}{2\pi m^3 c^5}\right)^{p/2} C(B\sin\alpha)^{(p+2)/2}\,\Gamma\left(\frac{3p+2}{12}\right)\Gamma\left(\frac{3p+22}{12}\right)\nu^{-(p+4)/2}.$$

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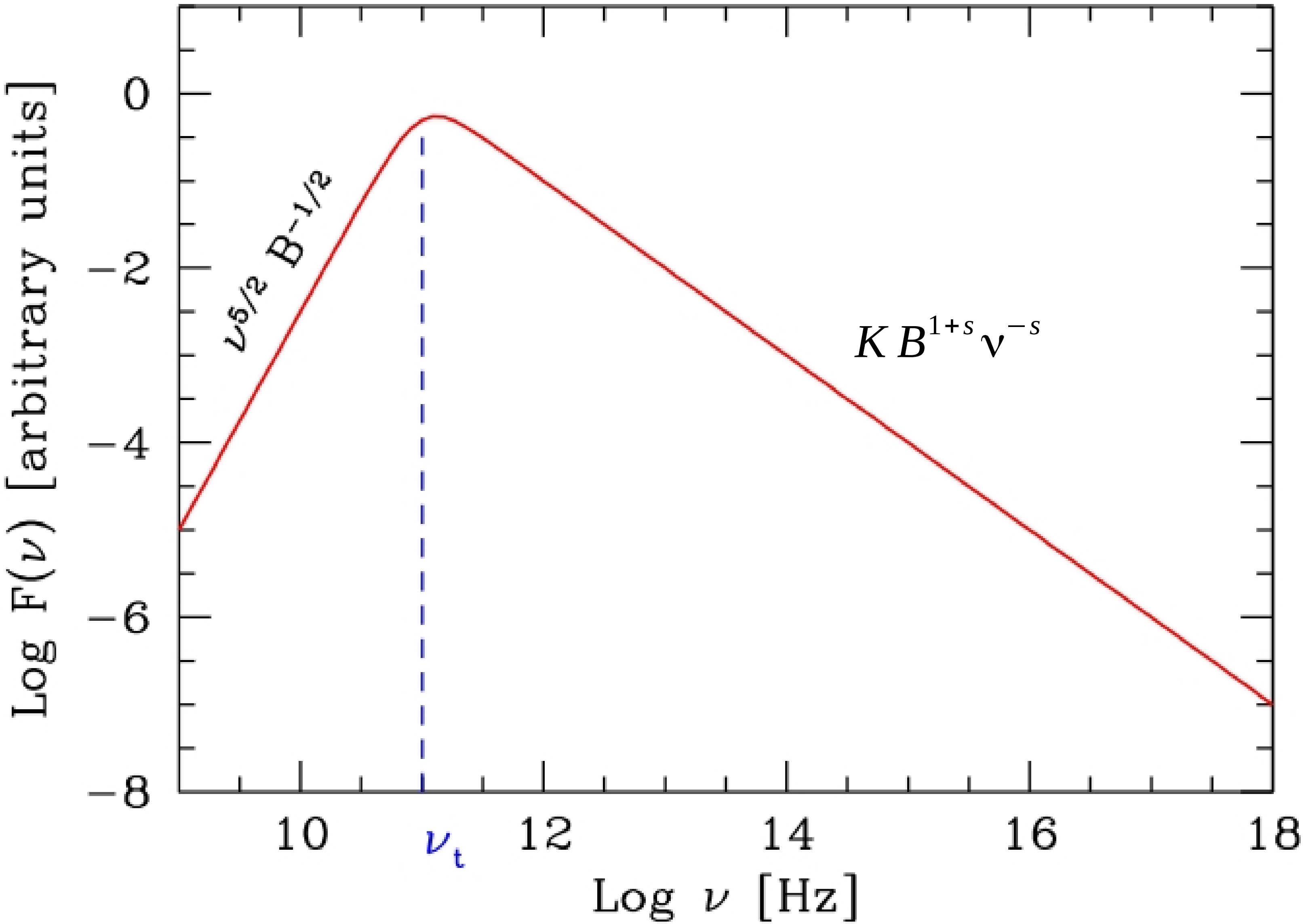
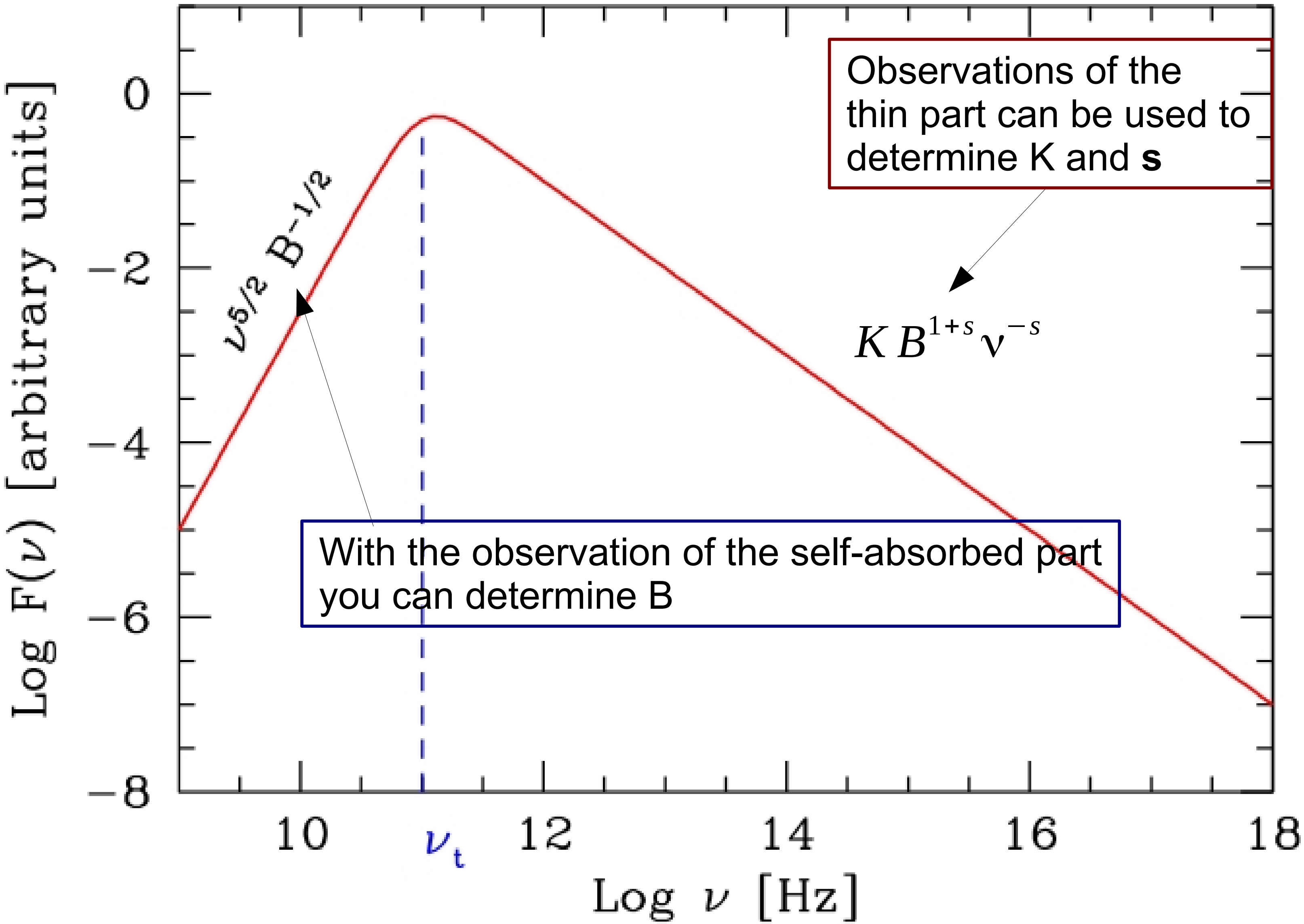
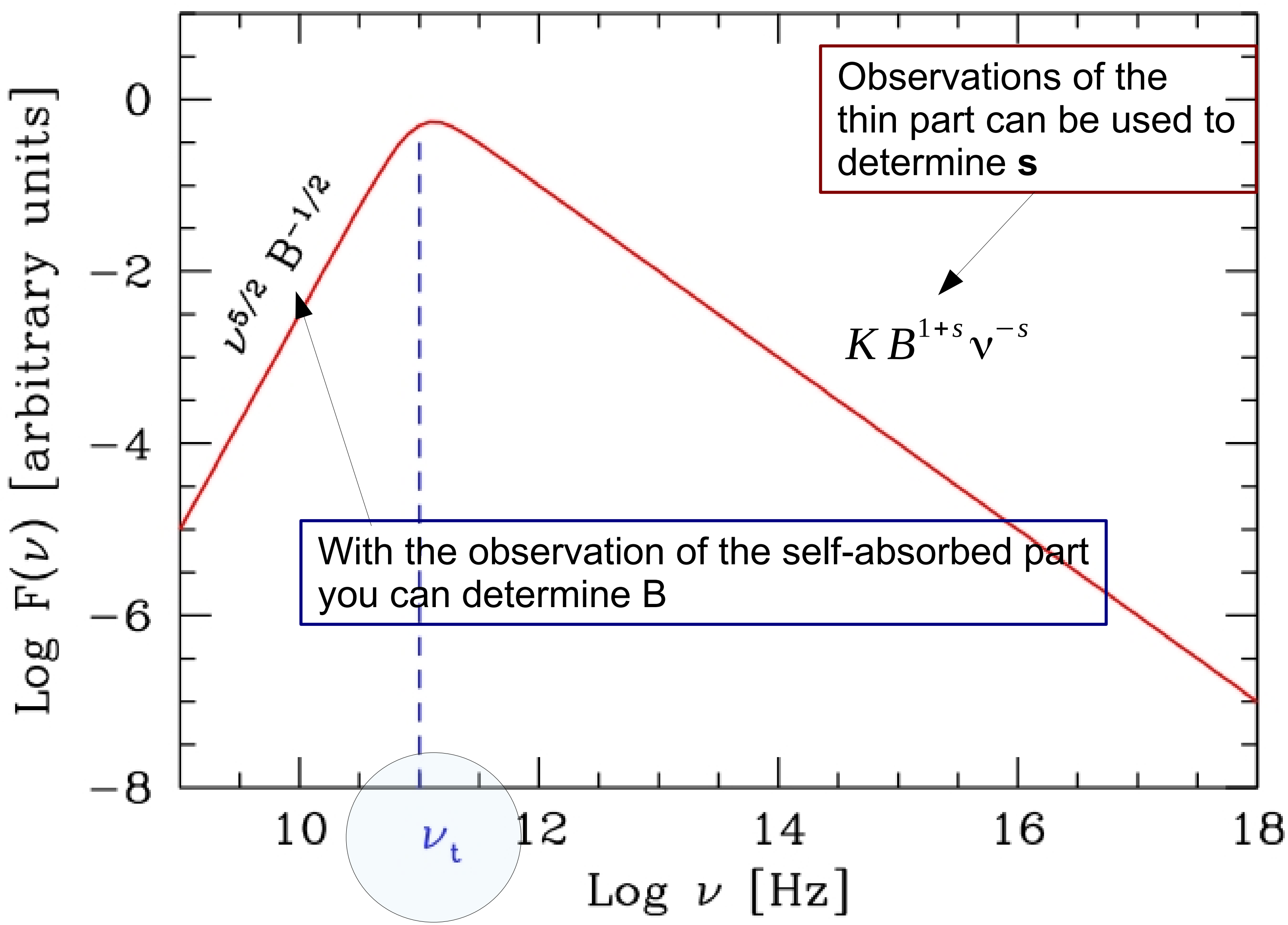
$$S_\nu = \frac{j_\nu}{\alpha_\nu} = \frac{P(\nu)}{4\pi\alpha_\nu} \propto \nu^{5/2}$$

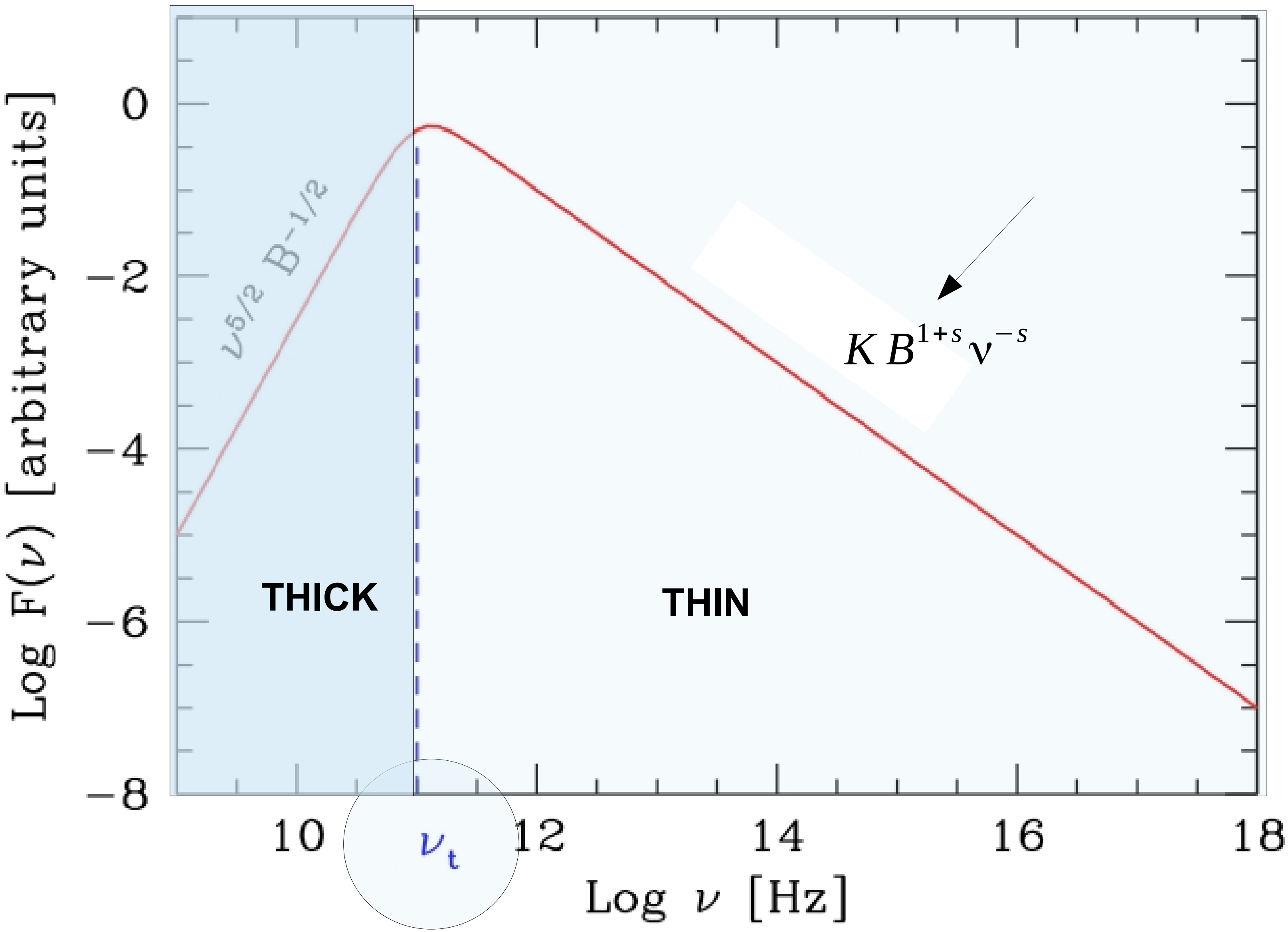
Note that the slope is NOT 2 as in the Rayleigh-Jeans regime, but it’s 5/2.
The reason is simple: here we do not have (and cannot have) a thermal distribution of particles since the B field prevents this to happen.


Log F(ν) [arbitrary units]
Log ν [Hz]
0
−2
−4
−6
−8
10
12
14
16
18
ν_t
ν^{5/2} B^{-1/2}
K B^{1+s} ν^{-s}

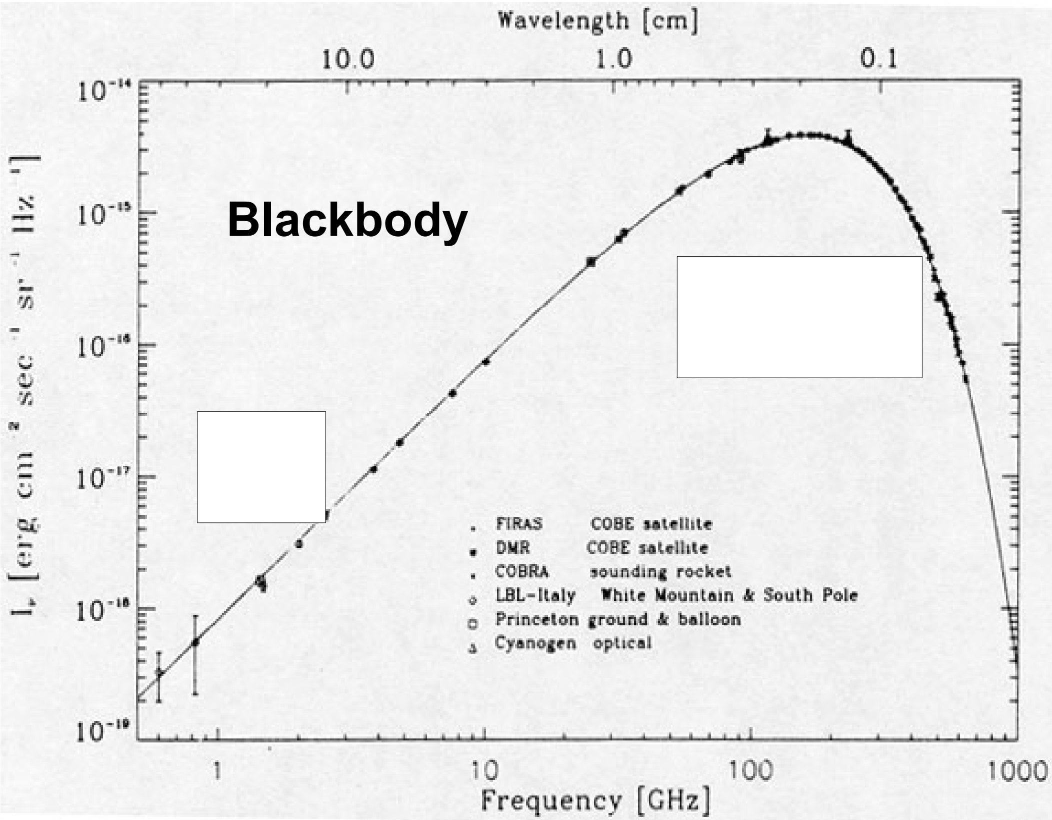

Observations of the thin part can be used to determine K and **s**
$K\,B^{1+s}\nu^{-s}$
$\nu^{5/2}\,B^{-1/2}$
With the observation of the self-absorbed part you can determine B
Log F(ν) [arbitrary units]
0
−2
−4
−6
−8
10
ν_t
12
14
16
18
Log ν [Hz]

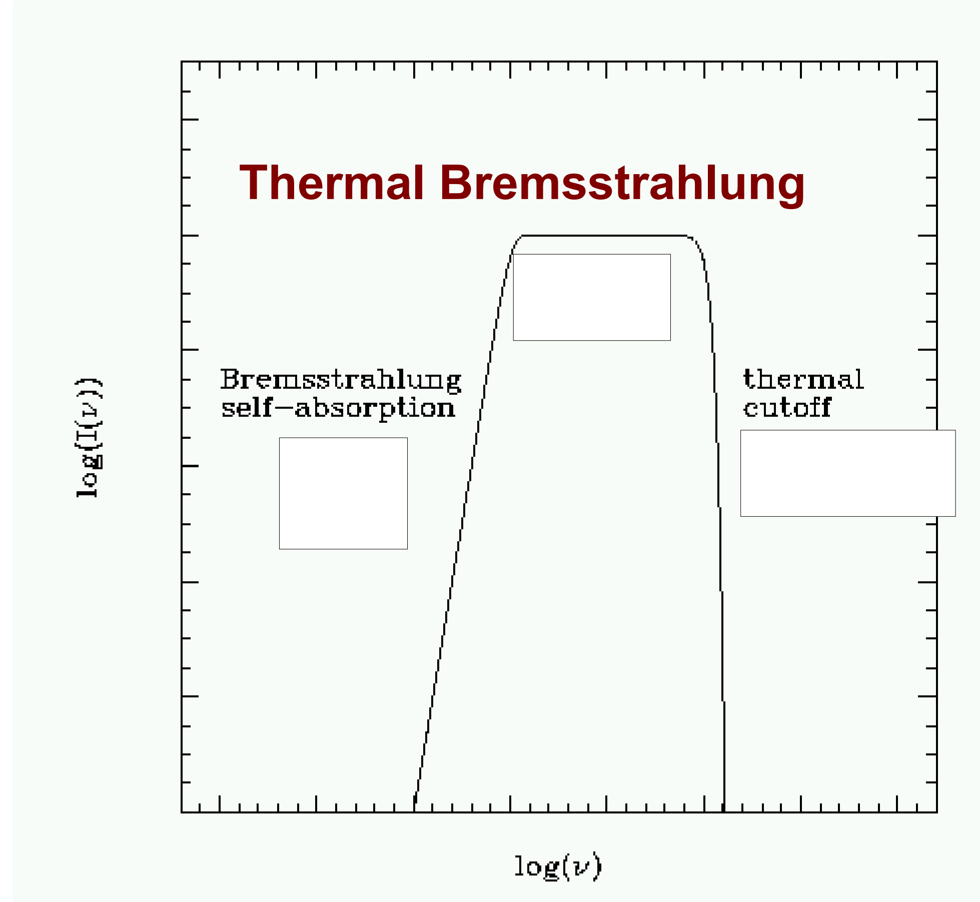


Self absorption frequency: marks the transition from optically thin to thick

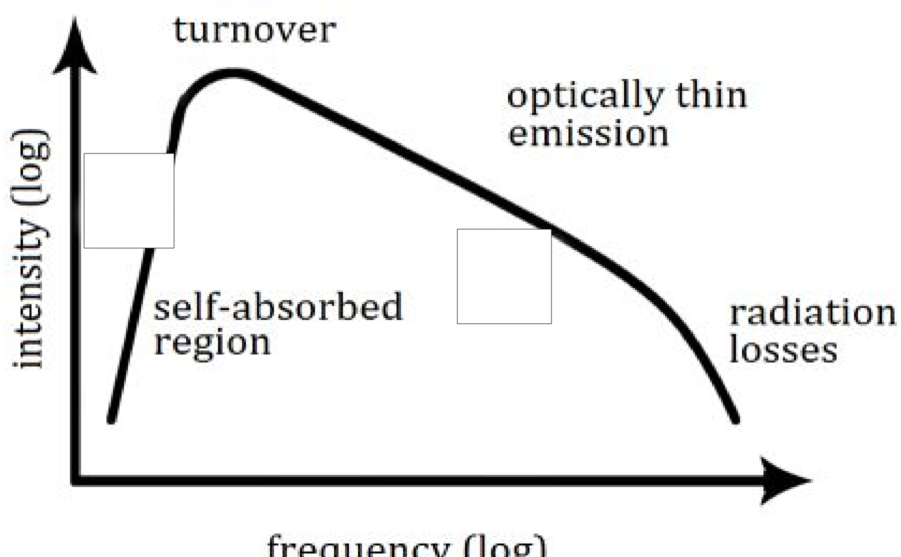


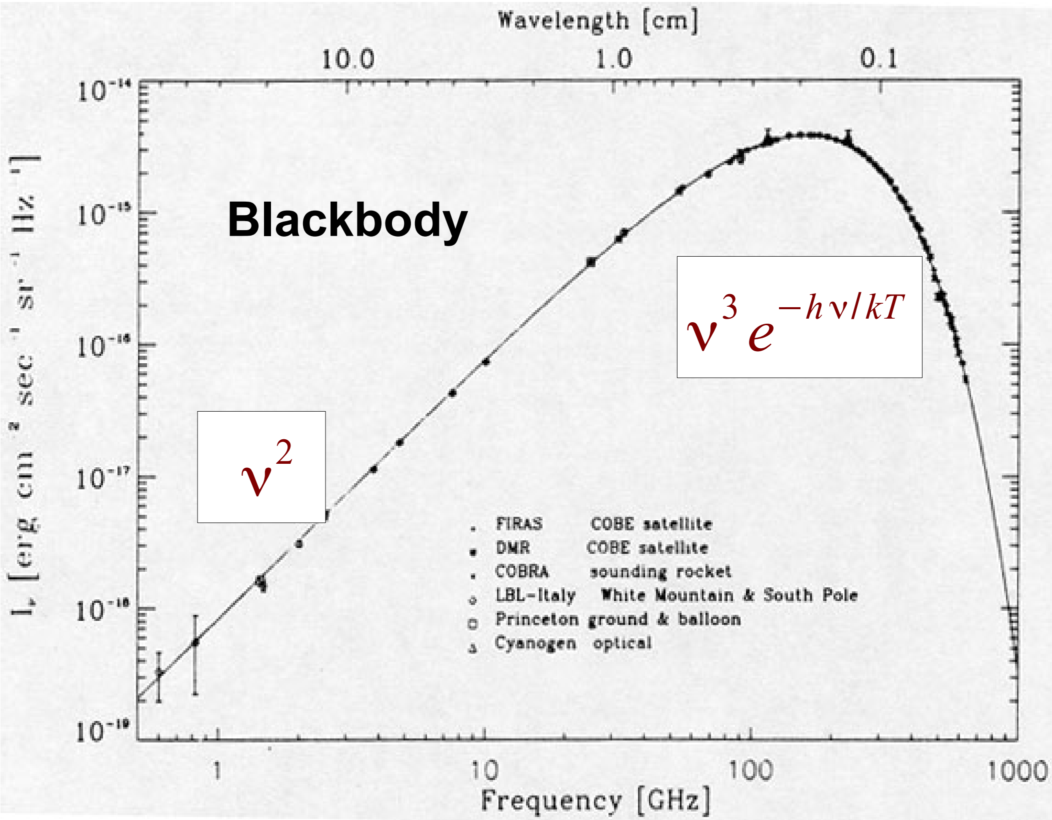
Self absorption frequency: marks the transition from <u>optically thin to thick</u>

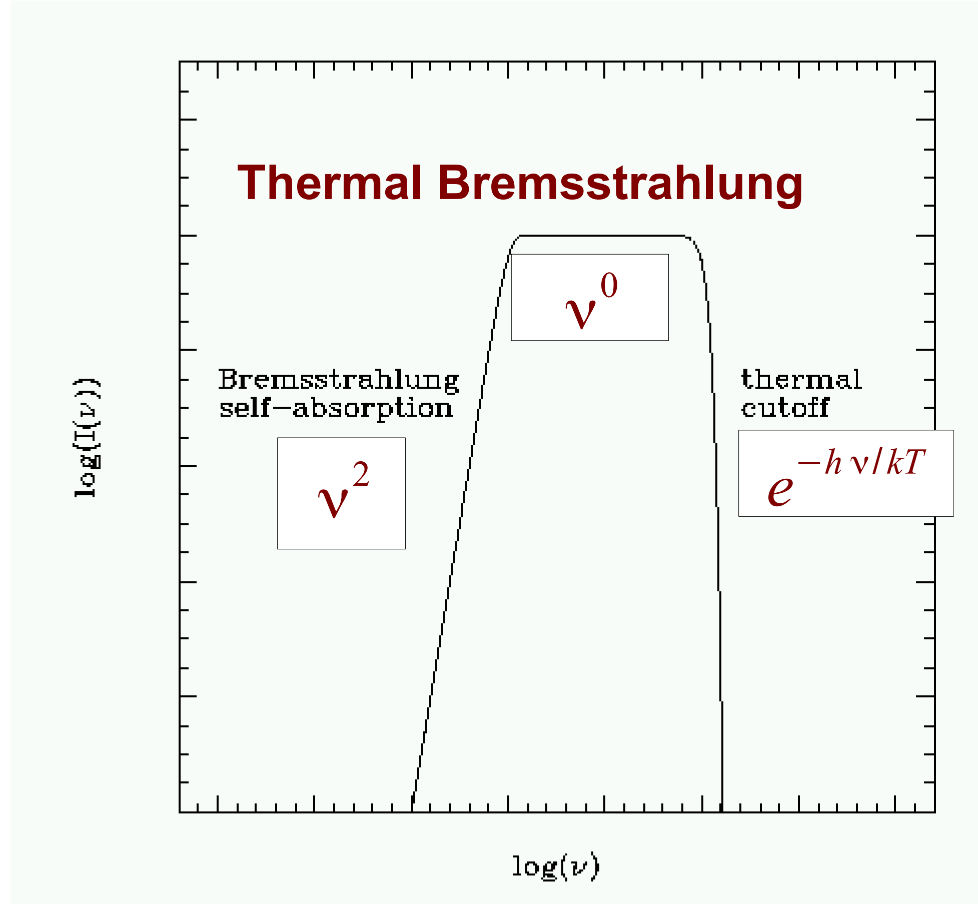




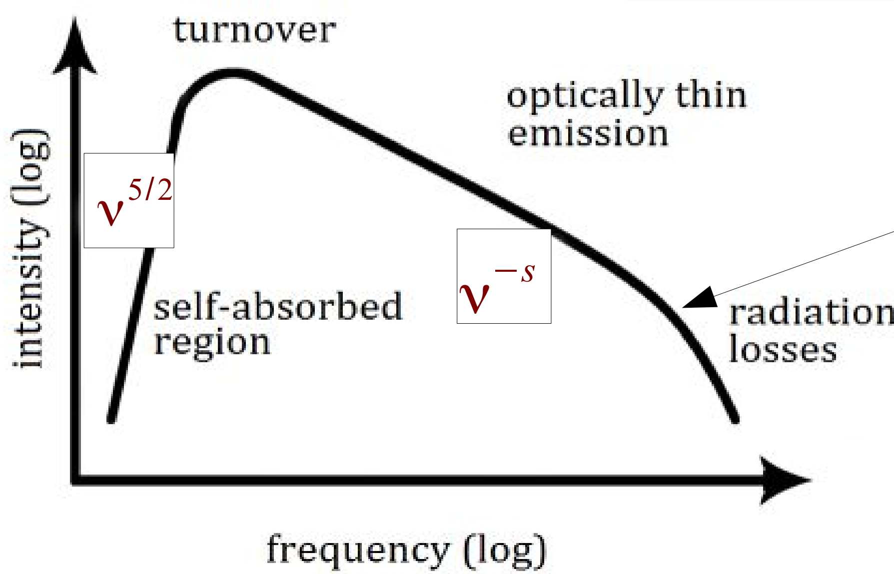
Synchrotron



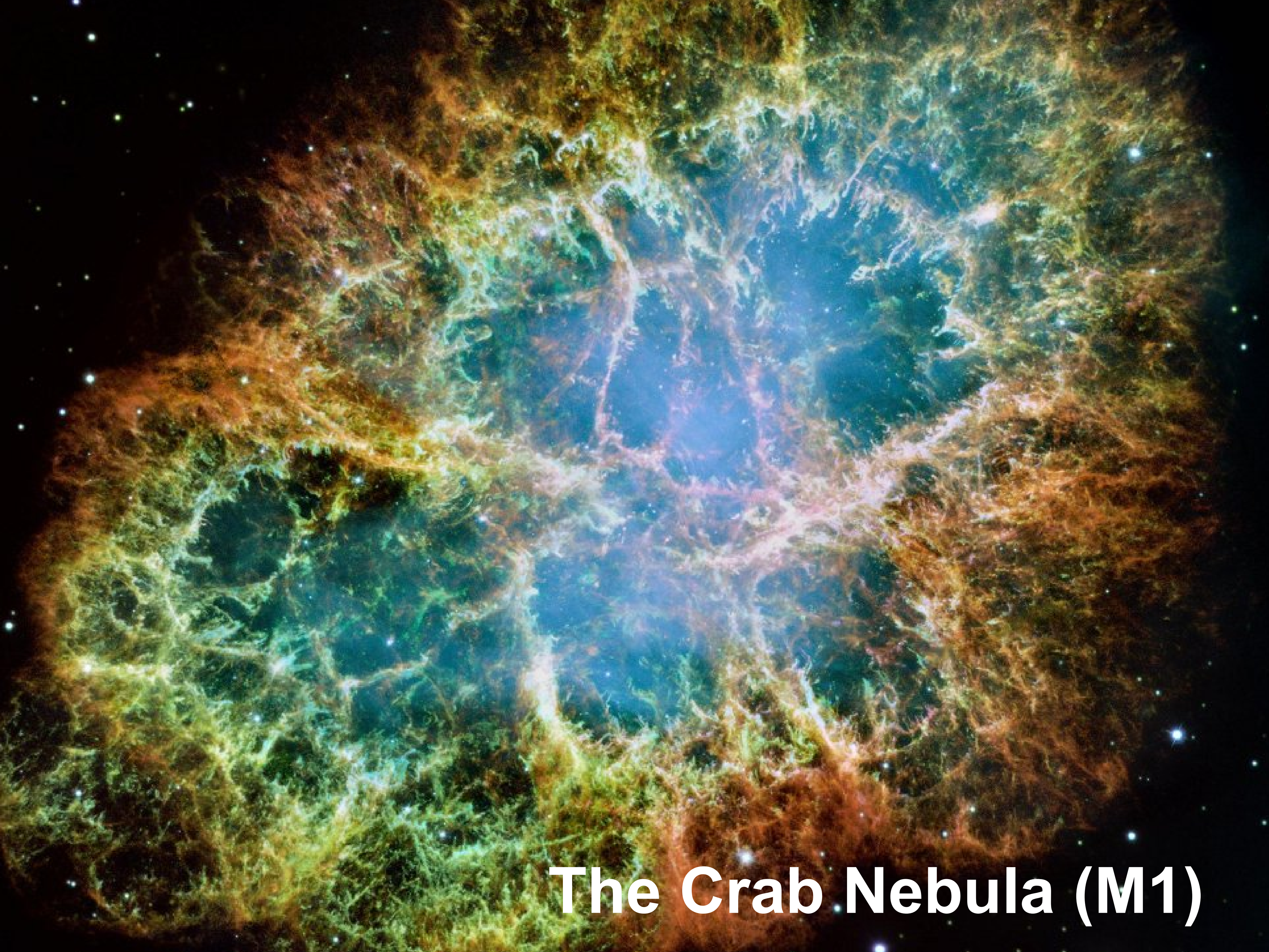


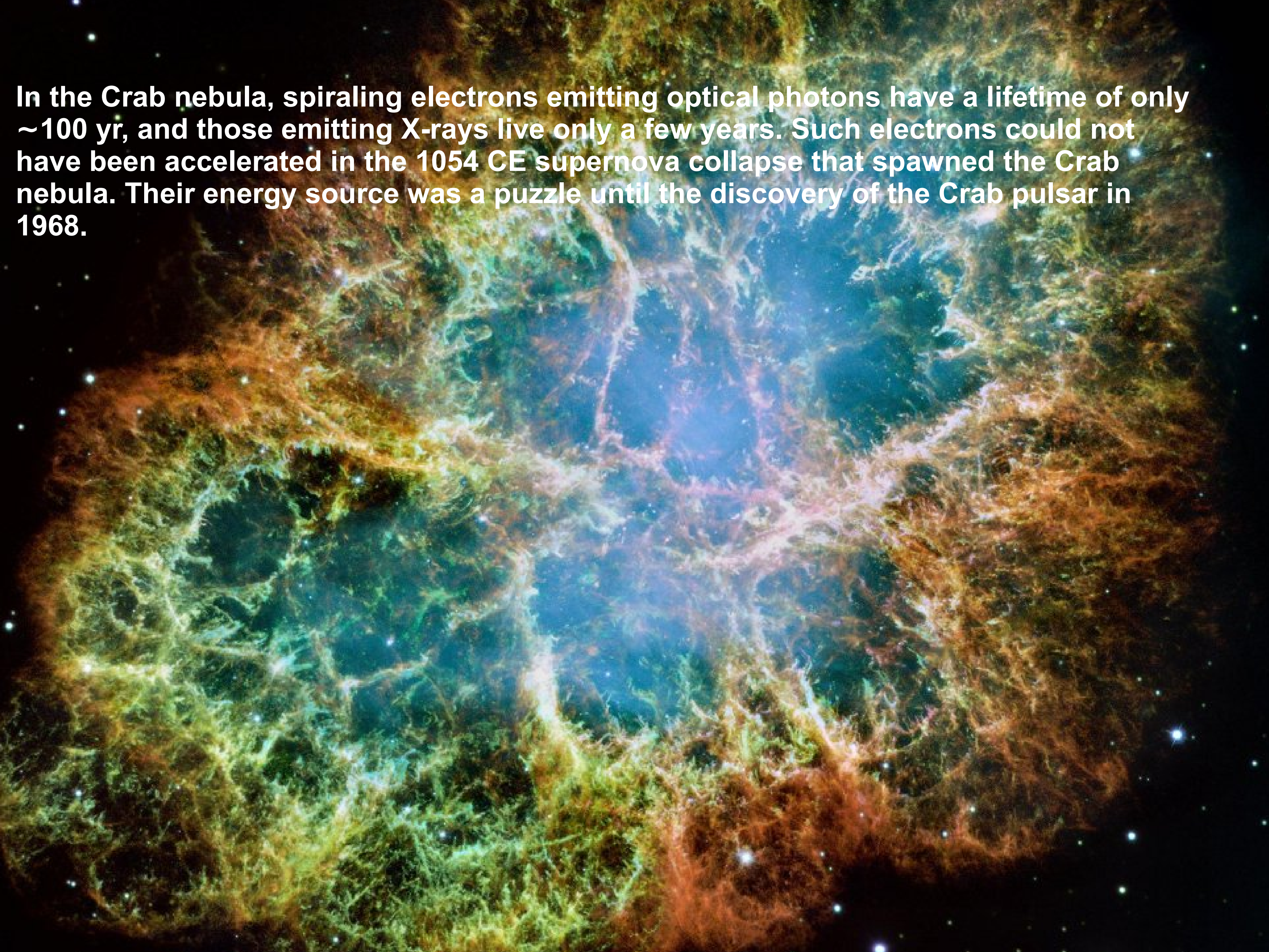


Synchrotron

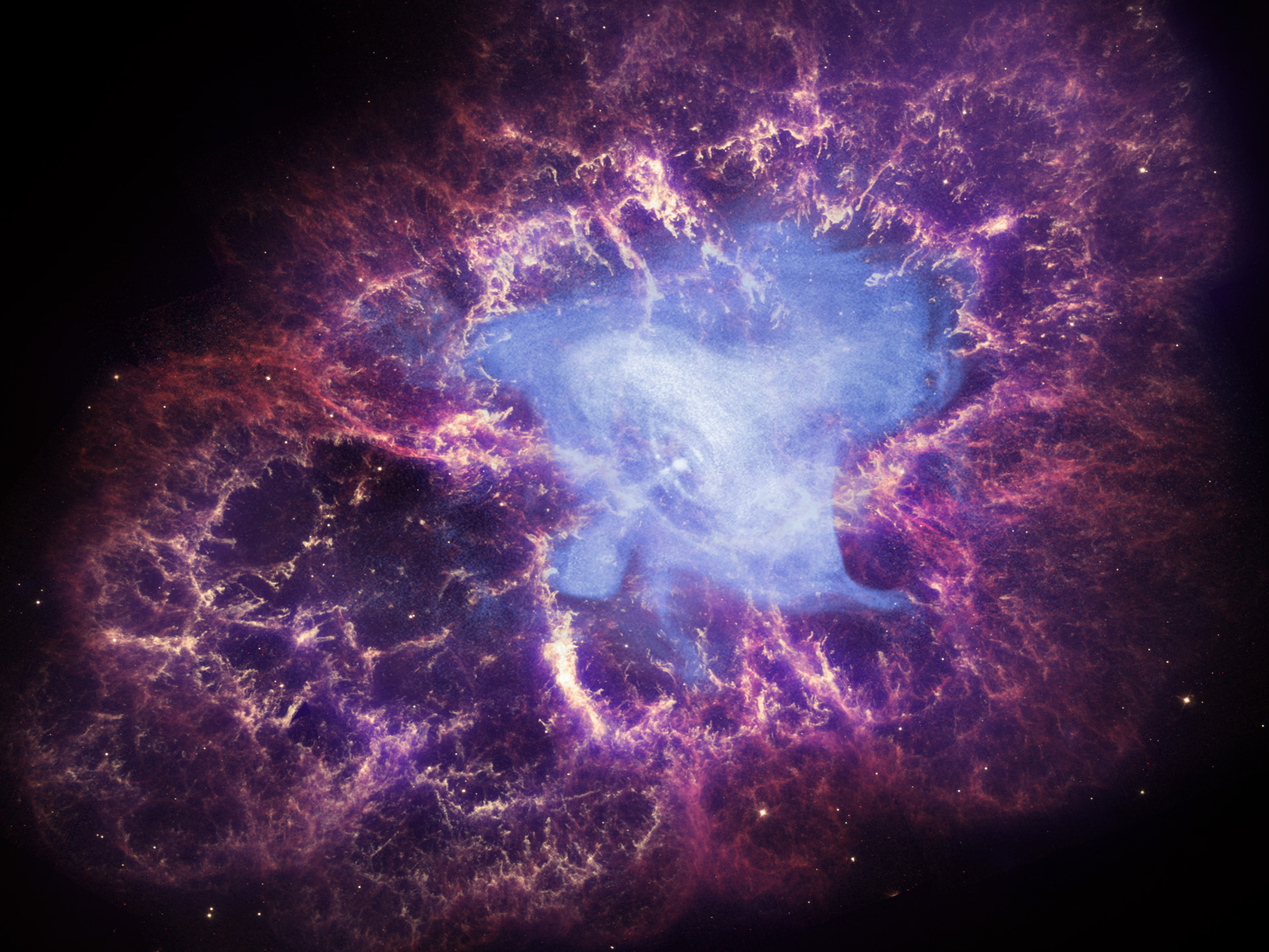


What happens here?


The Crab Nebula (M1)

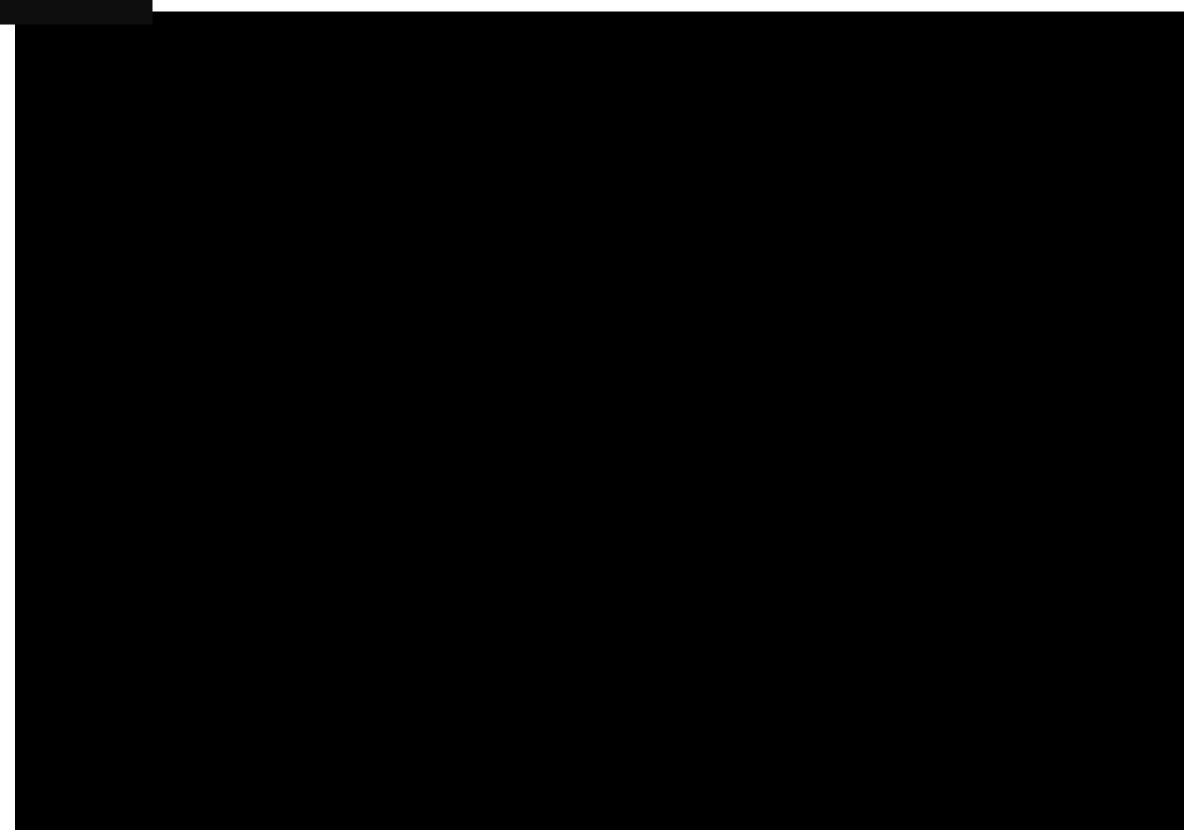


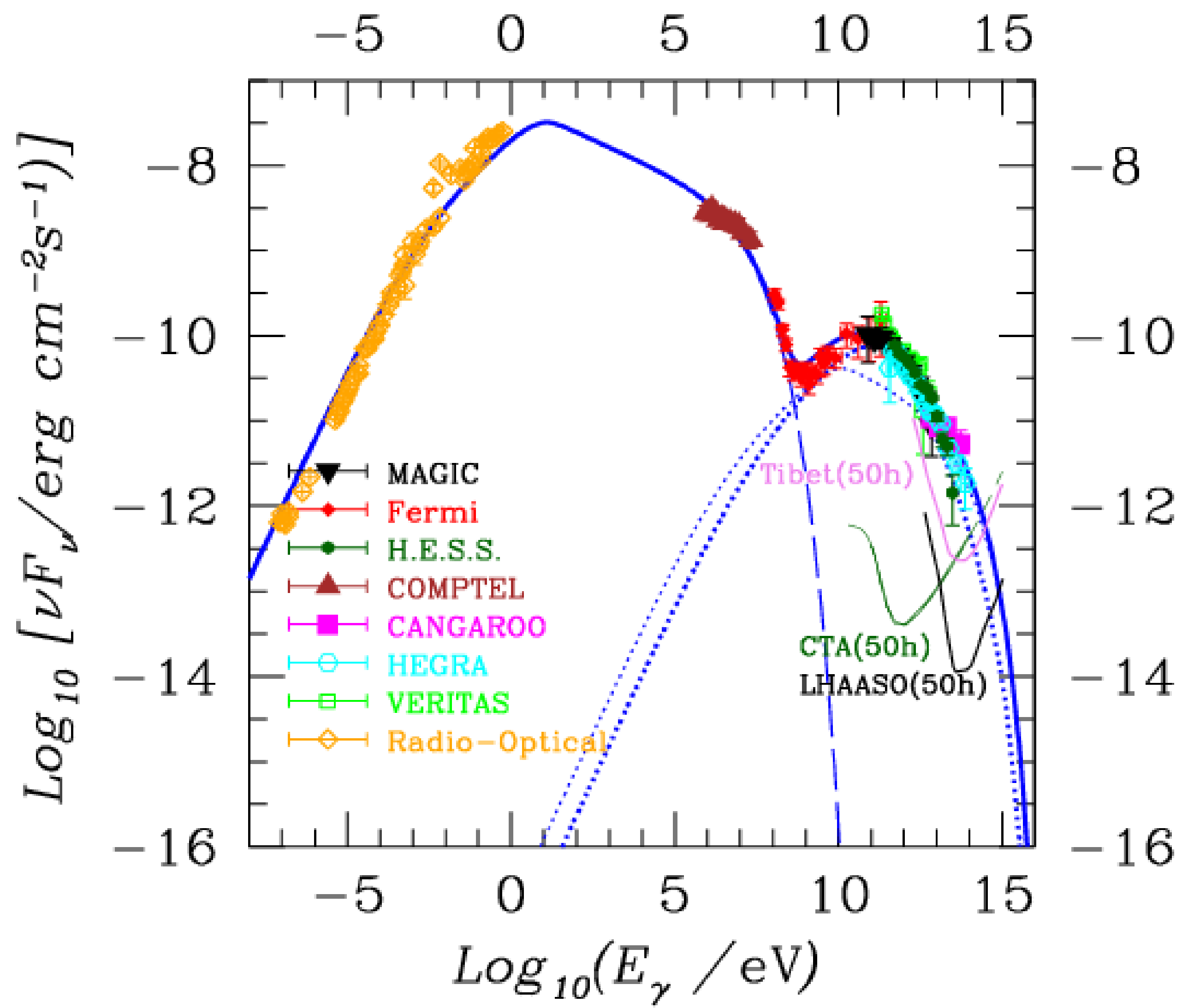
In the Crab nebula, spiraling electrons emitting optical photons have a lifetime of only ~100 yr, and those emitting X-rays live only a few years. Such electrons could not have been accelerated in the 1054 CE supernova collapse that spawned the Crab nebula. Their energy source was a puzzle until the discovery of the Crab pulsar in 1968.



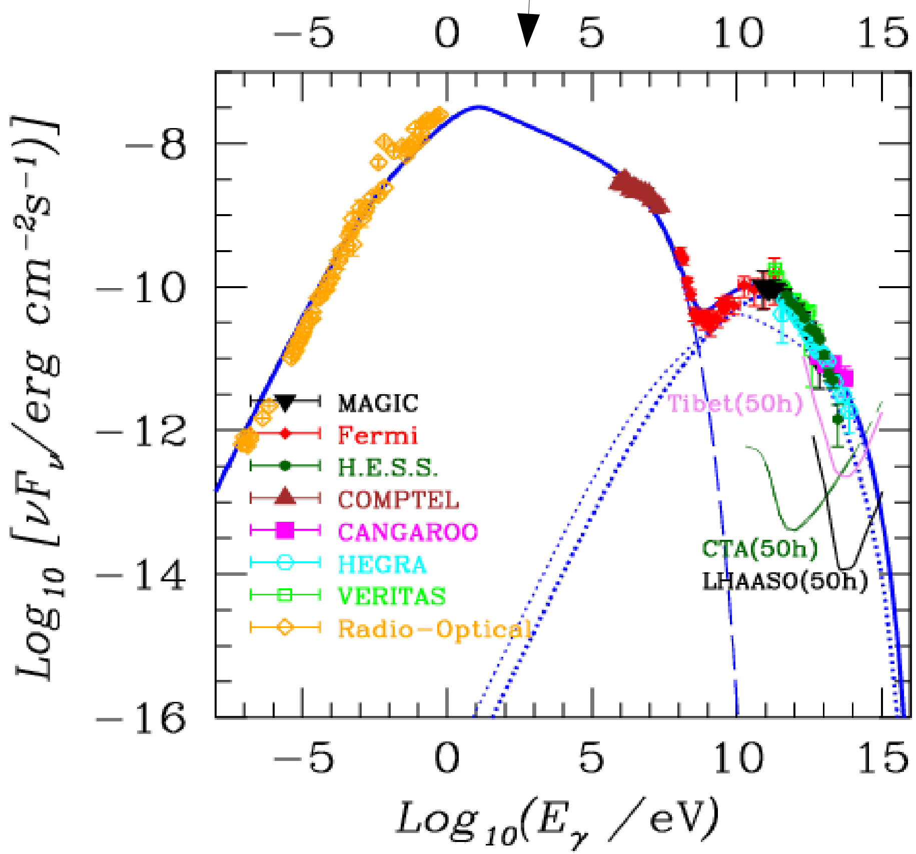
Chandra X-Ray Observatory
(0.3-8 keV)

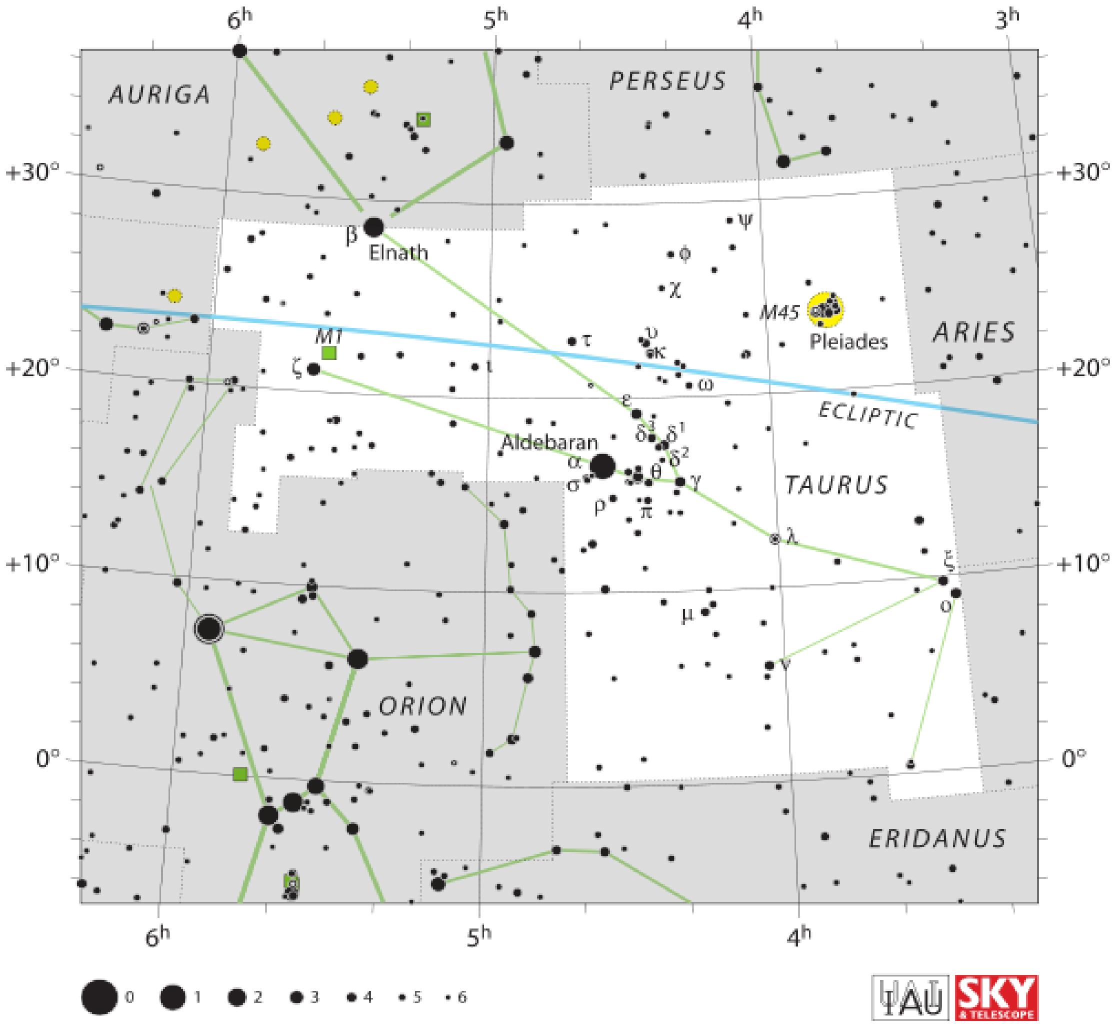
Hubble Space Telescope
(Optical)




MAGIC
Fermi
H.E.S.S.
COMPTEL
CANGAROO
HEGRA
VERITAS
Radio−Optical
Tibet(50h)
CTA(50h)
LHAASO(50h)
$Log_{10}\ [\nu F_\nu/erg\ cm^{-2}s^{-1})]$
$Log_{10}(E_\gamma\ /eV)$

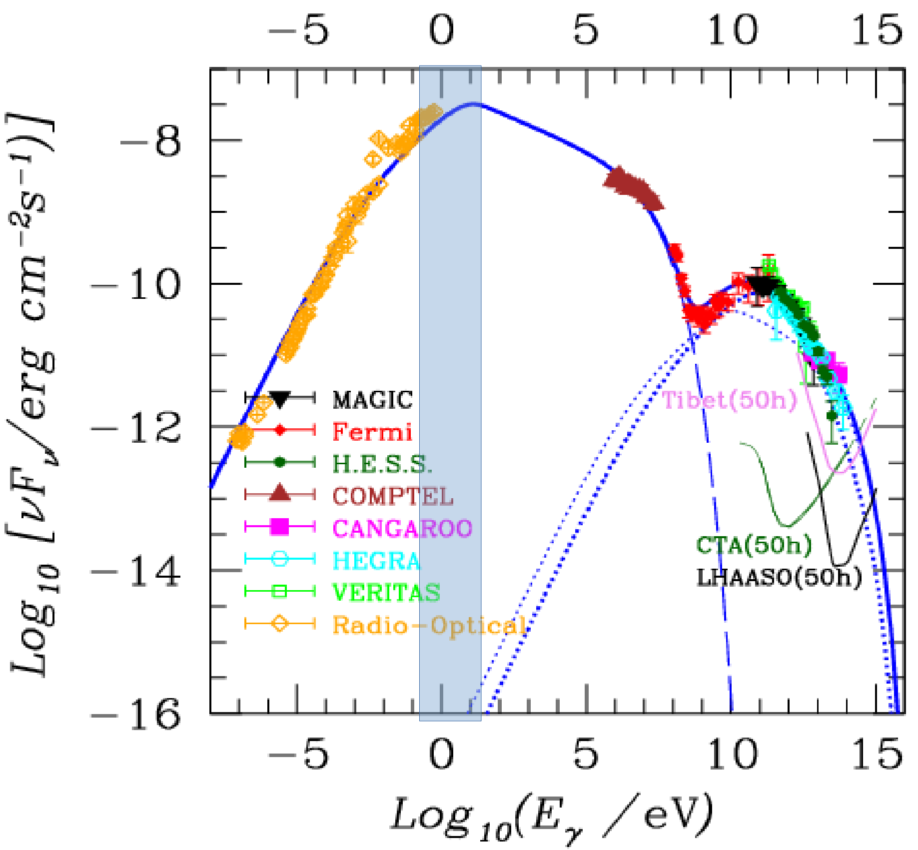
Where is the "visible" here?

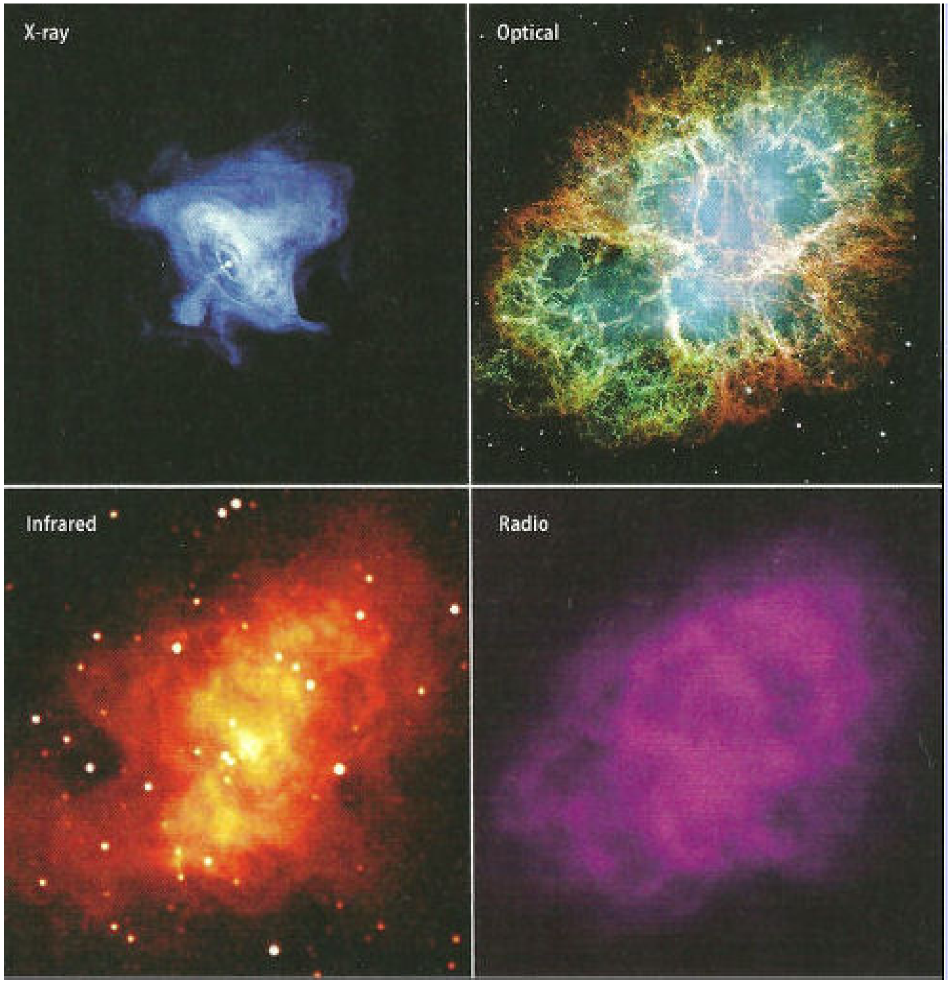


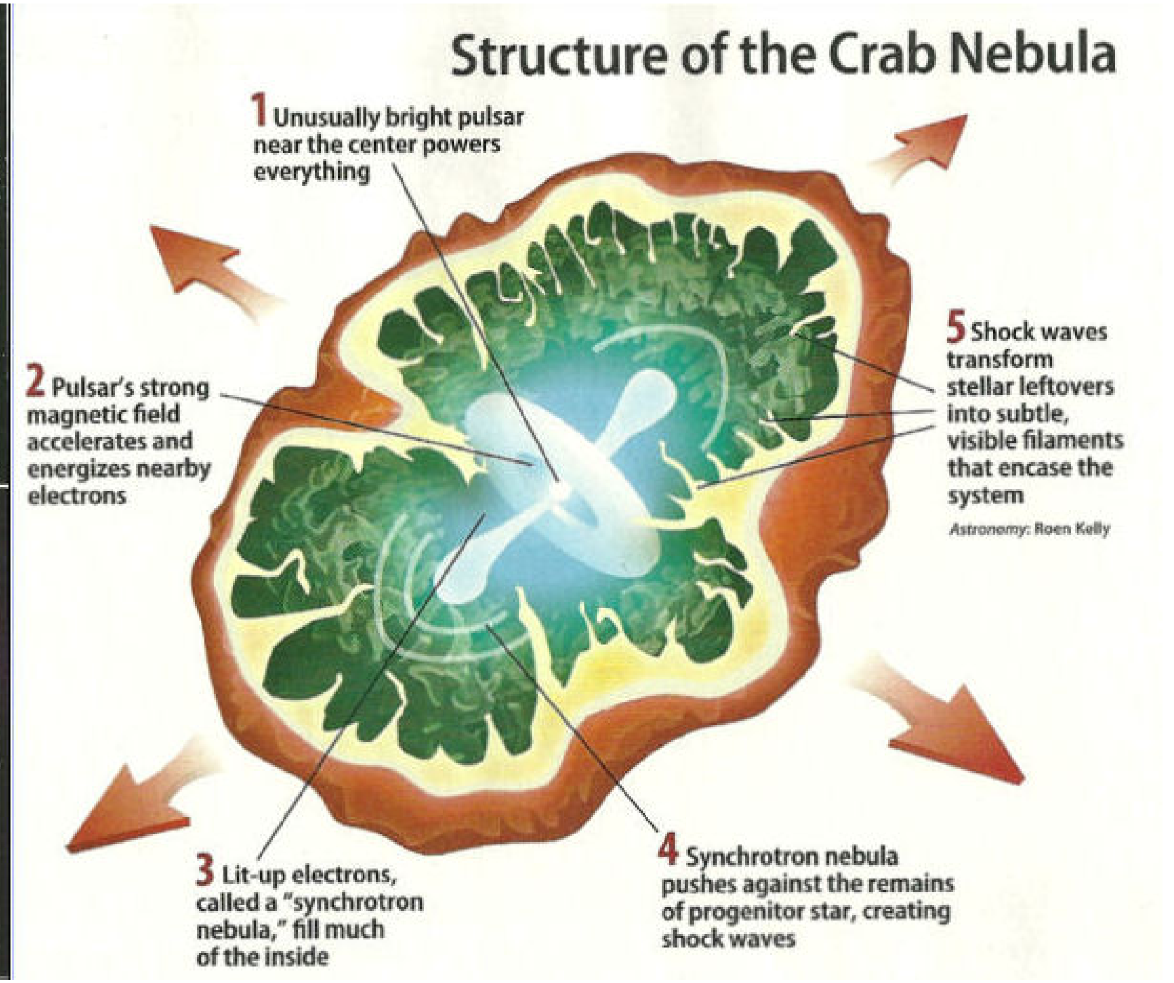


Very easily visible
Mag. ~ 7.5

You can “see” some synchrotron here!




X-ray
Optical
Infrared
Radio


Structure of the Crab Nebula
1 Unusually bright pulsar near the center powers everything
2 Pulsar's strong magnetic field accelerates and energizes nearby electrons
3 Lit-up electrons, called a "synchrotron nebula," fill much of the inside
4 Synchrotron nebula pushes against the remains of progenitor star, creating shock waves
5 Shock waves transform stellar leftovers into subtle, visible filaments that encase the system
Astronomy: Roen Kelly

Summary of Radiation Properties

	Thermal	**Blackbody**	**Bremsstrahlung**	**Synchrotron**	**Inverse Compton**
Optically thick	–	YES	NO	–	
Maxwellian distribution of velocities	YES	YES	–	NO	
Relativistic speeds	–	–	–	YES	
Main Properties	Matter in thermal equilibrium	Matter AND radiation in thermal equilibrium	Radiation emitted by accelerating particles	Radiation emitted by accelerated particles in **B** field.	

Rules of thumb:

1. Blackbody is always thermal, but thermal radiation is not always blackbody (e.g., thermal Bremsstrahlung)
2. Bremsstrahlung can be thermal or non-thermal.
3. Bremsstrahlung becomes blackbody when optical depth >>1.
4. Synchrotron emission exists only for non-thermal distribution of particles