Collapsing Cloud

Consider a sphere of ionized hydrogen plasma that is undergoing spherical gravitational collapse. The sphere is held at constant isothermal temperature T_0, uniform density and constant mass M_0 during the collapse, and has decreasing radius $R(t)$. The sphere cools by emission of bremsstrahlung radiation in its interior. At $t = t_0$ the sphere is optically thin.

(a) What is the total luminosity of the sphere as a function of M_0, $R(t)$ and T_0 while the sphere is optically thin?

(b) What is the luminosity of the sphere as a function of time after it becomes optically thick?

(c) Give an implicit relation, in terms of $R(t)$, for the time t_1 when the sphere becomes optically thick.

Collapsing sphere emitting bremsstrahlung radiation.

(a) To calculate the luminosity in the optically thin regime we can use the emission per unit volume of bremsstrahlung, given by,

$$\varepsilon^{ff} = 1.4 \times 10^{-27} T^{1/2} n_e n_i Z^2 \bar{g}_B \text{ erg s}^{-1} \text{ cm}^{-3}.$$

By multiplying this by the volume we can have the luminosity. Since we have a hydrogen plasma, we have $Z = 1$ and as usual we can use $\bar{g}_B = 1.2$. We also know that $n_e = n_i = M_0/m_p V$. Getting now everything in terms of T_0, M_0 and $R(t)$ we have,

$$L_{\text{thin}} = 1.6 \times 10^{20} M_0^2 T_0^{1/2} R^{-3}.$$

(b) In the optically thick case we know that the sphere will radiate as a black-body, then $L = 4\pi R^2 \sigma T^4$ or,

$$L_{\text{thick}} = 7.1 \times 10^{-4} T_0^4 R^2.$$

(c) We know that for large values of R we will have the optically thin case, while for small R we are in the optically thick regime. Considering this, at values of R for which both luminosities are approximately the same, will correspond roughly with the radius at which the transition occurs. Then,

$$\begin{aligned} 1.6 \times 10^{20} M_0^2 T_0^{1/2} R^{-3} &\approx 7.1 \times 10^{-4} T_0^4 R^2, \\ R^5 &\approx 2.3 \times 10^{23} M_0^2 T_0^{-7/2}, \\ R(t_1) &\approx 4.7 \times 10^4 M_0^{2/5} T_0^{-7/10}. \end{aligned}$$