Clarification on the Meaning of the Wien Bump in Comptonization

Notes on Lecture 9

As we have seen before, when radiation and matter are in thermal equilibrium then the emerging spectrum is that of a blackbody with the specific intensity given by the Planck function:

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}. \tag{1}$$

This function comes from the fact that photons are bosons, so the right statistical distribution to use is the Bose–Einstein distribution (as opposed to the Fermi-Dirac distribution for fermions, like electrons). The Bose–Einstein distribution has the form:

$$f(E) = \frac{1}{\exp\left(\frac{h\nu-\mu}{k_B T}\right) - 1} \tag{2}$$

where $E = h\nu$ is the energy of the photon and μ is the chemical potential defined as $\mu = \left(\frac{\partial U}{\partial N_i}\right)_{S,V,N_{j\neq i}}$. Here U is the internal energy, N the number of particles of the species i and S and V are the entropy and volume of the system. Remember that the chemical potential has dimensions of energy. Indeed it is the energy that can be absorbed or released due to a change of the particle number of the given species. When radiation is in thermal equilibrium, the number and energy density of radiation depend only on the temperature and the chemical potential is zero. If we take an enclosure at temperature T that has reached thermal equilibrium, and we perturb the radiation field, for example by adding new photons (i.e., adding new energy), then the number of photons adjust to a new equilibrium condition until the radiation density and photon number density depend only on temperature, this time with $T' > T$. This happens because photons are not conserved and therefore if the number and energy density of radiation are perturbed, the number of photons can adjust to the new condition by being destroyed and/or created (i.e., absorbed/emitted) by the medium. However, if we change the enclosure in such a way so that the photons cannot be absorbed by the medium then there is a mismatch between the number density of photons and the energy density of the radiation field. In this case the energy distribution of photons still follows the Bose-Einstein distribution (they are still bosons of course) but the chemical potential μ is not zero.

When does this happen? Suppose to have a cloud of electrons with a thermal distribution at temperature T and let's say that the cloud is optically thick, or, better, that the Compton y parameter is very large. For a large Comptonization parameter $y \gg 1$, Compton scattering becomes a dominant process, in the sense that almost all photons are scattered multiple times before escaping the medium and reaching the observer. Do we expect to see a blackbody in this case?

The answer is no because the photons here are *scattered* so that their number is *conserved*. In other words, there is no way for the radiation to reach thermal equilibrium because there is no absorption/emission process associated, i.e., photons cannot be created or destroyed. So the photons will follow the Bose-Einstein distribution with a non-zero chemical potential. When $\mu \gg 1$, i.e., when there is a strong deviation of the radiation field from thermal equilibrium, then the specific intensity can be approximated as:

$$I_\nu = \frac{2h\nu^3}{c^2} \exp(-\mu) \exp\left(\frac{-h\nu}{k_B T}\right). \quad (3)$$

so that the intensity of radiation closely follows the Wien exponential drop seen in black-body radiation at high frequencies ($\nu^3 \exp(-h\nu/k_B T)$), with the reduction of the intensity by a factor $\exp(-\mu)$. This is the so-called Wien bump in the saturated Comptonization spectrum.