Clarification on the meaning of $W(k, x)$

Today there has been a request for a clarification on what is the meaning of the *bulk energy* of nuclear matter $W(x, k)$.

First of all, we said in Lecture 7 that the liquid drop model used for example by Harrison & Wheeler (see S&T §2.6) is not realistic when considering nuclei in the inner crust. The reason of this follows from the existence of free neutrons above the neutron drip point. Free neutrons provide an extra source of pressure whose effect is to "compress" the nuclei that form the crystal lattice of the inner crust. The effect of free neutrons can also be thought as a modification of the surface tension of the nuclei. As an analogy, consider a drop of water immersed in the atmosphere. The water has a certain surface tension, which resists to a compression of an external force (for example the gravity pull exerted by an insect, like a "water strider" on the surface of a lake). Of course if you now place the same drop of water in a denser gas than the atmosphere, the surface tension is reduced, because the pressure of the gas is higher. As a limit, when the water drop is immersed in water, the surface tension disappears because now the surface tension equals the force that other water elements exert on the water drop. For nuclei the situation is the very similar: nuclei feel the presence of free neutrons which now exert a pressure on their surface, decreasing the surface tension of nuclei. Therefore the semiempirical mass formula used by Harrison & Wheeler needs to be modified to take into account this effect.

Now, the incompressible liquid drop model used by Harrison & Wheeler (see Eq. 2.6.4 in S&T) contained four terms: volume, surface, coulomb and symmetry term. How are these terms modified in the compressible liquid drop model ? The Coulomb term basically remains the same (W_C), with the difference that Baym, Bethe and Pethick included also the lattice energy (W_L, similarly to what done by Baym, Pethick and Suterland, see §2.7 in S&T).

The surface energy (W_S), is instead modified with respect to the incompressible liquid drop model. In the latter model, the surface energy was just proportional to $A^{2/3}$, where A is the atomic mass. Therefore the surface energy never vanishes. In the *compressible* liquid drop model instead, W_S is build explicitly to vanish when the density of the free neutron gas equals the nucleon density. Recall the analogy with water: if the water drop is immersed in an equal density material (e.g., other water) the surface tension vanishes.

Finally, there is the volume term that in the incompressible liquid drop module was proportional to A. For the compressible liquid drop model, we cannot just use the same expression proportional to A for the simple fact that the volume of the nucleus itself depends on the external pressure of the neutron gas. In other words, the volume of the nucleus can be squeezed or decompressed if the neutron gas pressure increases or decreases, respectively. Remember that the nucleons in nuclei interact via nuclear force. This force depends on the relative nucleon-nucleon distance (think at the simple Yukawa potential, which has a repulsive core and an attractive force, depending on the relative distance). Therefore if you compress a nucleus, the energy of each nucleon will change *even if in this case you are not changing the atomic mass number A*. Therefore the volume term in the compressible liquid drop model **cannot** be simply proportional to A. We call this "volume energy" the "bulk energy" $W(k, x)$ (the term "bulk" is more general than the term "volume" in a sense that will become clear in the lines below). This energy now depends on the relative number of protons and neutrons $x = \frac{Z}{A}$, and on the "momentum" k (we use $k = p/\hbar$ as the "momentum" instead of p for convenience) of the nucleons. The dependence on the momentum is a consequence of the Heisenberg principle (see Lecture 1). If you squeeze N particles in a volume V_N, the particles must move with momentum k. The dependence on x instead is a bit more subtle, but intuitively you can remember the concept of *symmetry* energy in the incompressible liquid drop model, which, as you have noticed, has disappeared as a separate term in the compressible liquid drop model (so far we used surface term W_S, Coulomb and lattice term W_{C+L} and "bulk" energy $W(k,x)$. In reality the symmetry energy is indeed included in $W(k,x)$ and gives the dependence of W on x.

The summarize: the most important property of $W(k,x)$ is therefore is that it **includes** the nucleon-nucleon interaction energy. In the laboratory, it has been possible to measure several properties of the bulk energy $W(k,x)$. Therefore, when you build an EoS and you choose a specific nucleon-nucleon interaction, you do not have the freedom of choosing any interaction whatsoever, because your model has to reproduce the measured experimental value of $W(k,x)$. In the lab, it is found that $W(k,x)$ has a minimum when $x = 1/2$, i.e., for symmetric nuclear matter. This energy is $W_V = -16$ MeV. Then if you take symmetric matter and you compress it, you need to introduce the *compression modulus* K, which is proportional to a function of the momentum k of the nucleons (because from the Heisenberg principle, if you squeeze a particle in a smaller volume it increases its momentum, and therefore its energy). How much energy do we need to provide to a nucleon such that it is significantly compressed in the nucleus ? It is $K = 240$ MeV for *symmetric nuclear matter*. If you change x then K will change accordingly. Finally, if we change the proton to neutron ratio x, we will increase W because of the symmetry energy effect. This effect is described by the term S_V.