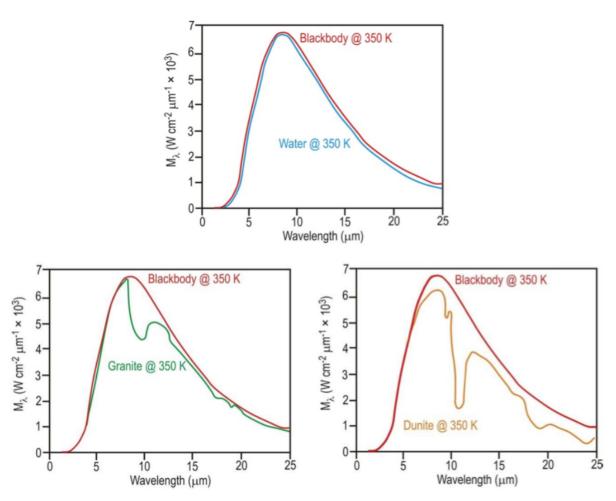


Special Relativity & Radiative Processes

Clarification on Kirchhoff's Law



$$j_{\nu} = \alpha_{\nu} B_{\nu}(T)$$

A good absorber is a good emitter A good emitter is a good absorber

It was proved for objects in fully thermodynamic equilibrium but it is applicable to any object in thermal equilibrium.

It is NOT applicable for non-thermal emitters (e.g., synchrotron, shocks, nuclear explosions)

Special Relativity

Special Relativity is a theory describing the motion of particles and fields at any speed. It is based on two principles:

- 1. All inertial frames are equivalent for all experiments i.e. no experiment can measure absolute velocity.
- 2. Maxwell's equations and the speed of light must be the same for all observers.

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Galilean Relativity

The laws of motion are the same in all inertial frames.

Applies to all inertial and non-inertial frames at *low speeds*.

Special Relativity

The laws of motion are the same in all **inertial** frames.

Applies to all inertial and non-inertial frames.

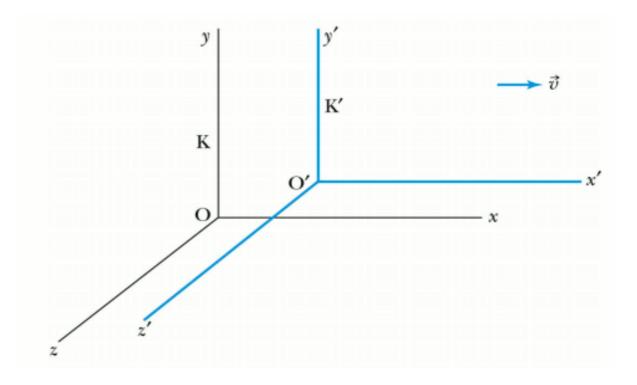
General Relativity

Applies to all inertial and non-inertial frames + gravitational fields.

Lorentz Transformations

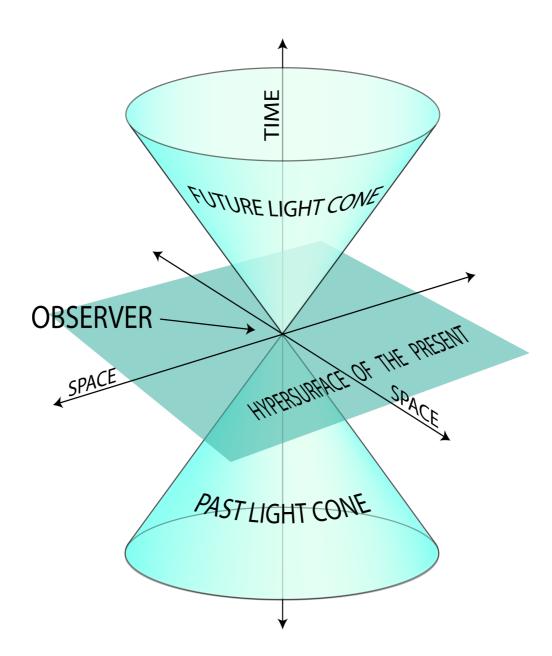
$$\gamma \equiv \frac{1}{\sqrt{1-v^2/c^2}}$$
 $x = \gamma (x'+vt')$
 $y = y'$
 $z = z'$
 $t = \gamma (t'+vx'/c^2)$

$$t' = \gamma \left(t - \frac{vx}{c^2} \right)$$
$$x' = \gamma \left(x - vt \right)$$
$$y' = y$$
$$z' = z$$

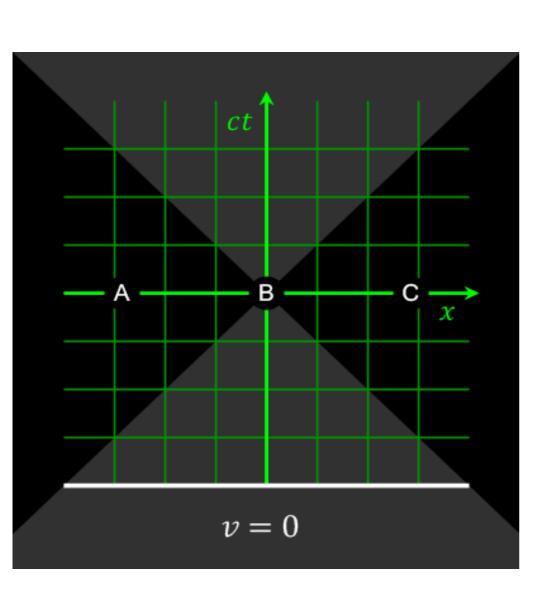


Lorentz Transformations: Both space and time are subject to transformation. The description of events occurring at a certain location in space and time depends on the particular reference frame of choice.

Light Cones



Relativity of Simultaneity

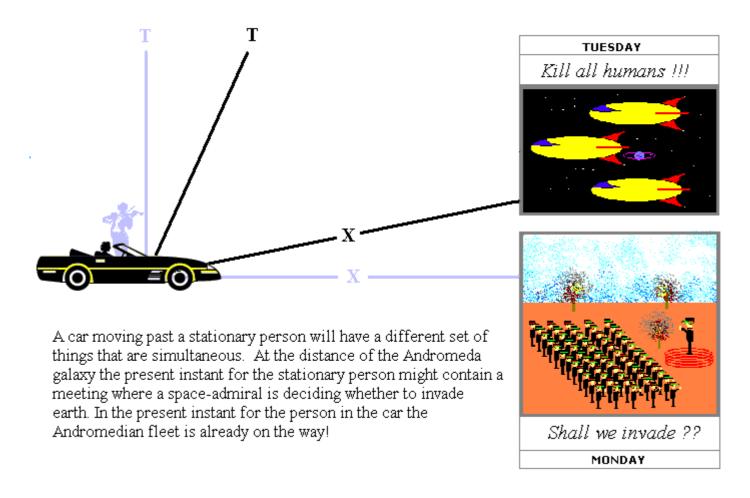


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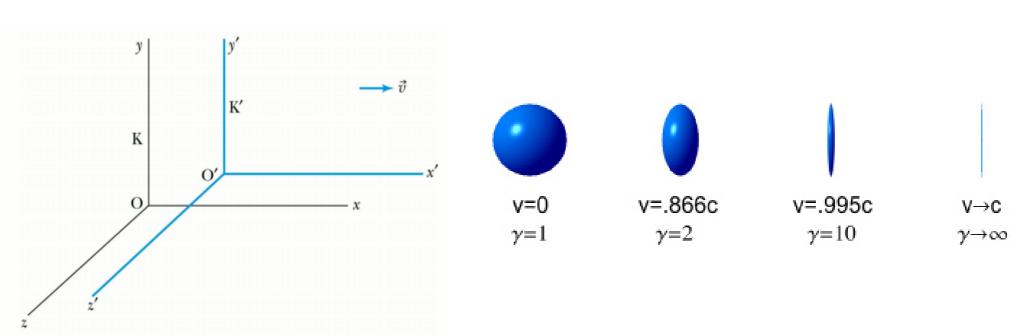
The Andromeda Paradox

The Andromeda Paradox



Formulated first by R. Penrose to illustrate the apparent paradox of relativity of simultaneity

Ruler: Measuring a bar



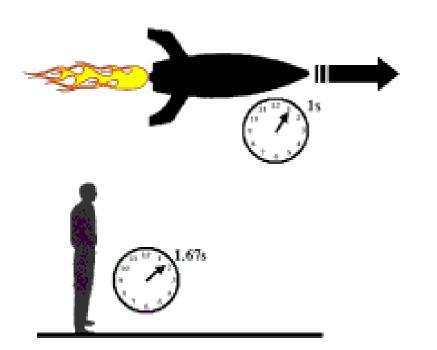
Lorentz-Fitzgerald Contraction

$$L = L' \sqrt{1 - \frac{v^2}{c^2}} = \frac{L'}{\Gamma}$$

L' = length of object in K' L = length of object in K Note: the two observers in K and K' would measure the same effect with respect to each other. How is that possible?

Solution: Lorentz transformation of time is **NOT** Lorentz invariant since it depends also on space. Therefore temporal simultaneity is **NOT** Lorentz invariant. Therefore each observer does not see the other carrying the measurement of the two ends of the stick at the *same time*.

Clocks: Time Intervals



Time dilation effect

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}} = \Delta t' \Gamma$$

Time in the lab frame flows faster than in the moving frame.

Same story here: both observers will see the each other's clock slowing down. Each would object that the clocks used by the other to measure the time interval were not synchronized.

Observability of Lorentz contraction and Time dilation

Question for you: Lorentz contraction and time dilation assume that you are carrying your measurements with rods and clocks, i.e., you can carry the measurement "in place".

But what happens when you use photons?

This is the situation we encounter in astronomy, basically all information is carried by photons and we make measurements by collecting photons on a detector (either by taking a picture or by recording the photons' time of arrivals).

Observability of Lorentz contraction and Time dilation

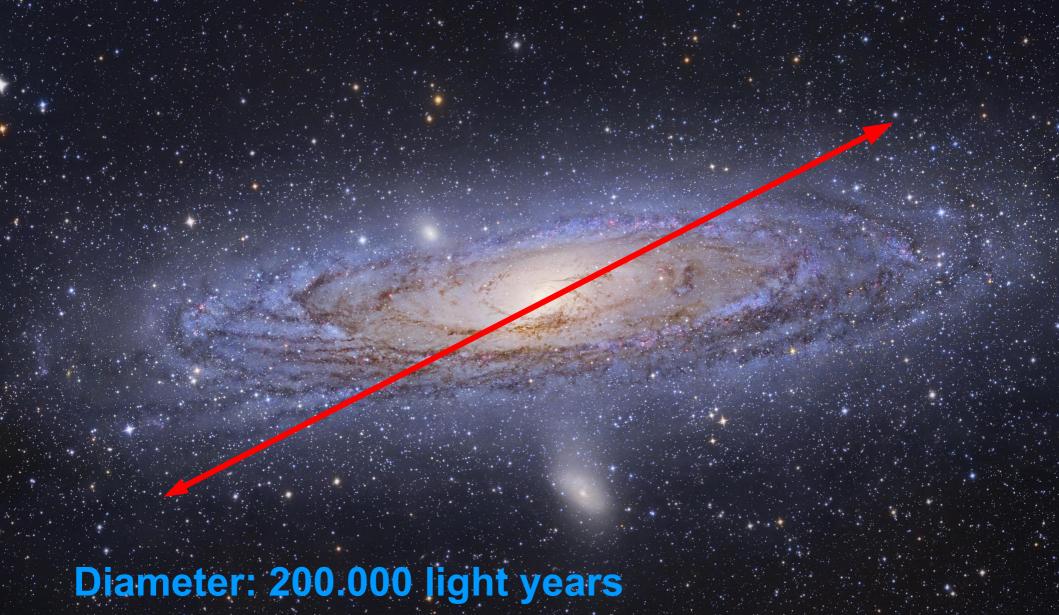
Question for you: Lorentz contraction and time dilation assume that you are carrying your measurements with rods and clocks, i.e., you can carry the measurement "in place".

But what happens when you use photons?

This is the situation we encounter in astronomy, basically all information is carried by photons and we make measurements by collecting photons on a detector (either by taking a picture or by recording the photons' time of arrivals).

We will see now that the fact that we use photons instead of rulers and clocks "in place" changes completely the effect observed. This doesn't mean of course that the Lorentz contraction and/or time dilation do not occur. It means simply that the finite propagation speed of light introduces distortions in the effect measured when using photons.





This picture does not represent an "instant" of the Andromeda Galaxy. Indeed the photons you're recording were emitted with up to 200,000 years difference.

Diameter: 200.000 light years

What you're seeing are photons arriving at the same time, but NOT emitted at the same time.

Invisibility of the Lorentz Contraction*

JAMES TERRELL Los Alamas Scientific Laboratory, University of California, Los Alamas, New Mexico (Received June 22, 1959)

It is shown that, if the apparent directions of objects are plotted as points on a sphere surrounding the observer, the Lorentz transformation corresponds to a conformal transformation on the surface of this sphere. Thus, for sufficiently small subtended solid angle, an object will appear-optically-the same shape to all observers. A sphere will photograph with precisely the same circular outline whether stationary or in motion with respect to the camera. An object of less symmetry than a sphere, such as a meter stick, will appear, when in rapid motion with respect to an observer, to have undergone rotation, not contraction. The extent of this rotation is given by the aberration angle $(\theta - \theta')$, in which θ is the angle at which the object is seen by the observer and θ' is the angle at which the object would be seen by another observer at the same point stationary with respect to the object. Observers photographing the meter stick simultaneously from the same position will obtain precisely the same picture, except for a change in scale given by the Doppler shift ratio, irrespective of their velocity relative to the meter stick. Even if methods of measuring distance, such as stereoscopic photography, are used, the Lorentz contraction will not be visible, although correction for the finite velocity of light will reveal it to be present.

INTRODUCTION

VER since Einstein presented his special theory of L relativity^t in 1905 there seems to have been a general belief that the Lorentz contraction should be visible to the eye. Indeed, Lorentz stated2 in 1922 that the contraction could be photographed. Similar statements appear in other references too numerous to be mentioned, and even Einstein's first paper leaves the impression, perhaps unintentionally, that the contraction due to relativistic motion should be visible. The usual statement is that moving objects "appear contracted," which is somewhat ambiguous. The special theory predicts that the contraction can be observed by a suitable experiment, and the words "observe" and "see" seem to be used interchangeably in this connection.

There is, however, a clear distinction between observing and seeing. An observation of the shape of a fastmoving object involves simultaneous measurement of the position of a number of points on the object. If done by means of light, all the quanta should leave the surface simultaneously, as determined in the observer's system, but will arrive at the observer's position at different times, Similar restrictions would apply to the use of radar as an observational method. In such observations the data received must be corrected for the finite velocity of light, using measured distances to various points of the moving object. In seeing the object, on the other hand, or photographing it, all the light quanta arrive simultaneously at the eye (or shutter), having departed from the object at various earlier times. Clearly this should make a difference between the contracted shape which is in principle observable and the actual visual appearance of a fast-moving

CONFORMALITY OF ABERRATION

The basic question of the visibility of the Lorentz contraction may be stated as that of the appearance of a rapidly moving object in an instantaneous photograph. The object, of known shape when at rest, is assumed to have a high uniform speed relative to the camera. The camera is assumed to be at rest in a Galilean (unaccelerated) frame of reference. Of course it would make no difference if the camera were, instead, considered to move at high speed past the stationary object, but the photograph produced must be examined at rest, so it is simpler to consider the camera as stationary. The mechanism of the camera must be such as to give it essentially instantaneous shutter speed and sharp focus over the necessary depth of field.

The questions of whether to use photographic film which lies in a plane or is curved so that all points are at the same distance from the lens (or pinhole), and whether to use a lens corrected to eliminate optical distortions, could be troublesome. To simplify matters, it is assumed that the object subtends a visual solid angle sufficiently small that these matters need not be considered. It is assumed that the camera is pointed directly at the apparent position of the object, so that the light rays strike the film in a perpendicular direction, producing an image in the center of the photographic film. The camera is assumed, also for simplicity, J. Terrell: "Invisibility of the Lorentz Contraction"

R. Penrose: "The Apparent Shape of a Relativistically Moving Sphere"

These papers were published in 1959. The effects of the finite speed of light seem maybe obvious to you, but that's how long it took to realize this (special relativity was first published in 1905...).

Actually A. Lampa (Austrian) realized this earlier, in 1924. But for some reasons his work was mostly ignored.

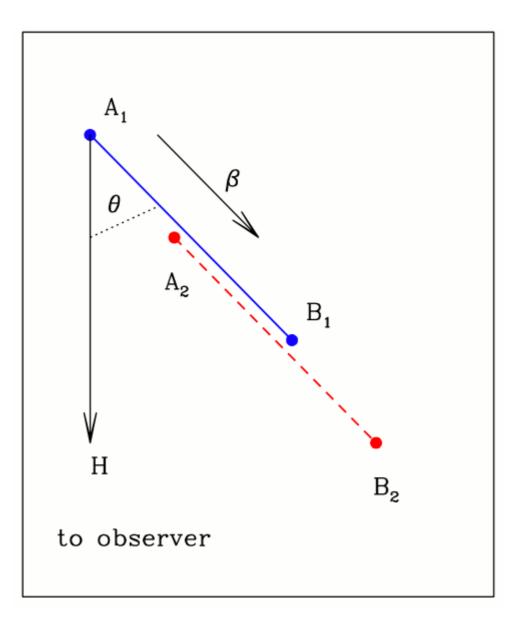
A. Lampa: ""Wie erscheint nach der Relativitätstheorie ein bewegter Stab einem ruhenden Beobachter?"

^{*}This work was supported by the U. S. Atomic Energy

A. Einstein, Ann. Physik 17, 891 (1905).

⁵ H. A. Lorentz, Lechures on Theoretical Physics (Macmillan and Company, Ltd., London, 1931; translated from Dutch edition of 1922), Vol. 3, p. 203.
⁵ In reference 1 [English translation from The Principle of Relativity (Dover Publications, Inc., New York, reprinted from The Principle of Relativity). 1923 Methuen edition) Einstein stated: "A rigid body which, measured in a state of rest, has the form of a sphere, therefore has in a state of motion-viewed [betrachtet] from the stationary system-the form of an ellipsoid of revolution with the axes $R(1-\sigma^2/\sigma^2)^4$, R_cR . Thus, whereas the V and Z dimensions of the sphere (and therefore of every rigid body of no matter what form) do not appear [nicht erscheinen] modified by the motion, the X dimension appears [erscheint] shortened in the natio $1:(1-\vartheta/e^2)^2$, i.e., the greater the value of v, the greater the shortening. For v=c all moving objects—viewed [betrachtet] from the "stationary" system—shrivel up into plane figures."

Photons: Measuring a bar



$$A_1B_1=L$$

If you use a "ruler" you will see $L = \frac{L'}{\Gamma}$

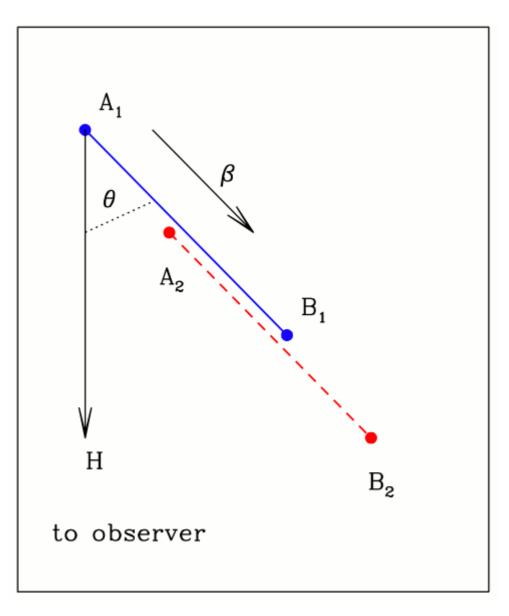
However, if you take a picture (i.e., you use photons) you will see something very different.

Call H the point reached by the photon emitted in A1 after a time Δt , such that the extreme B1 of the rod will be at B2 (i.e., H and B2 have the same distance to the observer).

A photon emitted in H and B2 will thus reach the observer simultaneously (i.e., create the "picture" on the camera). The length A1B1 is thus measured as HB2. Is HB2 = L'/Γ ?

No!

Photons: Measuring a bar



$$A_1B_1=L$$

$$B_1B_2=\beta c \Delta t$$

$$A_1 H = c \Delta t$$

With a bit of algebra one can find:

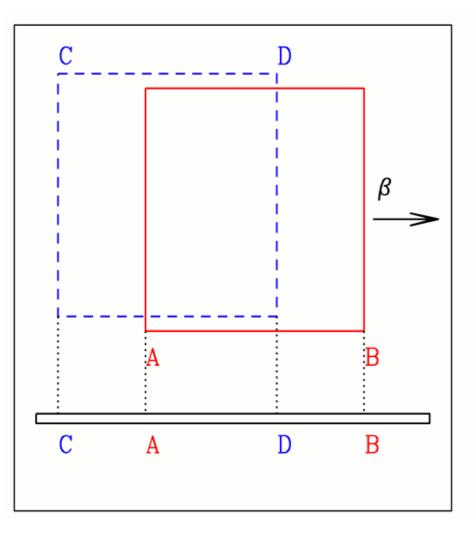
$$A_1 H = A_1 B_2 \cos \theta = \frac{L' \cos \theta}{c \Gamma (1 - \beta \cos \theta)}$$

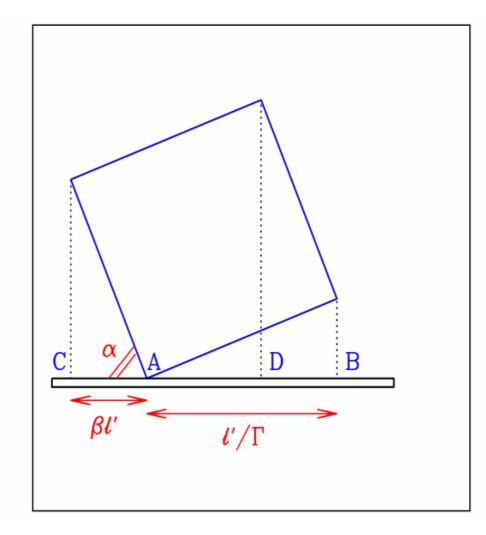
Then:

$$A_1 B_2 = \frac{A_1 H}{\cos \theta} = \frac{L'}{\Gamma(1 - \beta \cos \theta)} = \delta L'$$

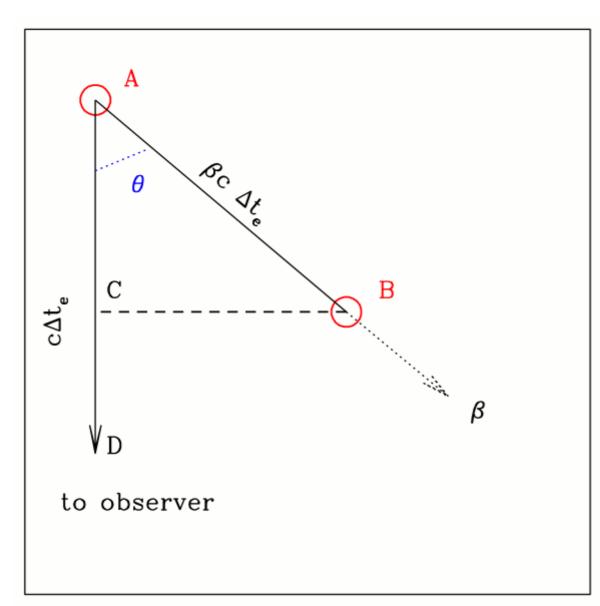
$$\Lambda = H B_2 = A_1 B_2 \sin \theta = L' \delta \sin \theta$$

Note the difference between the ruler and photon case





Here the square represents an object of finite size and extension. In other words any *real* object we can observe. The net effect is a rotation of the object. Not a contraction



We show now that the time dilation effect is completely reversed when you do your measurements with photons instead of using "in place" clocks.

From what we said before we expect a time dilation.

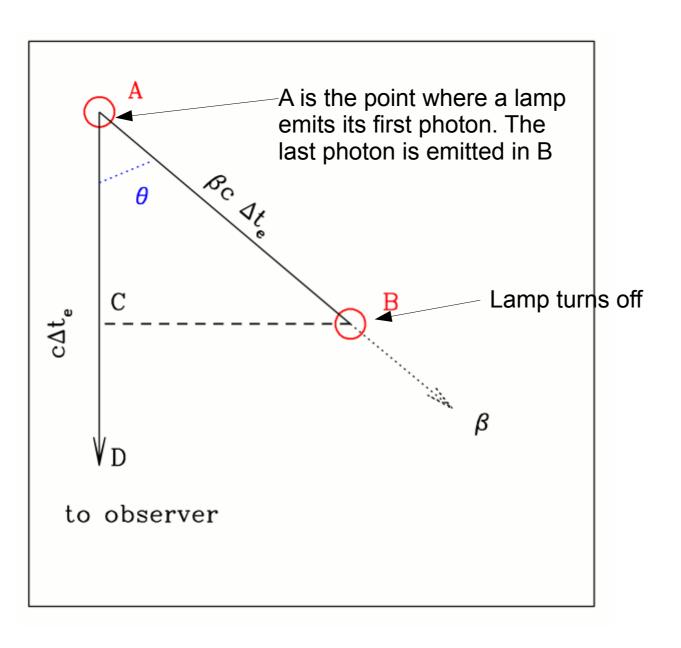
We will show now that, when using photons, we find a time contraction:

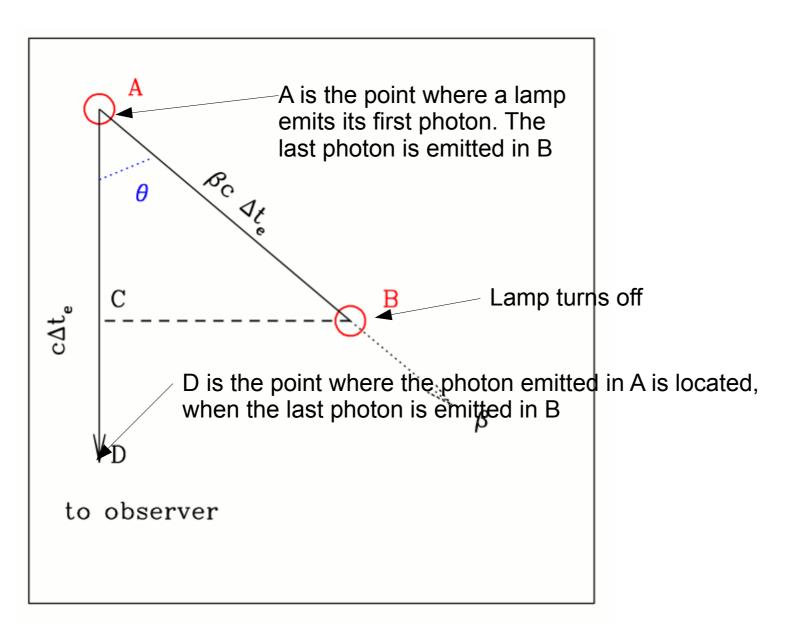
$$\Delta t_a = \frac{\Delta t_e'}{\delta}$$

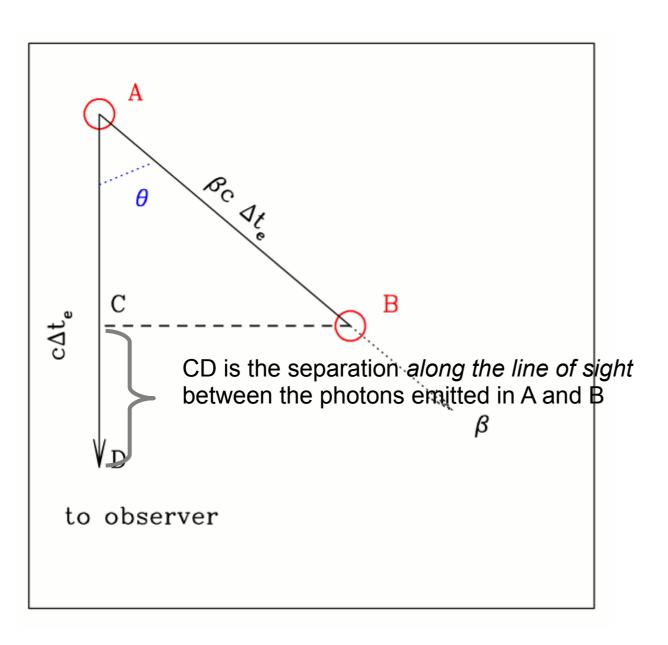
Here the subscript "a" and "e" refer to the "arrival" and emission time. Note that:

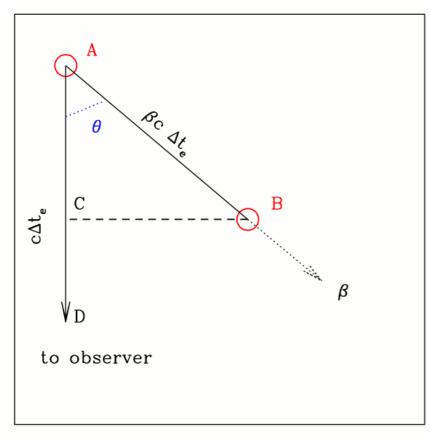
$$\Delta t_e = \Delta t_e' \Gamma$$

is not measurable (with photons) but we measure $\Delta\,t_a$. Again, this is just an effect due to the use of photons and the fact that they propagate with a finite speed









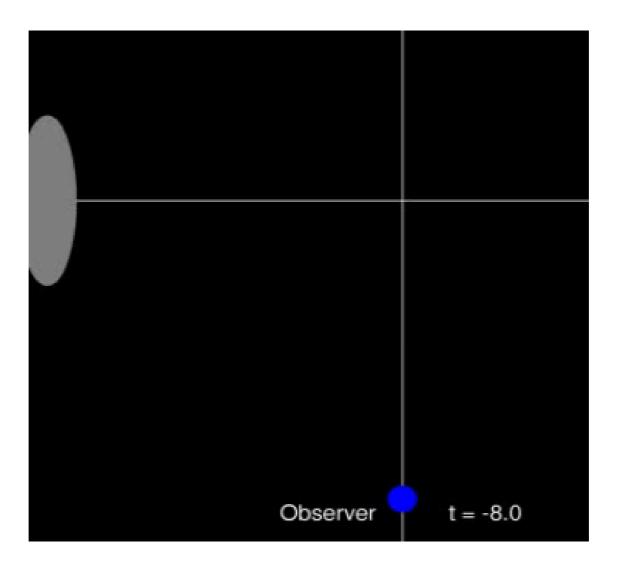
An observer will thus measure the interval CD/c, where c is the speed of light.

$$\Delta t_a = \frac{CD}{c} = \frac{AD - AC}{c} = \Delta t_e - \beta \Delta t_e \cos \theta = \Delta t_e (1 - \beta \cos \theta)$$

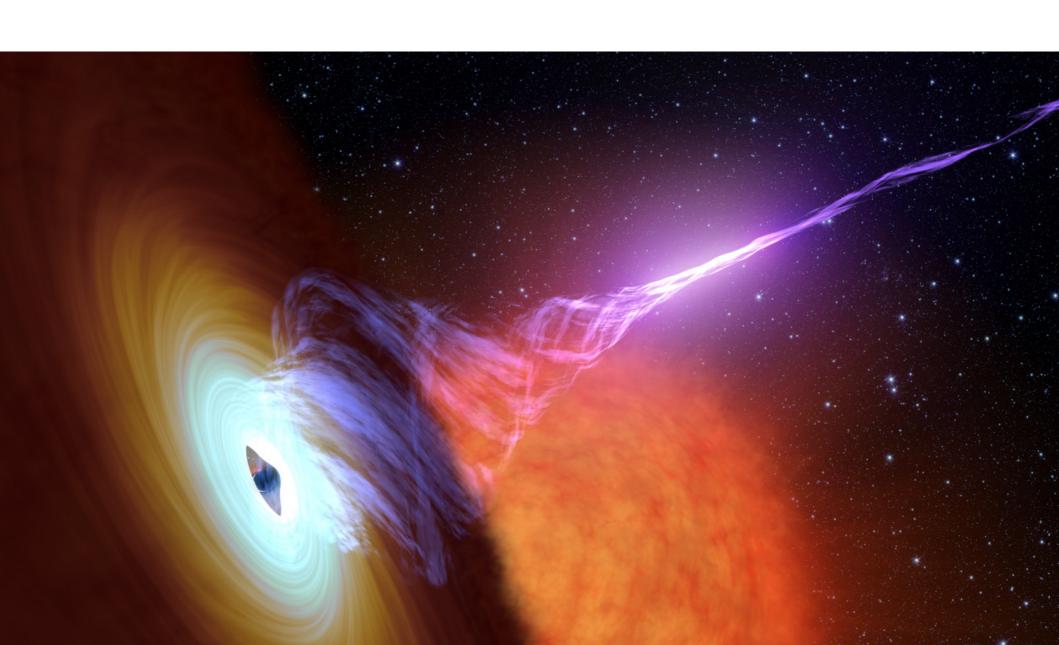
$$\Delta t_e (1 - \beta \cos \theta) = \Gamma \Delta t'_e (1 - \beta \cos \theta) = \frac{\Delta t'_e}{\delta}$$

Remember what delta is:

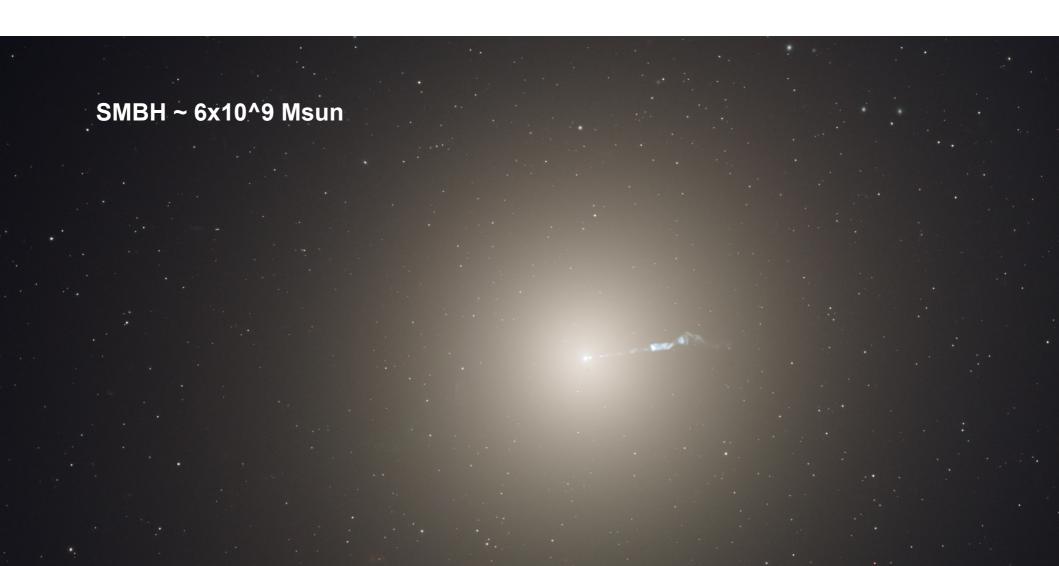
$$\delta = \frac{1}{\Gamma(1 - \beta \cos \theta)}$$



Astrophysical Jets



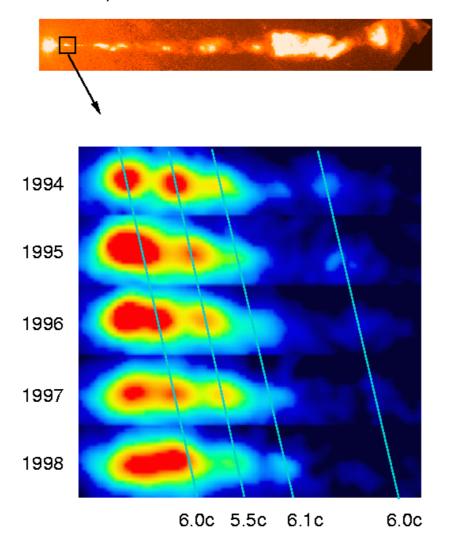
M87



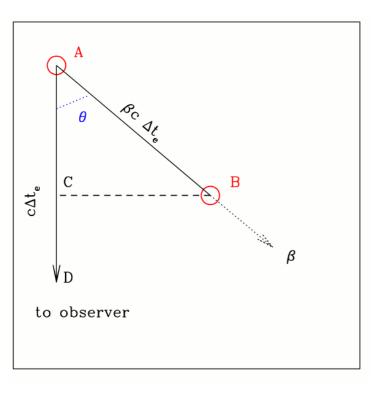
A practical application: Superluminal Motion

Special Relativity states that the speed of light cannot be crossed. So how do you explain the following image?

Superluminal Motion in the M87 Jet



To answer this question look at the exercise 4.7 of the R&L



$$\Delta t_e$$
 Time to move from **A** to **B** (in reference frame K)

B is closer to observer than **A**, therefore:

$$\Delta t_{a} = \Delta t_{e}^{'} \gamma (1 - \beta \cos \theta) = \Delta t_{e} (1 - \beta \cos \theta)$$

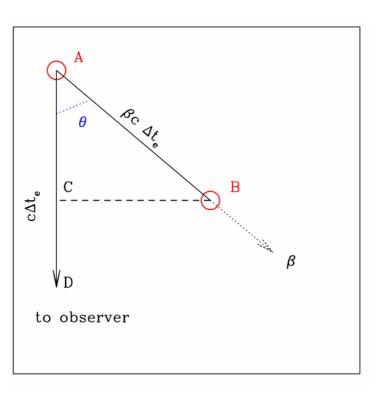
Observer

The displacement $\mathbf{C} \rightarrow \mathbf{B}$ is $v \sin \theta \Delta t_e$

Therefore the apparent velocity must be:

$$v_{app} = \frac{v \sin \theta \Delta t_e}{\Delta t_a} = \frac{v \sin \theta}{1 - \beta \cos \theta}$$

To answer this question look at the exercise 4.7 of the R&L

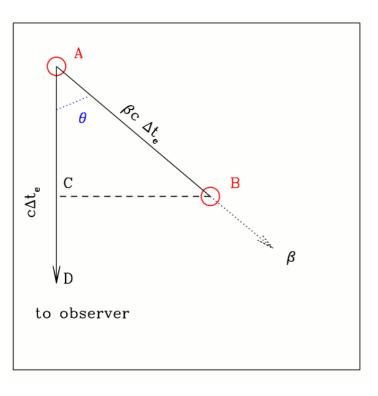


How can we now find the maximum of this apparent velocity?

$$v_{app} = \frac{v \sin \theta \Delta t_e}{\Delta t_a} = \frac{v \sin \theta}{1 - \beta \cos \theta}$$

Observer

To answer this question look at the exercise 4.7 of the R&L



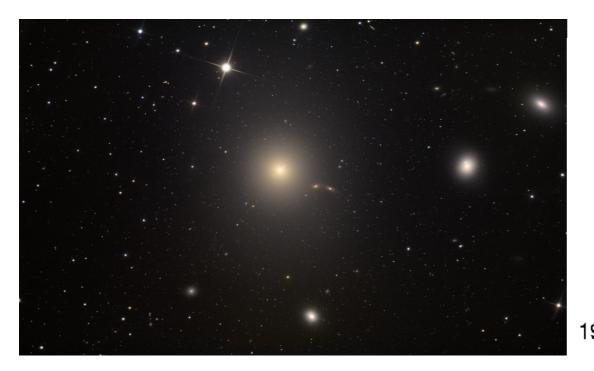
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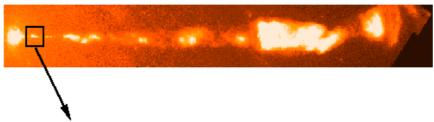
Differentiate (wrt the angle theta) and set the expression to zero:

$$v_{app}^{max} = \frac{v\sqrt{1-\beta^2}}{1-\beta^2} = \gamma v$$

So if the velocity v is large and gamma is >>1 you can easily go to apparent velocities >> c



Superluminal Motion in the M87 Jet



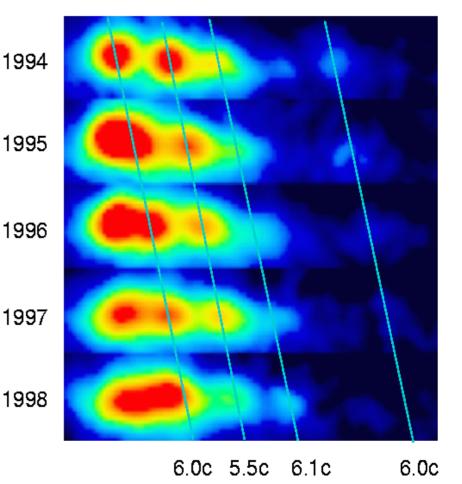
Apparent superluminal motion is a very well known and widespread phenomenon in astronomy!

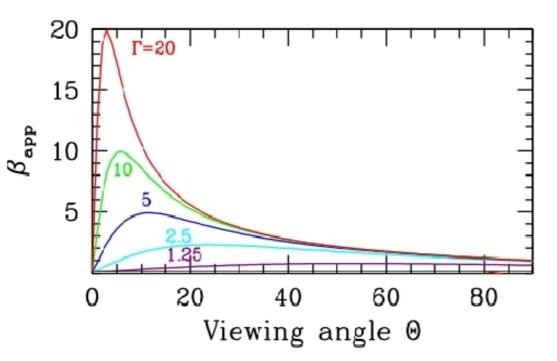
Again, this is an effect due to the way in

which we perform the measurement: we

speed. Nothing is really moving at v>c.

use photons which have a finite propagation



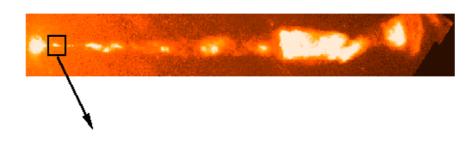


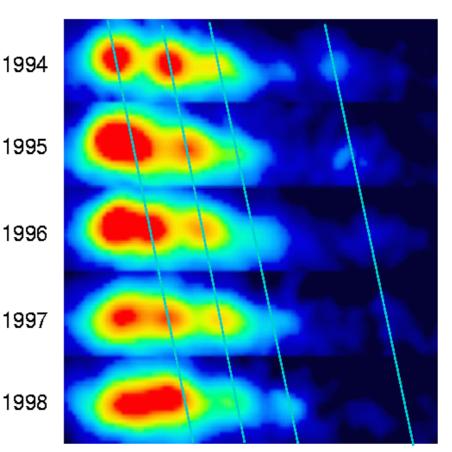
Apparent superluminal motion is a very well known and widespread phenomenon in astronomy!

Example: suppose v~0.99c, which is relativistic, but still relatively far from c.

Then $\Gamma \approx 7$ and you get a "superluminal" motion for basically almost any viewing angle from zero to 60 degrees. (angle zero \rightarrow object moves directly towards you) (angle 90 deg \rightarrow object moves orthogonal to you)

Superluminal Motion in the M87 Jet

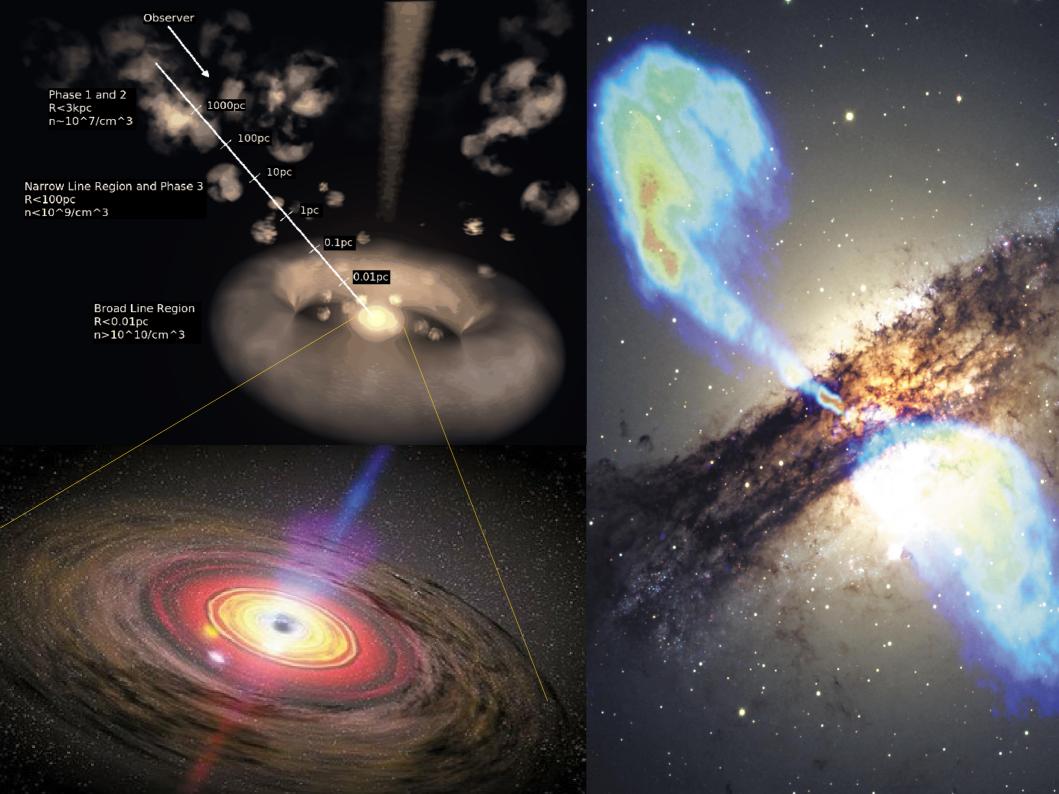




6.0c 5.5c

6.1c

6.0c



Relativistic Doppler Boost

Let's start from this expression we derived before:

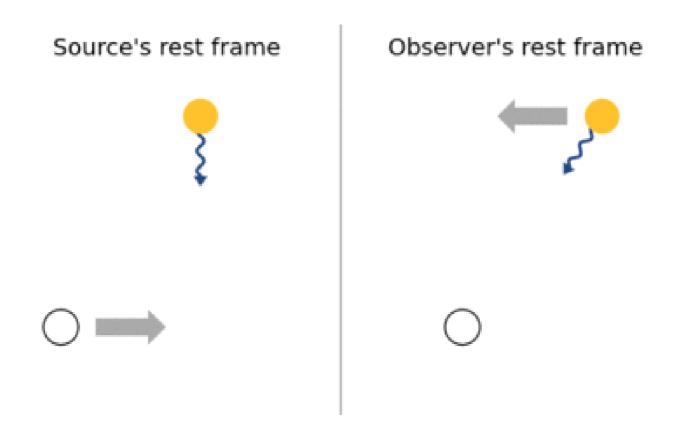
$$\Delta t_a = \Gamma (1 - \beta \cos \theta) \Delta t'_e$$

We know that frequency is the inverse of time so we can write:

$$v = \frac{v'}{\Gamma(1 - \beta \cos \theta)} = v' \delta$$

This is the relativistic Doppler effect, based on the time dilation AND the finite time for light propagation.

Aberration of Light



Lorentz Transformations of Velocities

Call v the velocity of a reference frame K' in K (as usual).

Call **u'** the velocity of an object in K'. What is **u** in K? First let's check the easy case: **v** is along the x-axis.

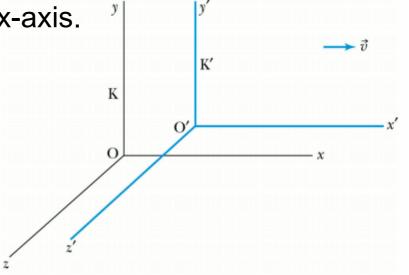
$$dx = \gamma (dx' + v dt'), \quad dy = dy'$$

$$dz = dz',$$
 $dt = \gamma \left(dt' + \frac{v}{c^2} dx' \right).$

$$u_x = \frac{dx}{dt} = \frac{\gamma(dx' + v dt')}{\gamma(dt' + v dx'/c^2)} = \frac{u_x' + v}{1 + v u_x'/c^2},$$

$$u_y = \frac{u_y'}{\gamma (1 + v u_x'/c^2)},$$

$$u_z = \frac{u_z'}{\gamma (1 + v u_x'/c^2)}.$$



There are the velocity transformations when the velocity is on the x-axis direction. What about a more general form?

Take **v** along an arbitrary direction. Take the parallel and perpendicular components of **u** to **v**.

$$u_{\parallel} = \frac{u'_{\parallel} + v}{\left(1 + vu'_{\parallel}/c^2\right)}, \qquad u_{\perp} = \frac{u'_{\perp}}{\gamma\left(1 + vu'_{\parallel}/c^2\right)}$$

$$\tan \theta = \frac{u_{\perp}}{u_{\parallel}} = \frac{u' \sin \theta'}{\gamma (u' \cos \theta' + v)}$$
 Aberration formula

$$\tan \theta = \frac{\sin \theta'}{\gamma(\cos \theta' + v/c)}$$

$$\cos \theta = \frac{\cos \theta' + v/c}{1 + (v/c)\cos \theta'}$$

Aberration of light (u = c)

Lorentz Transformations of Velocities: Beaming Effect

Let's now ask the question of what happens when $\theta' = \pi/2$ i.e. the photon is emitted at right angles to **v** in K'.

Start from the aberration of light formula:

$$\tan \theta = \frac{\sin \theta'}{\gamma(\cos \theta' + v/c)}$$

$$\cos \theta = \frac{\cos \theta' + v/c}{1 + (v/c)\cos \theta'}$$

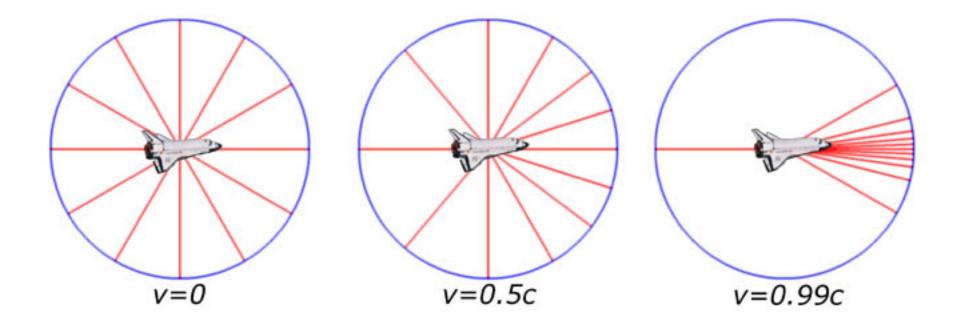
$$\tan \theta = \frac{c}{\gamma v}$$

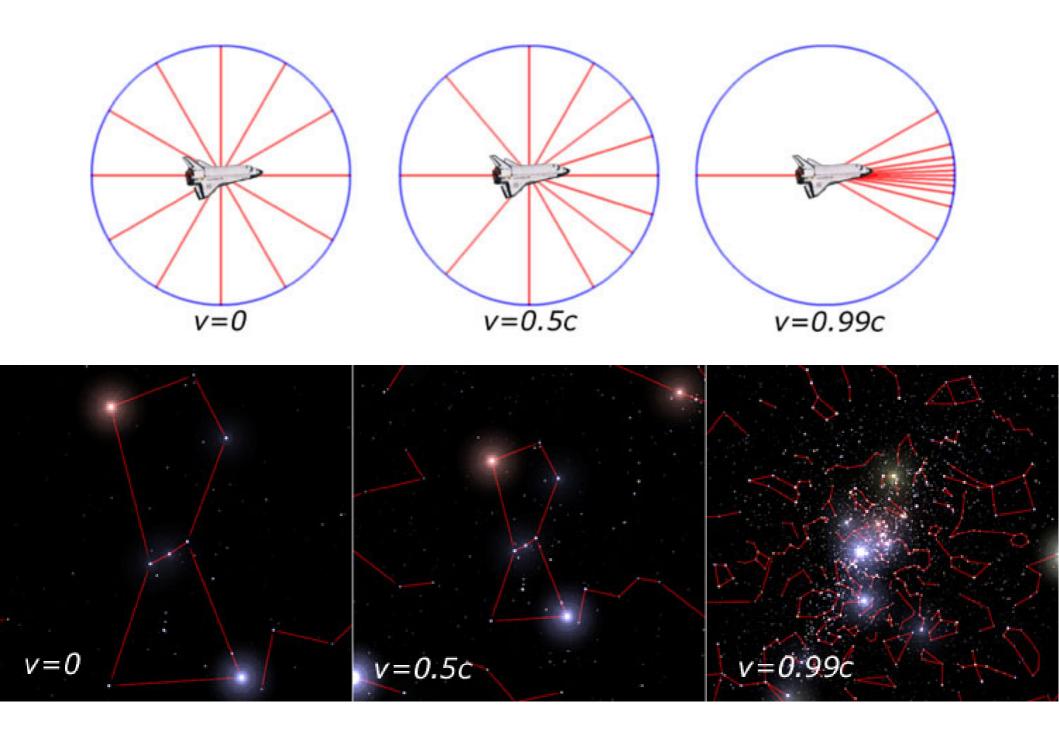
$$\sin \theta = \frac{1}{\gamma}$$

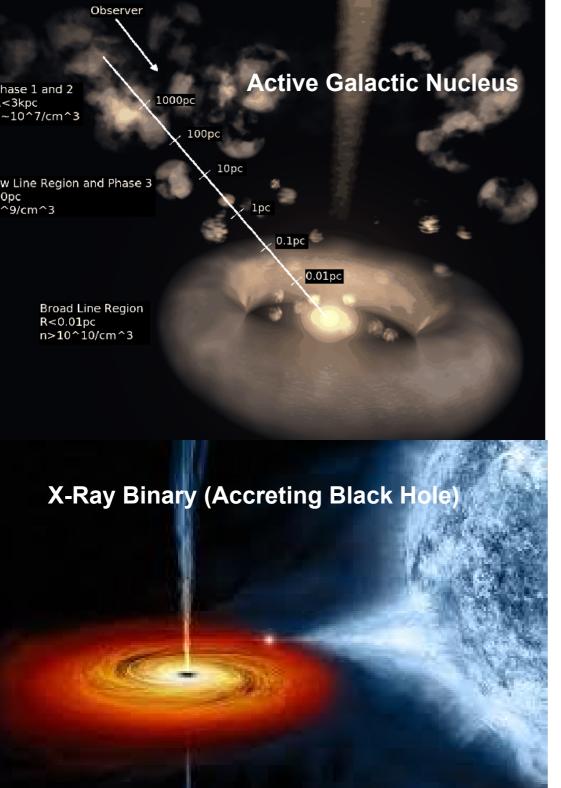
If we are in the highly relativistic regime, v~c (gamma>>1) then theta must be small and so:

$$\theta \approx \frac{1}{\gamma}$$

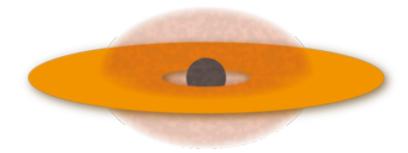
Terefore if photons are emitted isotropically in K' then in K you will see half of them within an angle 1/gamma

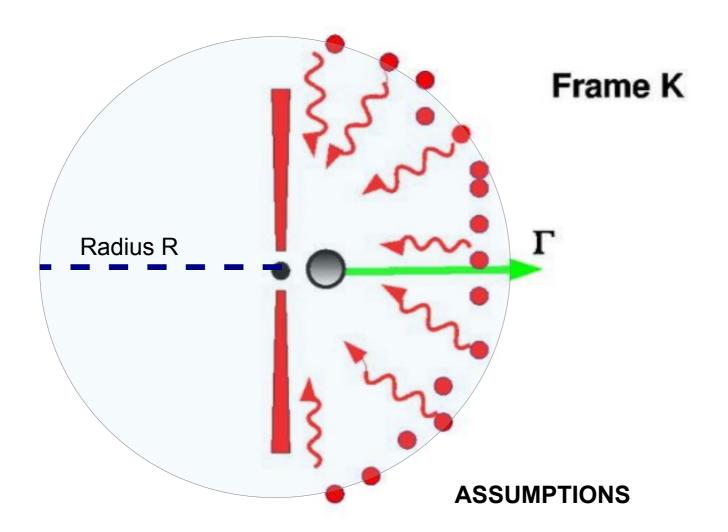




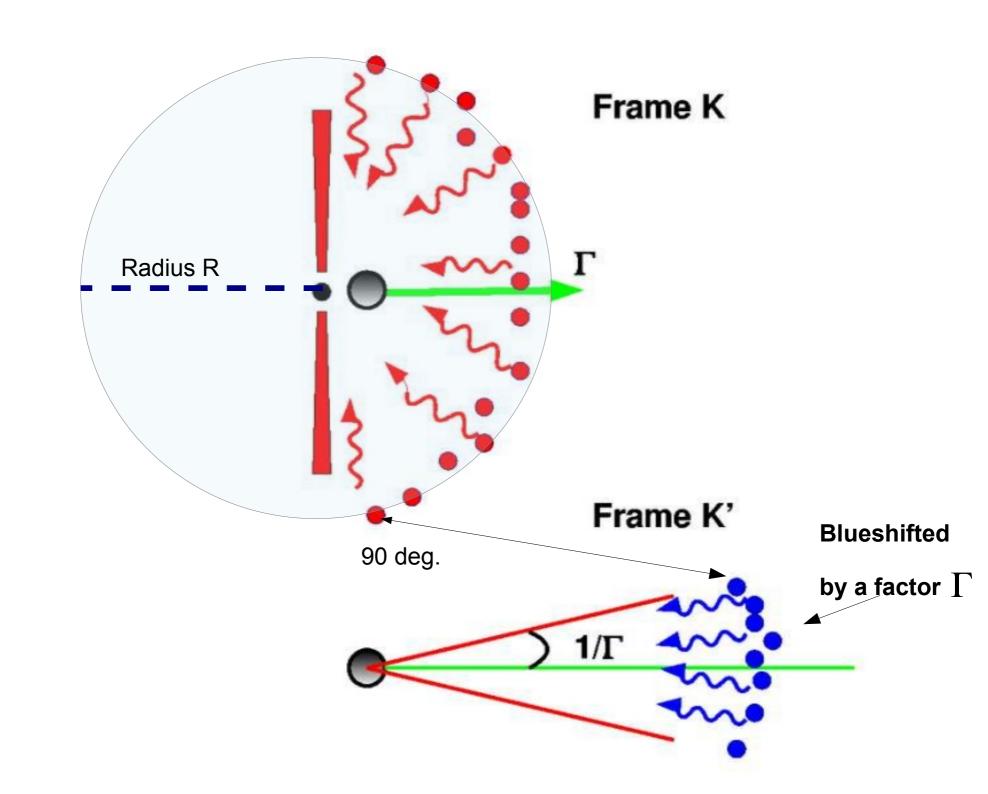


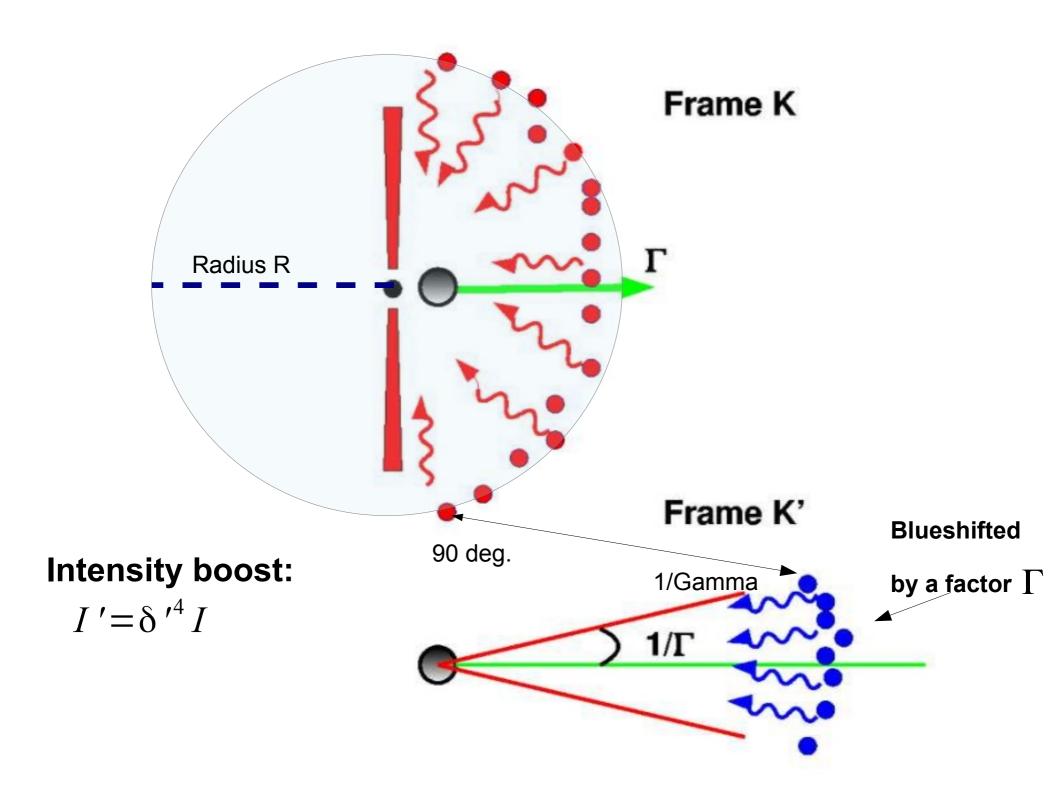
It is instructive to consider a blob of plasma ejected from around a black hole (both supermassive or stellar). In Active Galactic Nuclei, were a supermassive BH is present, the blob is surrounded by clouds of gas emitting radiation (so-called broad line regions). In stellar mass BHs instead, there is a large cloud of hot electrons that produce high energy radiation via inverse Compton scattering.

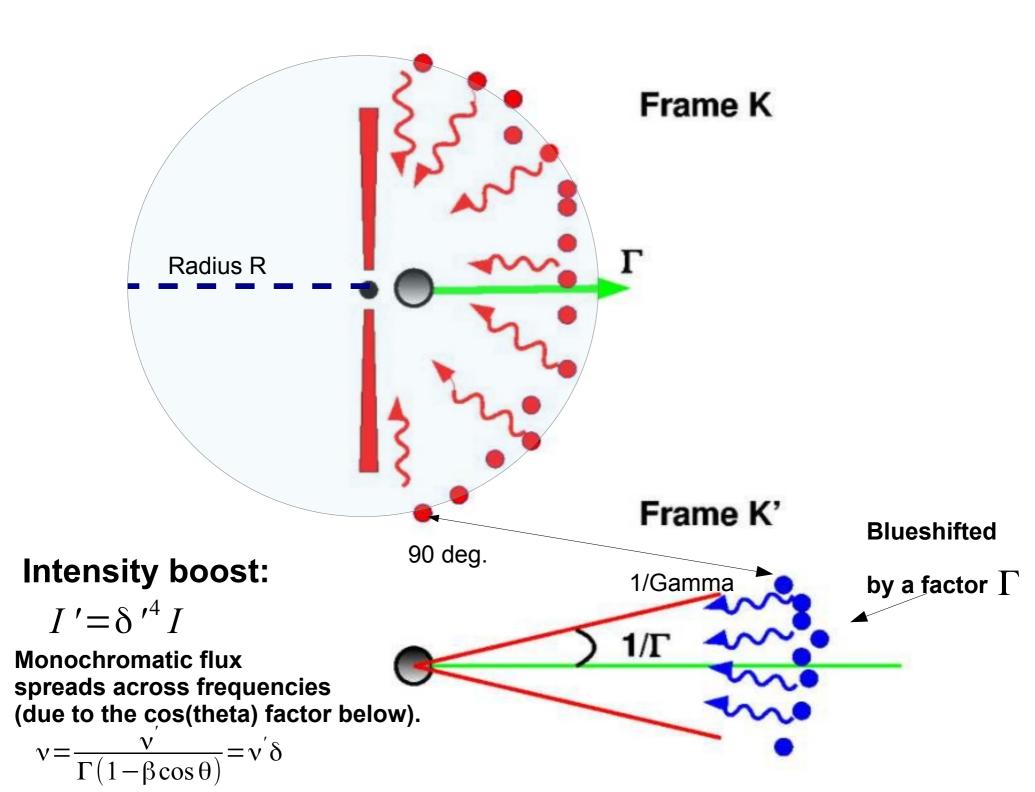


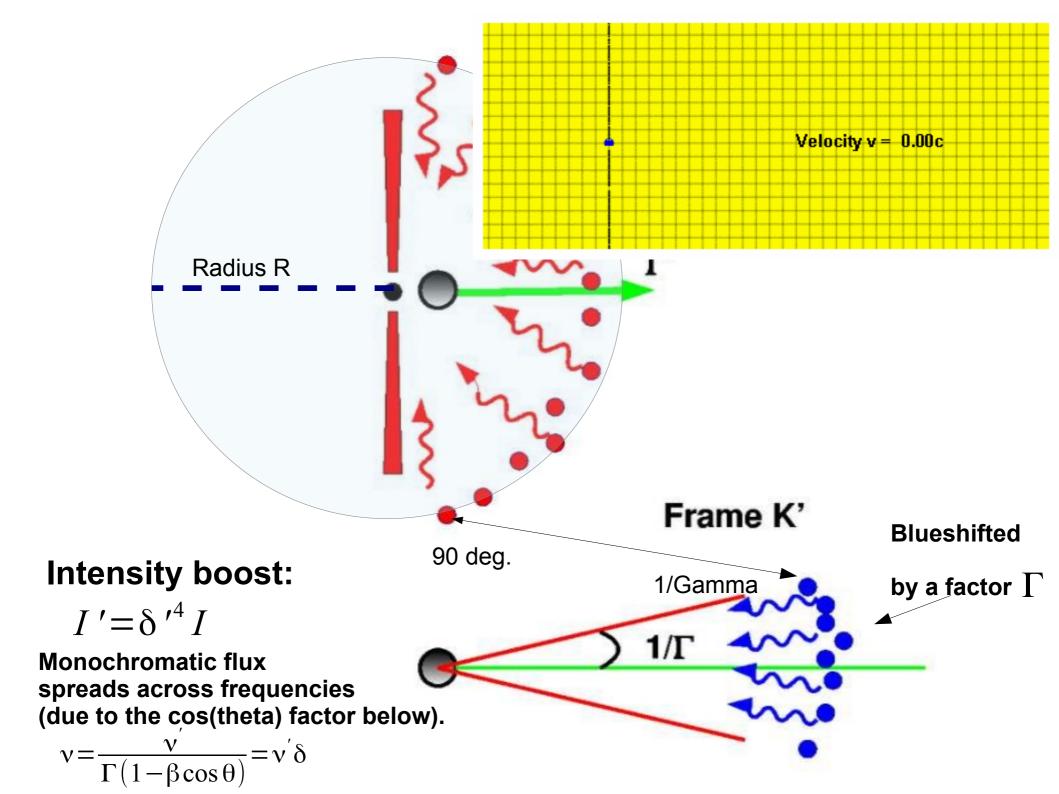


- 1. Jet is moving with bulk Lorentz factor Gamma
- 2. Broad Line Region's photons are produced in a sphere of radius R
- 3. The radiation is monochromatic









Lorentz Transformations of Solid Angles

$$\cos \theta = \frac{u_{\parallel}}{\sqrt{u_{\perp}^2 + u_{\parallel}^2}} = \frac{u_{\parallel}}{c}, \qquad \cos \theta' = \frac{u_{\parallel}'}{c}$$

$$\cos \theta = \frac{\cos \theta' + v/c}{1 + v/c \cos \theta'}$$

So $d\Omega = d\cos\theta d\phi$ and $d\Omega' = d\cos\theta' d\phi'$

$$d \phi = d \phi'$$

$$d\Omega = d\Omega' \frac{1}{\gamma^2 (1 + \beta \cos \theta')^2}$$

$$d \Omega = d \Omega' \delta^2$$

Photons: Intensity, opacity and emissivity Transformation

$$I(\nu) = h\nu \frac{dN}{dt \, d\nu \, d\Omega \, dA}$$

$$= \delta h\nu' \frac{dN'}{(dt'/\delta) \, \delta d\nu' \, (d\Omega'/\delta^2) \, dA'}$$

$$= \delta^3 I'(\nu') = \delta^3 I'(\nu/\delta)$$

$$\frac{I_{\nu}}{\nu^3}$$
 = Lorentz Invariant

$$\frac{S_{\nu}}{v^3}$$
 = Lorentz Invariant

Intensity, opacity and emissivity Transformation

Suppose you have a moving absorbing medium with velocity v in K. The medium has a certain optical depth. Theta is the angle that the photons (crossing the medium) make with the velocity of the medium. How does the optical depth transform?

Since exp(-tau) gives the fraction of photons passing through the material, the optical depth **must be** a Lorentz Invariant (i.e., simple counting does not change the outcome in any reference frame).

$$\tau = \frac{l \, \alpha_{\nu}}{\sin \theta} = Lorentz \, Invariant$$

Similar arguments can be used to show that also the emissivity divided by the frequency squared is a Lorentz Invariant:

$$\frac{j_{\nu}}{v^2}$$
 = Lorentz Invariant



