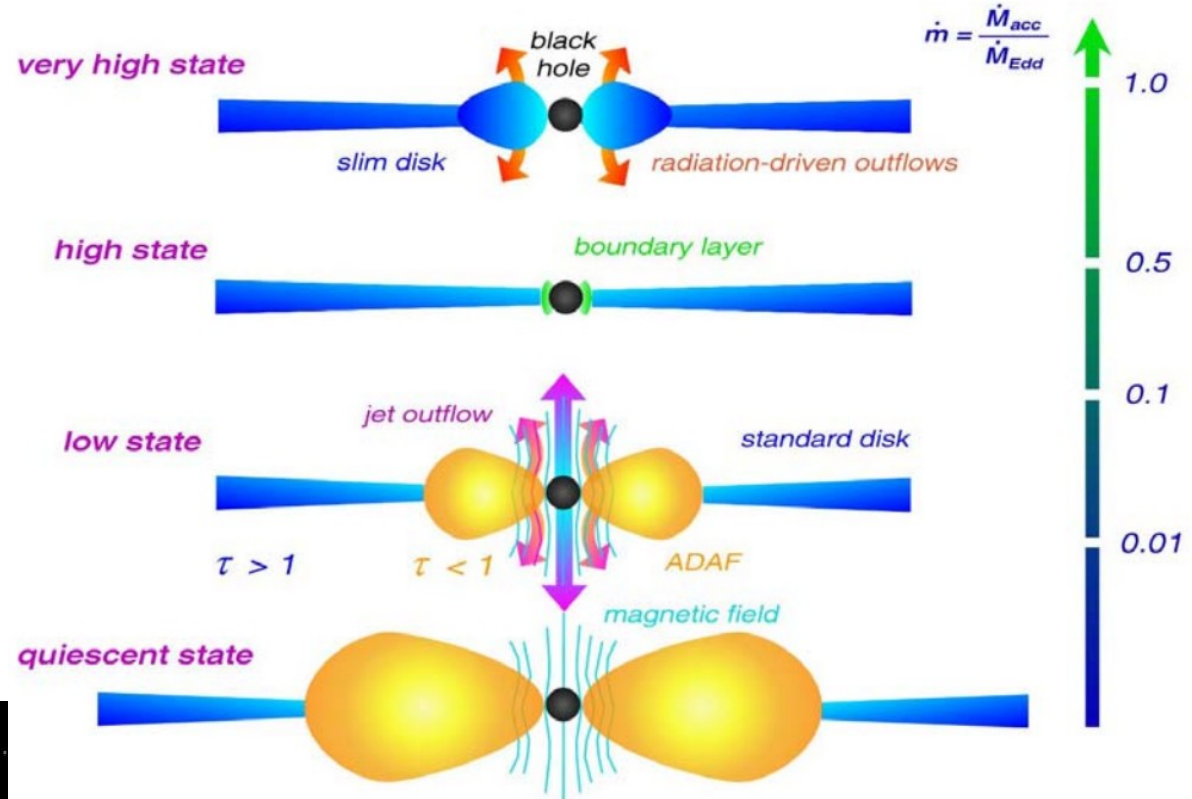
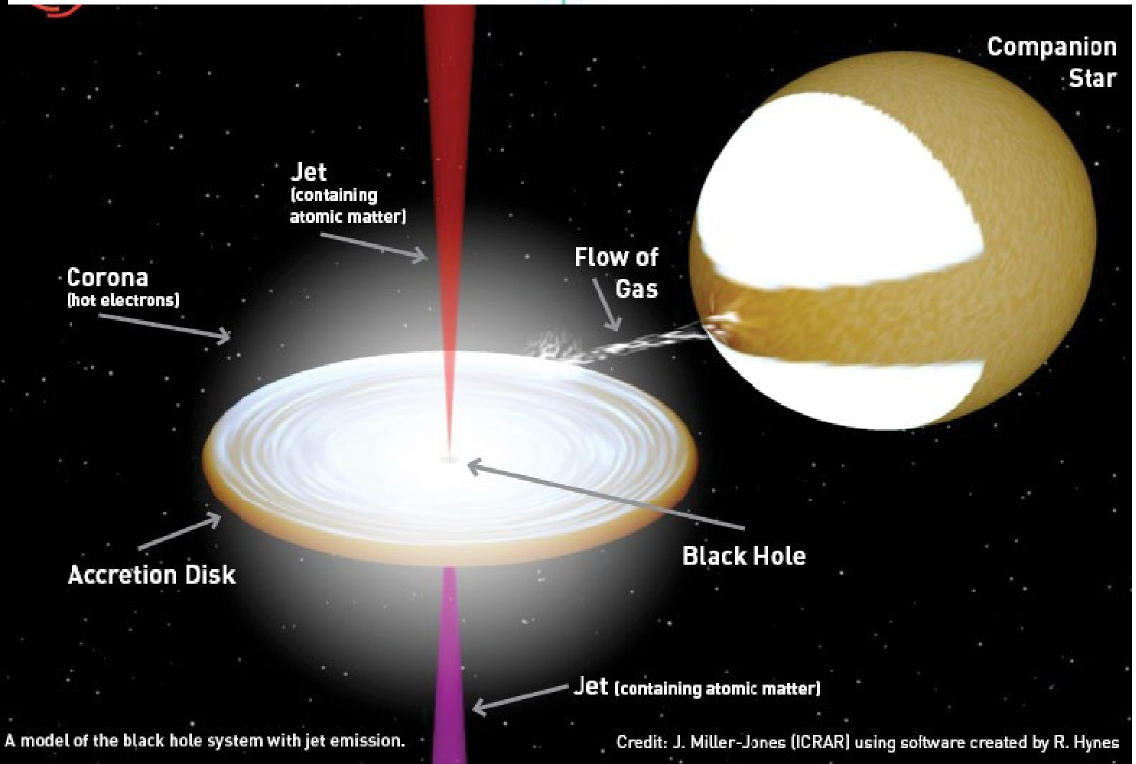
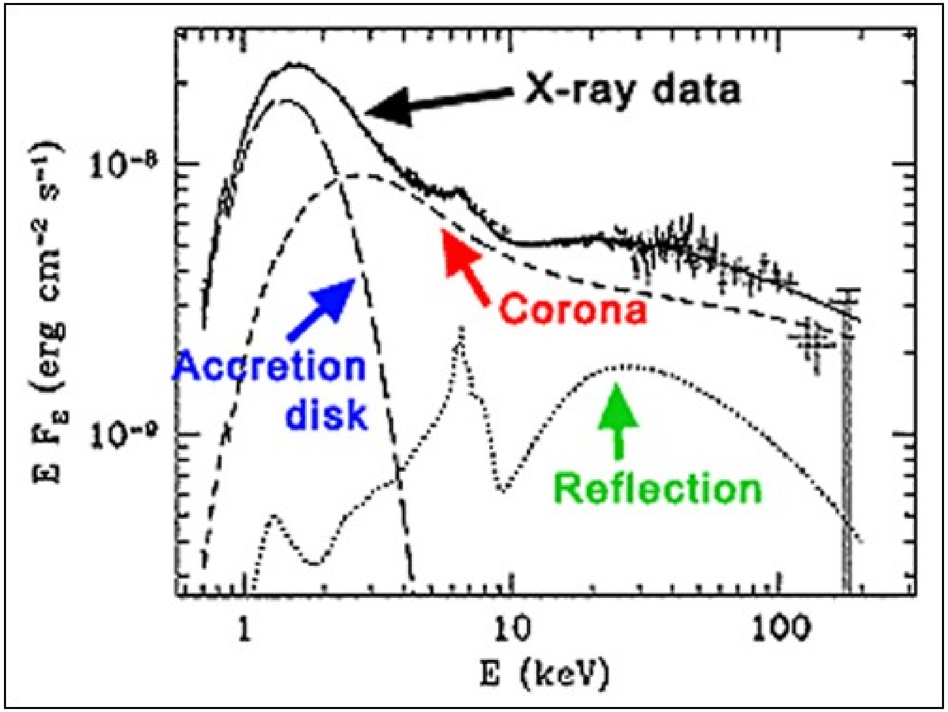
Inverse Compton Scattering II



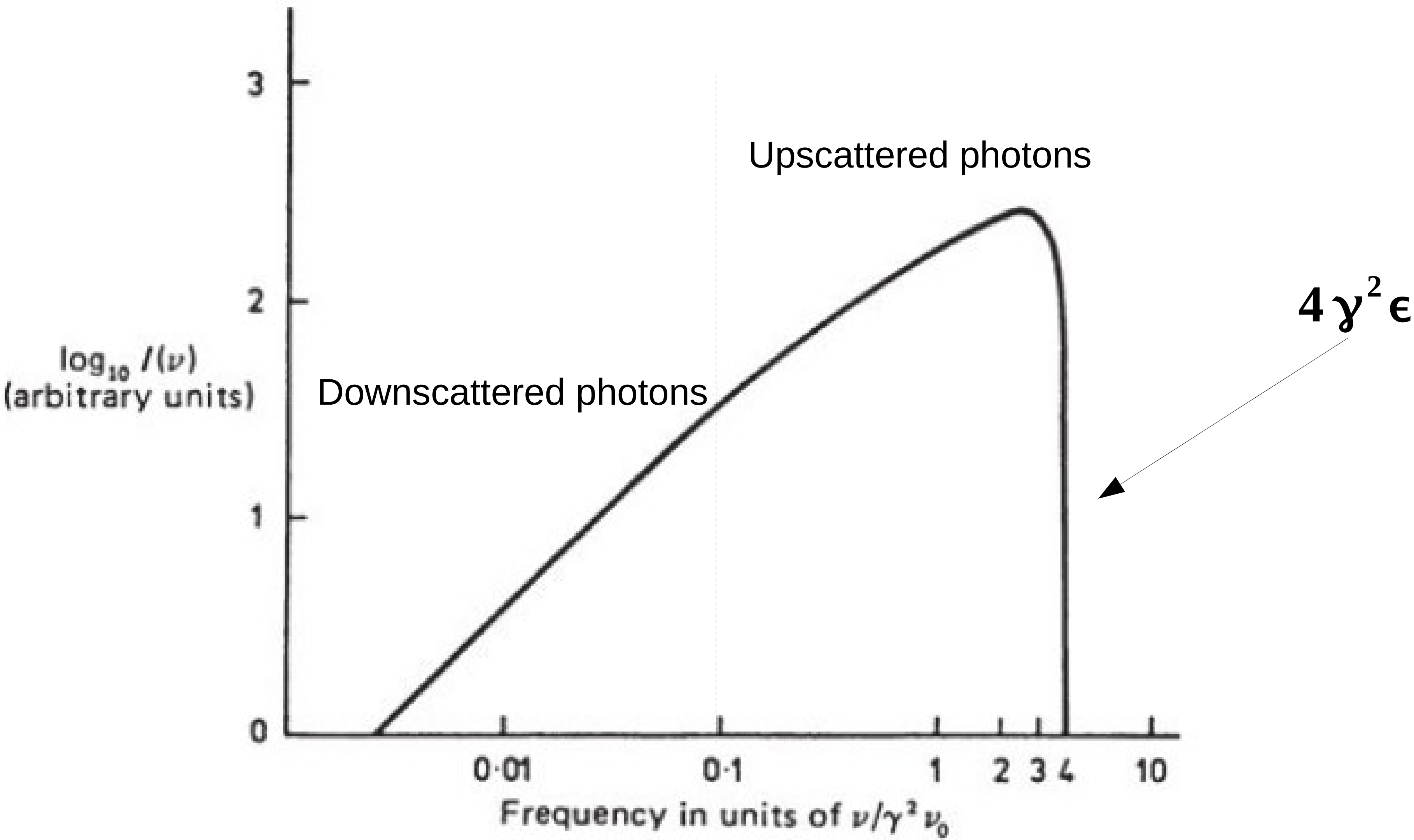




Photon scattering by electrons - Overview

	Low energy photons $\hbar\omega \ll m_e c^2$	**High energy photons** $\hbar\omega \geq m_e c^2$
$v \ll c$	Thomson scattering Classical treatment frequency unchanged	Compton scattering Quantum treatment incorporating photon momentum frequency decreases
$v \sim c$	$\gamma\hbar\omega \ll m_e c^2$ Inverse Compton Photons gain energy from relativistic electrons Approximate with classical treatment in electron rest frame Frequency increases	$\gamma\hbar\omega \geq m_e c^2$ Inverse Compton Quantum treatment in electron rest frame Photons gain energy from relativistic electrons

Single Particle Spectrum



Multiple Scatterings ($\gamma h\nu \ll mc^2$)

$$y = [\text{average \# of scatt.}] \times [\text{average fractional energy gain for scatt.}]$$

Comptonization parameter

Important: If y>1 then Comptonization is important!
(This does not necessarily mean
that there are many scatterings...)

Multiple Scatterings ($\gamma h\nu \ll mc^2$)

$$y = [\text{average \# of scatt.}] \times \boxed{[\text{average fractional energy gain for scatt.}]}$$

Let's first evaluate the average fractional energy change per scattering

Average energy

We have seen last week that when (monochromatic) photons scatter (once) off **one** relativistic electron, then:

$$\langle \epsilon_1 \rangle = \frac{4}{3} \gamma^2 \epsilon$$

Gamma depends on the energy of the electron and has a single value since there is only one electron. So the question now is what is <gamma^2> ? In other words, if we have many (relativistic) electrons, what happens to the average gain in energy after one scattering?

We will start first by assuming *a thermal distribution of electrons.* The reason for this is that this case is the simplest and will help us to find a few general parameters.

Thermal Compton (relativistic case)

We have the following starting assumptions:

1. thermal distribution of electrons
2. electrons are relativistic
3. $\gamma h\nu \ll mc^2$

The relativistic Maxwell-Boltzmann distribution has the form:

$$N(\gamma) = \gamma^2 e^{-\gamma/\Sigma}$$

$$\Sigma = \frac{kT}{mc^2}$$

We want to find $\langle \gamma^2 \rangle$ and so we can use the MB distribution to calculate this average.

Thermal Compton (relativistic case)

$$\langle \gamma^2 \rangle = \frac{\int \gamma^2 \gamma^2 e^{-\gamma/\Sigma} d\gamma}{\int \gamma^2 e^{-\gamma/\Sigma} d\gamma} = 12\Sigma^2 = 12\left(\frac{kT}{mc^2}\right)^2$$

Therefore we now know that when there is a thermal distribution of relativistic electrons, the average gain in energy of the photons per scattering is proportional to the square of the thermal energy of the electrons

$$y = [\text{average \# of scatt.}] \times \boxed{[\text{average fractional energy gain for scatt.}]}$$

Thermal Compton (relativistic case)

Let's now calculate the average number of scatterings

$$y = \boxed{[\text{average \# of scatt.}]} \times [\text{average fractional energy gain for scatt.}]$$

To calculate the average number of scattering you should think at the path that the photon does in the cloud of electrons as a "random walk" (see Section 1.7 on the R&L).

Multiple Scatterings ($\gamma h\nu \ll mc^2$)

$$y = \boxed{[\text{average \# of scatt.}]} \times [\text{average fractional energy gain for scatt.}]$$

$$\tau_T = \sigma_T n R$$

n = electron density.
R = source size

Case 1: tau>1

D = $c\Delta t = \tau_T^2 \dfrac{R}{\tau_T} = \tau_T R$

Total path traveled by photon
(valid when tau>1)

Case 2: tau<1

$$avg.Nr.scatt. \approx \tau_T$$

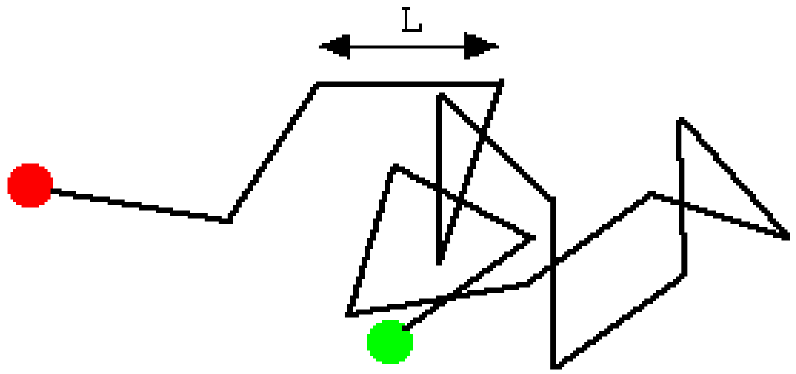
Multiple Scatterings ($\gamma h\nu \ll mc^2$)

$$y = \boxed{[\text{average \# of scatt.}]} \times [\text{average fractional energy gain for scatt.}]$$

$$\tau_{\mathrm{T}} = \sigma_{\mathrm{T}} n R$$

n = electron density.
R = source size

D = $c\Delta t = \tau_{\mathrm{T}}^2 \dfrac{R}{\tau_{\mathrm{T}}} = \tau_{\mathrm{T}} R$



Average nr.
of scatterings

Mean free path **L**

Multiple Scatterings ($\gamma h\nu \ll mc^2$)

$$y = \boxed{[\text{average \# of scatt.}]} \times [\text{average fractional energy gain for scatt.}]$$

Case 1: tau>1	Case 2: tau<1
Average # of scatt. = τ_T^2	Average # of scatt. = τ_T

We have everything we need now for calculating the spectrum for photons doing multiple scatterings off a thermal distribution of relativistic electrons (when we are in the Thomson limit).

Before continuing though, let's first check what happens when the electrons are non relativistic. The average # of scatt. does not change wrt the previous case. We need to calculate only the change in fractional energy.

$$y = [\text{average \# of scatt.}] \times \boxed{[\text{average fractional energy gain for scatt.}]}$$

NOTE: Non-relativistic here means that $kT \ll mc^2$

Thermal Compton (non rel. case)

If we are in the non-relativistic case then we are considering also the direct Compton case. However, there will be both electrons that gain energy from the photons and electrons that give energy to the photons. Why?

Photon scattering by electrons - Overview

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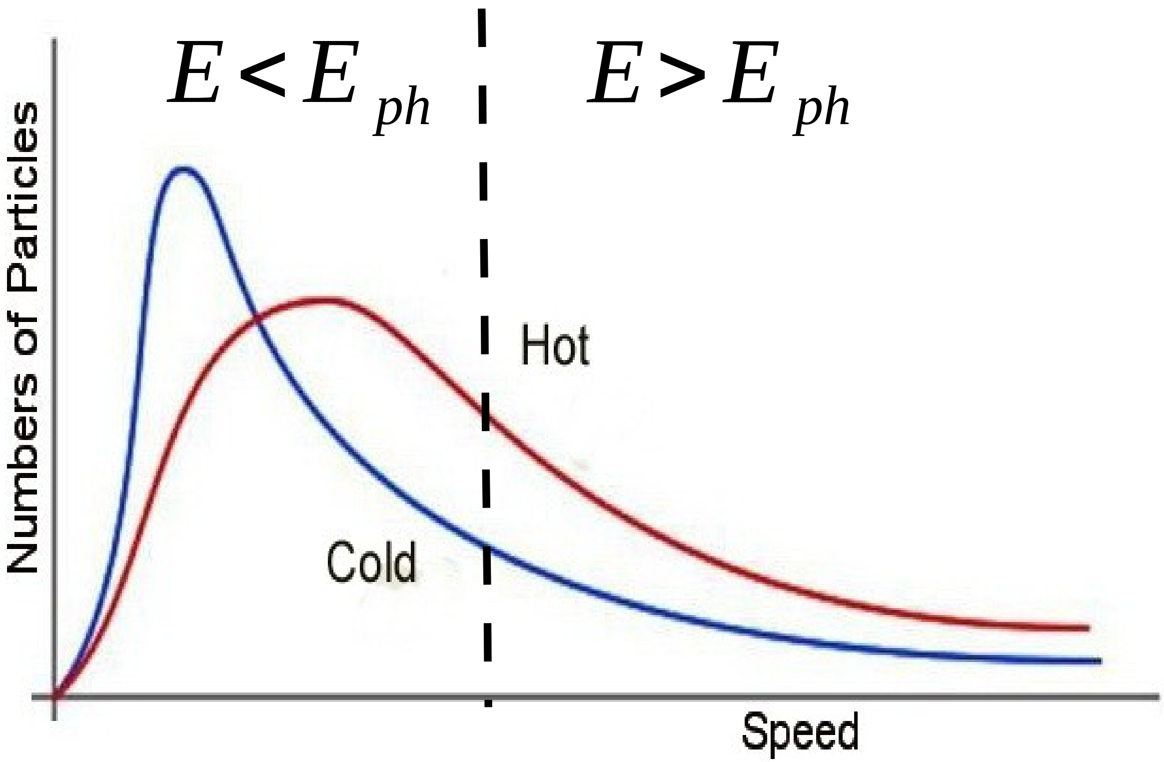
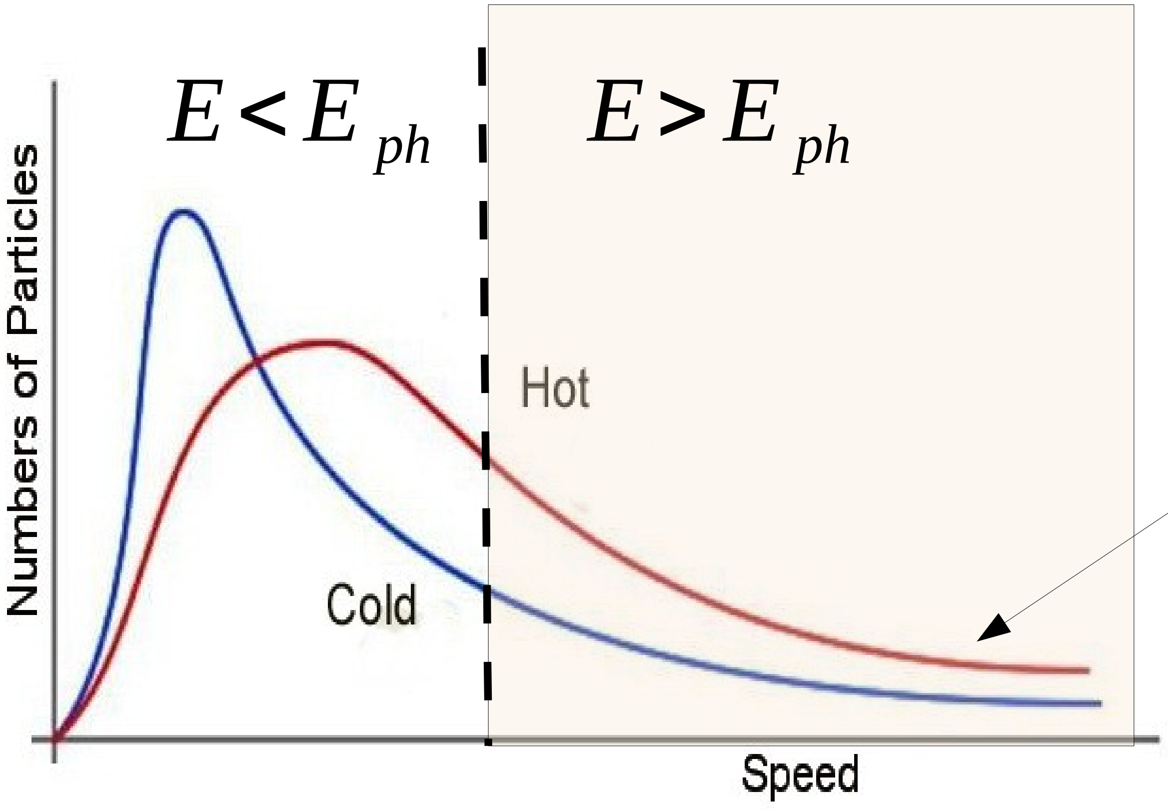
Thermal Compton (non rel. case)

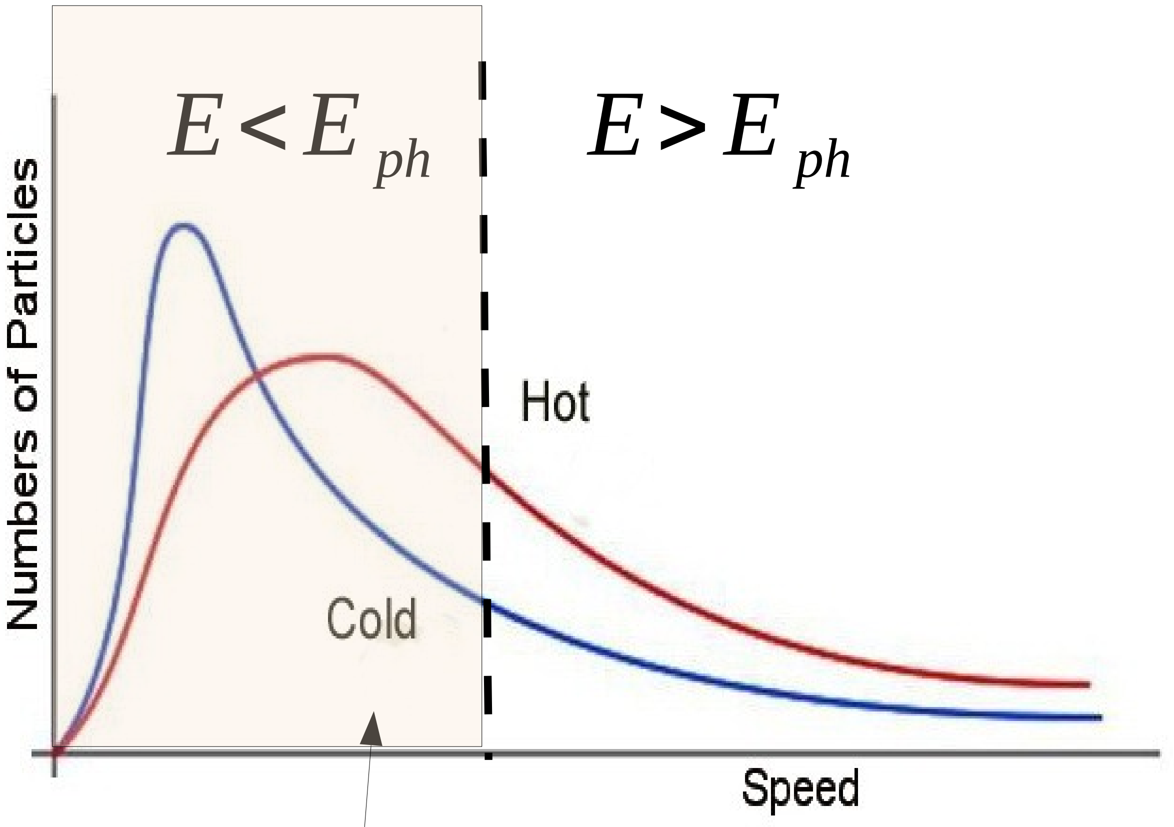


Thermal Compton (non rel. case)



These are the electrons that give energy to the photons.

Thermal Compton (non rel. case)



These are the electrons that gain energy from the photons.

Thermal Compton (non rel. case)

Let's take the energy change from one scattering and average it over a MB-distribution (as we did before for the relativistic case).

From what we said before, in the non-relativistic case we expect:

$$\frac{\Delta\epsilon}{\epsilon} = \alpha\Sigma - \frac{\epsilon}{mc^2}$$

Here alpha is a constant to be determined.
The “-epsilon” term corresponds to the energy removed in the direct Compton, whereas $\alpha\Sigma$ is the energy gained by the photons due to the thermal energy of the electrons.

Where does this come from and what is alpha?

Thermal Compton (non rel. case)

Remember what we said last week:

$$P_{compt} = \frac{4}{3} \sigma_T c \gamma^2 \beta^2 U_{ph}$$

Since we are in the non-relativistic limit, then gamma~1 and beta<<1
the energy loss per unit time for a non-relativistic electron becomes:

$$P_{compt} \approx \frac{4}{3} \sigma_T c \beta^2 U_{ph} = \frac{4}{3} \sigma_T c \beta^2 n_{ph} \epsilon$$

where n_{ph} is the number of photons and epsilon their energy.

The number of collisions that the electron suffers per unit time is:

$$\frac{dN}{dt} = n_{ph} \sigma_T c$$

Thermal Compton (non rel. case)

The mean energy loss per collision, for the electron, i.e. the mean energy **gain**,for the photon, becomes:

$$\langle \Delta \epsilon \rangle = \frac{P_{Compton}}{dN/dt} = \frac{4}{3} \beta^2 \epsilon$$

This shows that the mean energy **gain** is of the order of the velocity squared (since beta=v/c). But in a thermal distribution we know that:

$$\frac{1}{2} m v^2 \approx \frac{3}{2} kT$$

Therefore the mean energy **gain** must be:

$$\langle \Delta \epsilon \rangle = \frac{P_{Compton}}{dN/dt} = 4 \frac{kT}{mc^2} \epsilon \rightarrow \alpha = 4$$

Summary

Relativistic Electrons	Non-Relativistic Electrons
• Gain in energy of photons prop to (kT)^2 • All photons gain energy	• Gain in energy of photons prop to (kT) • Some photons gain energy (**upscattering**) • Some photons lose energy (**downscatt.**)
$$\frac{\Delta\epsilon}{\epsilon}=16\left(\frac{kT}{mc^2}\right)^2$$	$$\frac{\Delta\epsilon}{\epsilon}=\frac{(4kT-\epsilon)}{mc^2}$$

tau>1	**tau<1**
Average # of scatt. = τ_T^2	Average # of scatt. = τ_T

Compton Spectrum (rel.)

If the electron distribution is a power-law or a thermal one the result will be still a power-law. Let's first see the case for a thermal distribution. Call A the mean amplification of photon energy per scattering:

$$A=\frac{\epsilon_1}{\epsilon}\approx\frac{4}{3}\langle\gamma^2\rangle=16\Sigma^2=16\left(\frac{kT}{mc^2}\right)^2$$

Assume that an initial *photon* distribution has a mean energy :

$$\epsilon_i \ll \langle\gamma^2\rangle^{-1/2} mc^2$$

and intensity I(epislon_i) at epislon_i

Then after *k* scatterings the energy of a mean initial photon will be:

$$\epsilon_k \approx \epsilon_i A^k$$

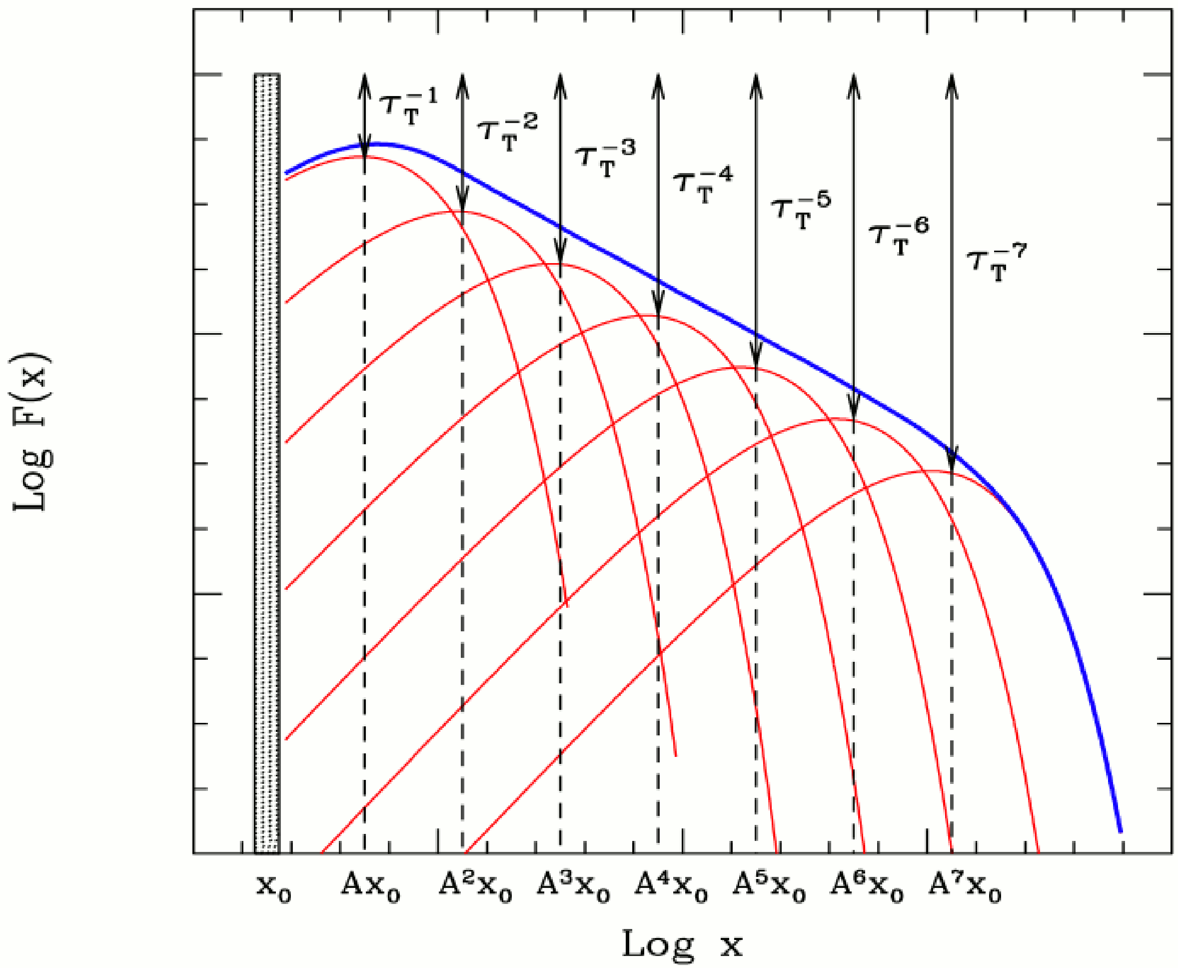
Compton Spectrum (rel. and tau<1)

Now, if the medium is of small optical depth the probability of a photon undergoing k scatterings before escaping the Comptonizing cloud is approximately:

$$Prob. \approx \tau^k$$

The intensity of the emerging Compton radiation will therefore decrease by a factor tau as a function of energy (or frequency) of the radiation.

The spectrum will look like in the figure to the right.



Compton scattering (non rel.)

When the photons diffuse along the energy axis (i.e. sometimes they lose, but more often they gain energy), then the time evolution is described by a diffusion equation. Most easily this equation is written in terms of the phase-space density (occupation number) n(x) of photons of energy x = epsilon/kT.

The diffusion equation is called in this case the **Kompaneet equation:**

$$\frac{\partial n}{\partial y} = \left(\frac{1}{x^2}\right) \cdot \frac{\partial}{\partial x}\left[x^4\left(\frac{\partial n}{\partial x} + n + n^2\right)\right] \qquad x \equiv \frac{h\nu}{k_B T}$$

Generally, the solutions of the equation **have to be found numerically**, but there are a number of cases in which analytic solutions can be found.

Compton scattering (non rel.)

$$\frac{\Delta\epsilon}{\epsilon} = \frac{(4kT - \epsilon)}{mc^2}$$

Recoil effect (Downscattering & Cooling)

Upscattering by
thermal electrons

Non-linear term (induced
scattering)

$$\frac{\partial n}{\partial y} = \left(\frac{1}{x^2}\right) \cdot \frac{\partial}{\partial x}\left[x^4\left(\frac{\partial n}{\partial x} + n + n^2\right)\right] \qquad x \equiv \frac{h\nu}{k_B T}$$

Generally, the solutions of the equation **have to be found numerically**, but there are a number of cases in which analytic solutions can be found.

Solutions to the Kompaneet Equation

A family of (analytical) solutions for the Kompaneet Equation can be found when the amount of Comptonization is either weak (y<<1), relatively strong (y~1) or very strong (y>>1).

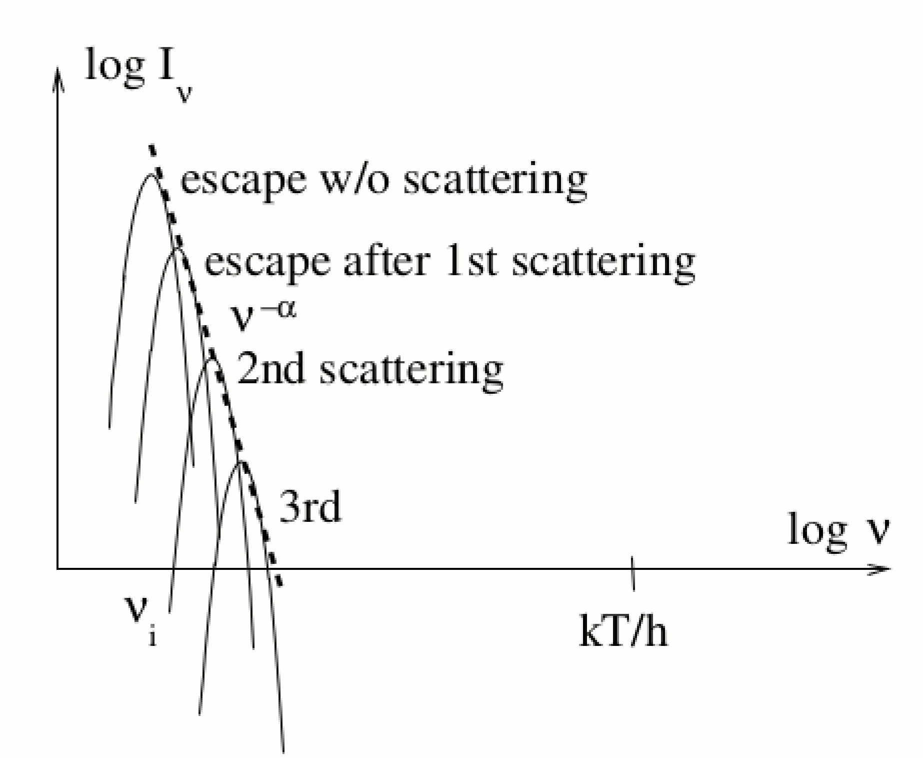
In the first two cases the solution is a power-law spectrum with spectral index alpha:

$$\alpha = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{4}{y}}$$

The positive root is a solution when y~1
The negative root is a solution when y<<1

Case y<<1: Very unsaturated

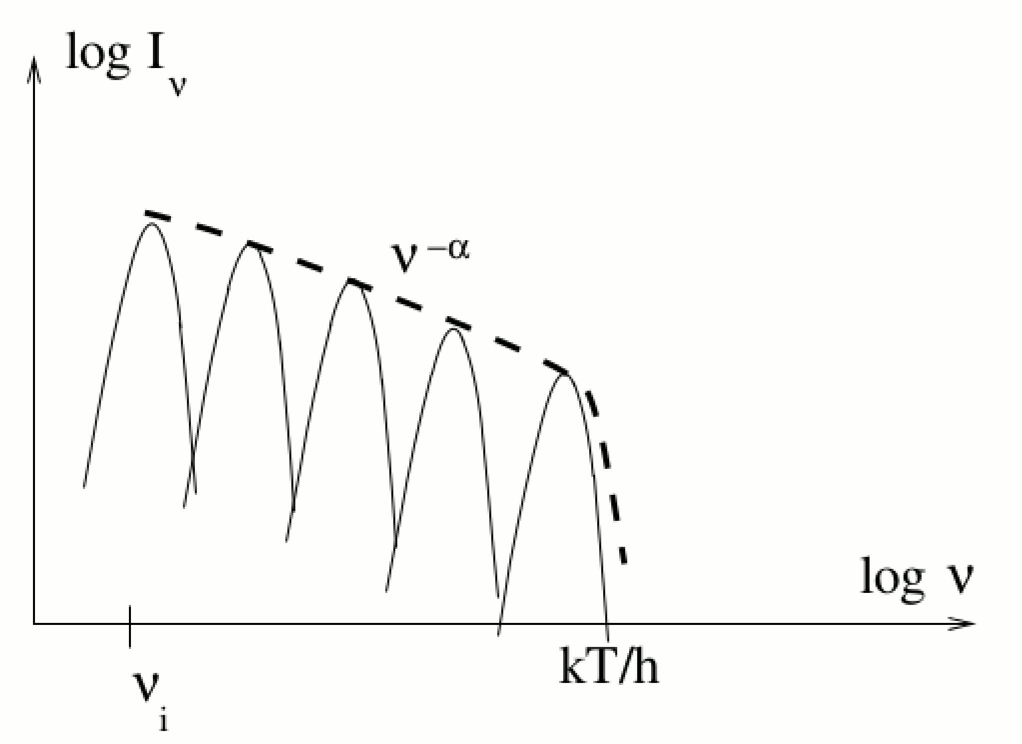
In this case alpha~2/sqrt(y)>>1



Only a small fraction of photons interact and are Comptonized. A steep power-law is created

Case y~1: unsaturated

In this case alpha~1



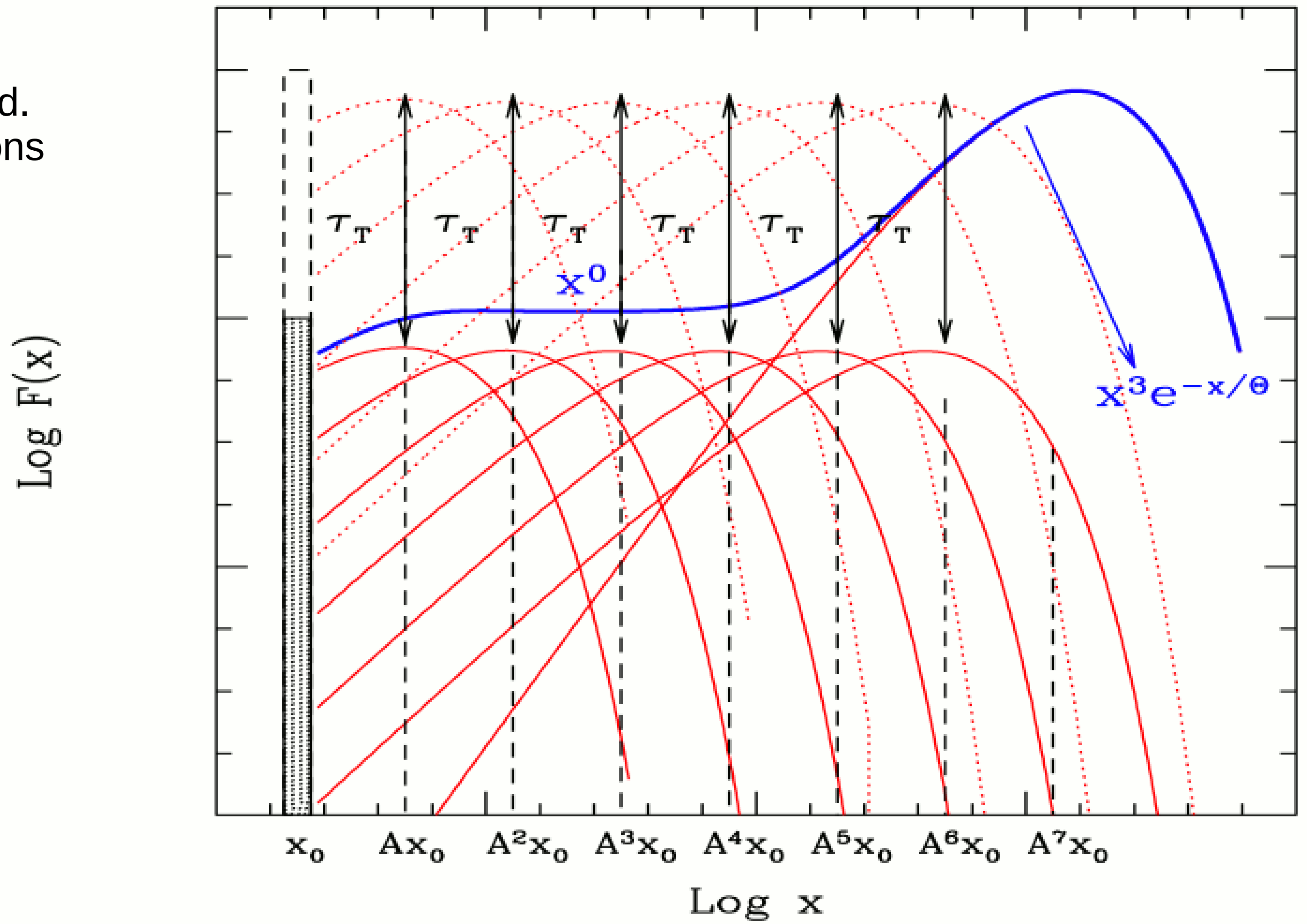
Examples: hard X-ray spectra
of Galactic black hole candidates,
accreting neutron stars,
some active galactic nuclei (Seyferts).

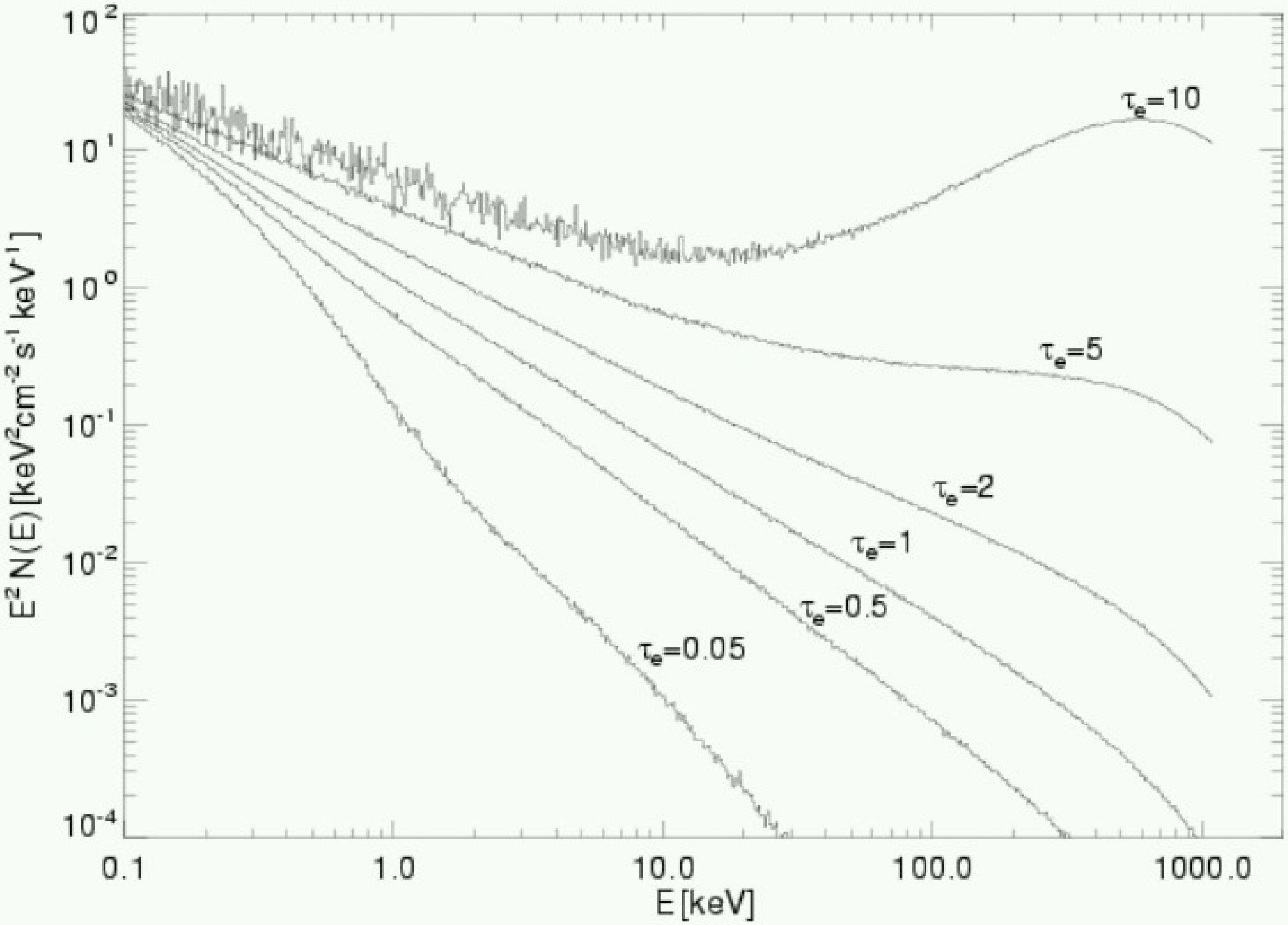
Case y>>1: Saturation

For the first scattering order, nearly all photons are scattered. Therefore the number of photons escaping at each scattering order is the same.

However, this cannot be going on indefinitely, since there is a limit from the energy of the electrons.

When this limit is reached, the photons show a “bump” which is nothing but the Wien spectrum.





Sphere with $kT_e = 0.7 m_e c^2$ ($\sim$ 360 keV), seed photons come from center of sphere.

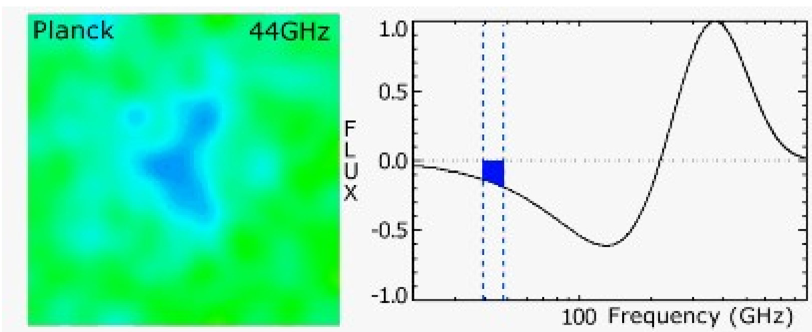
$y \ll 1$: pure power-law spectrum.

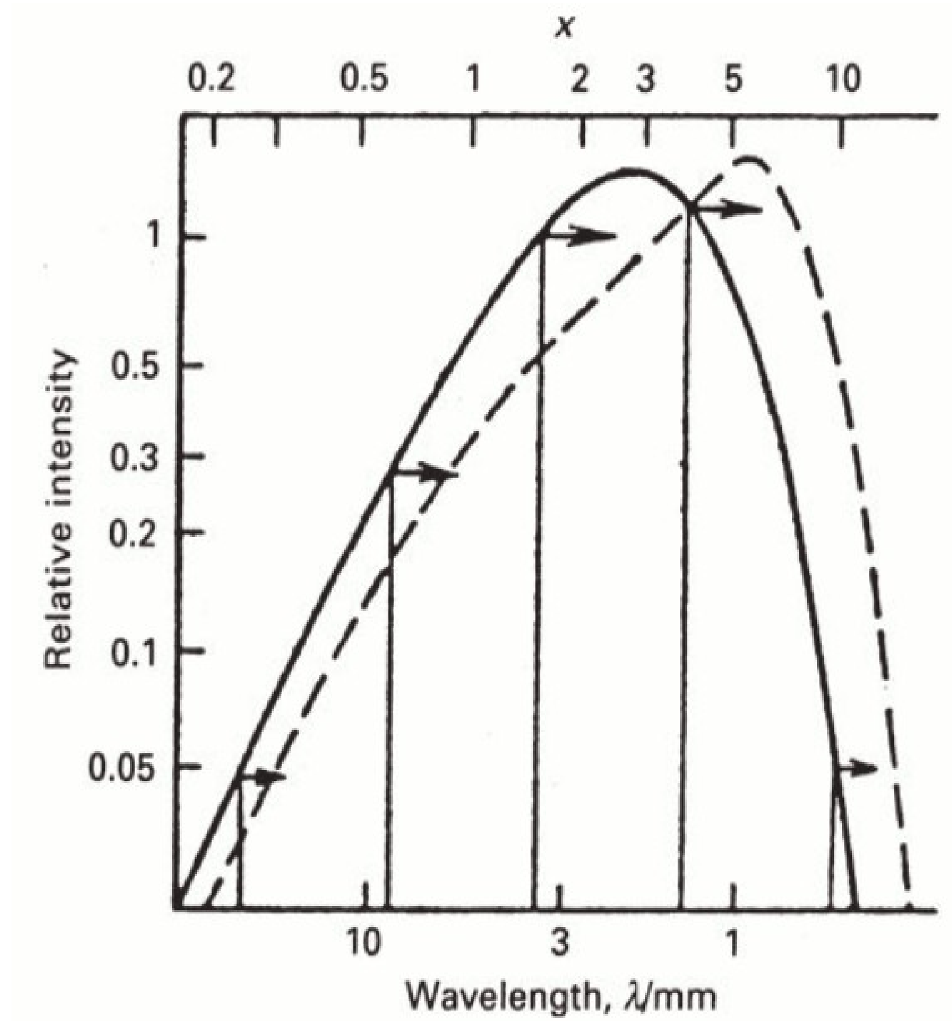
$y < 1$: power-law with exp. cut-off.

$y \gg 1$: "Saturated Comptonization".

Comptonization

(a) Very unsaturated, i.e. $y \ll 1$



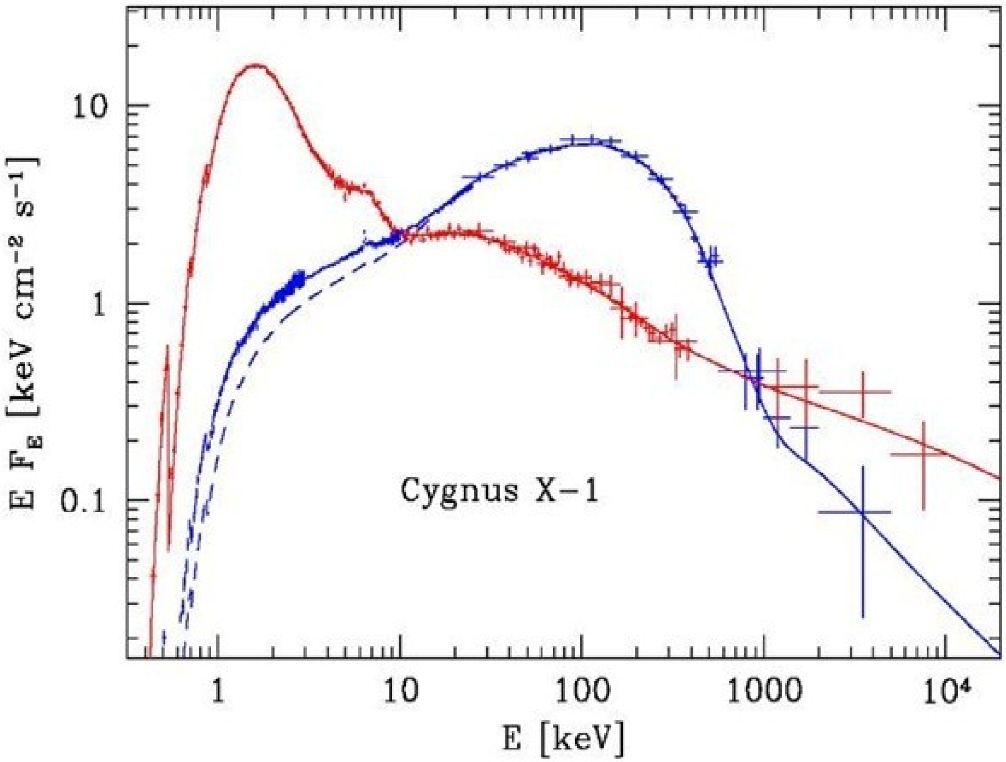


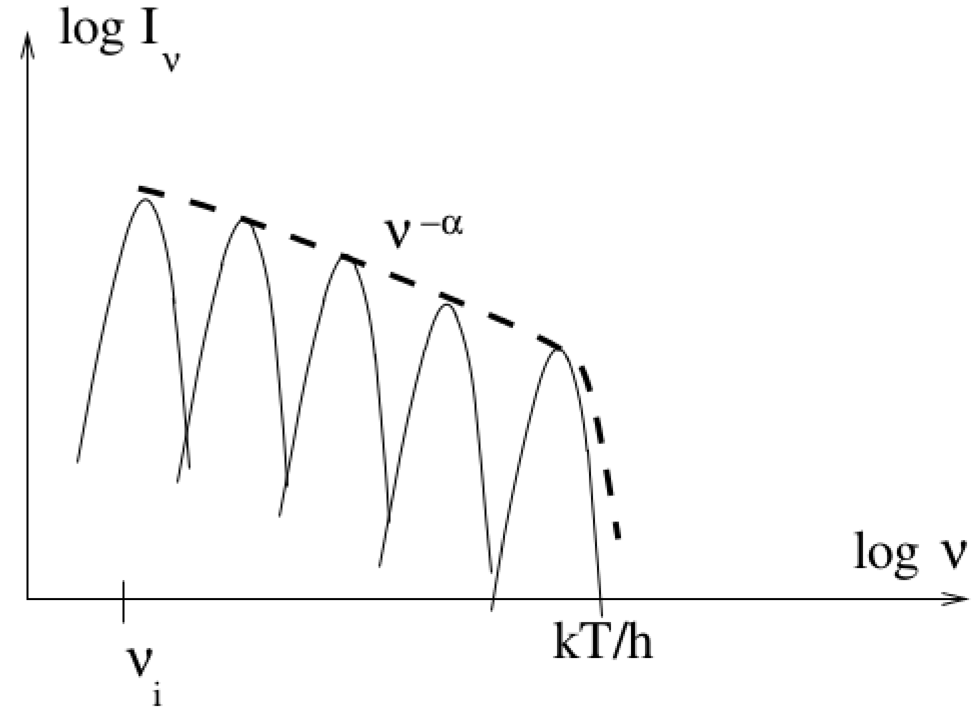
Sunyaev & Zeldovich effect

CMB photons have in general a ~1% probability to interact with a hot electron in the ICM (intracluster medium)

Comptonization

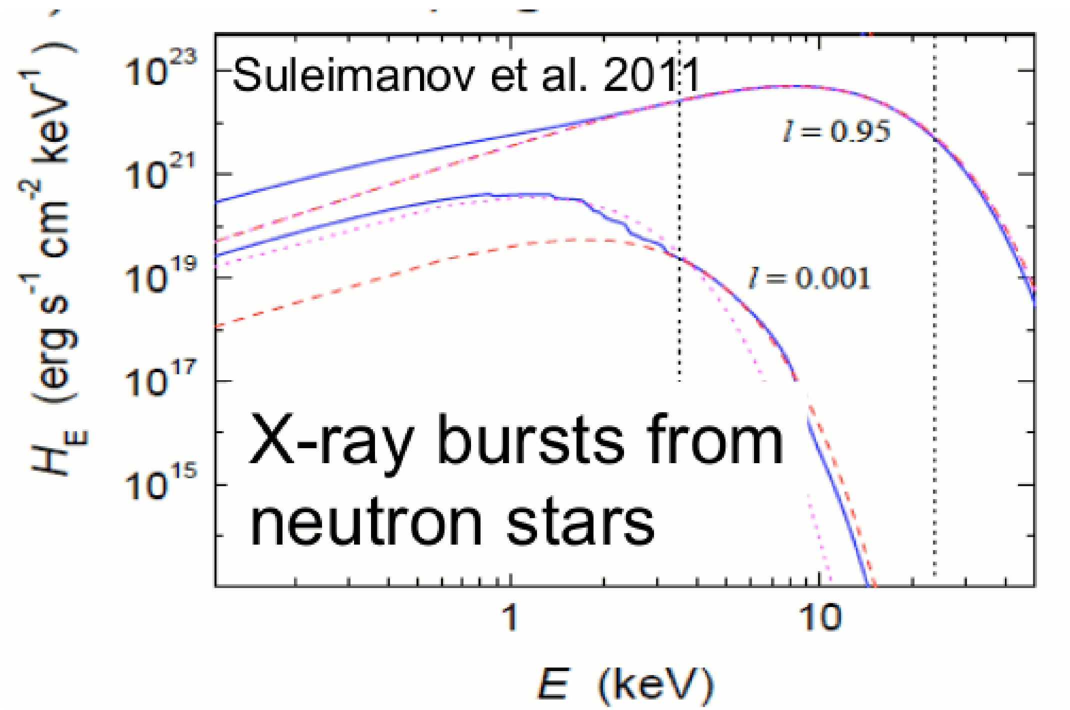
(b) Unsaturated, $y \approx 1$

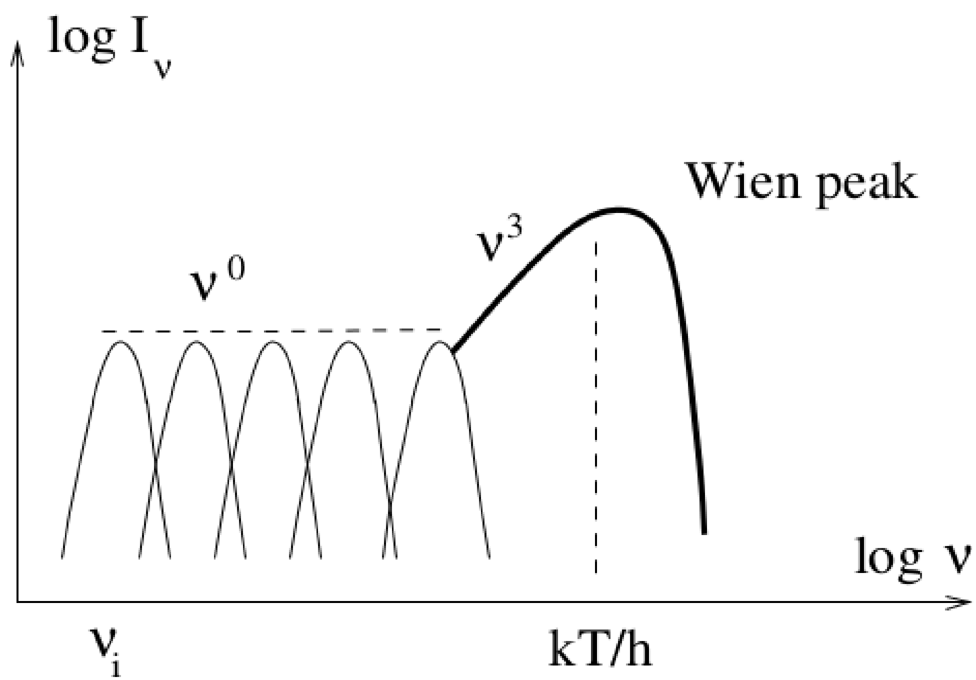




Comptonization

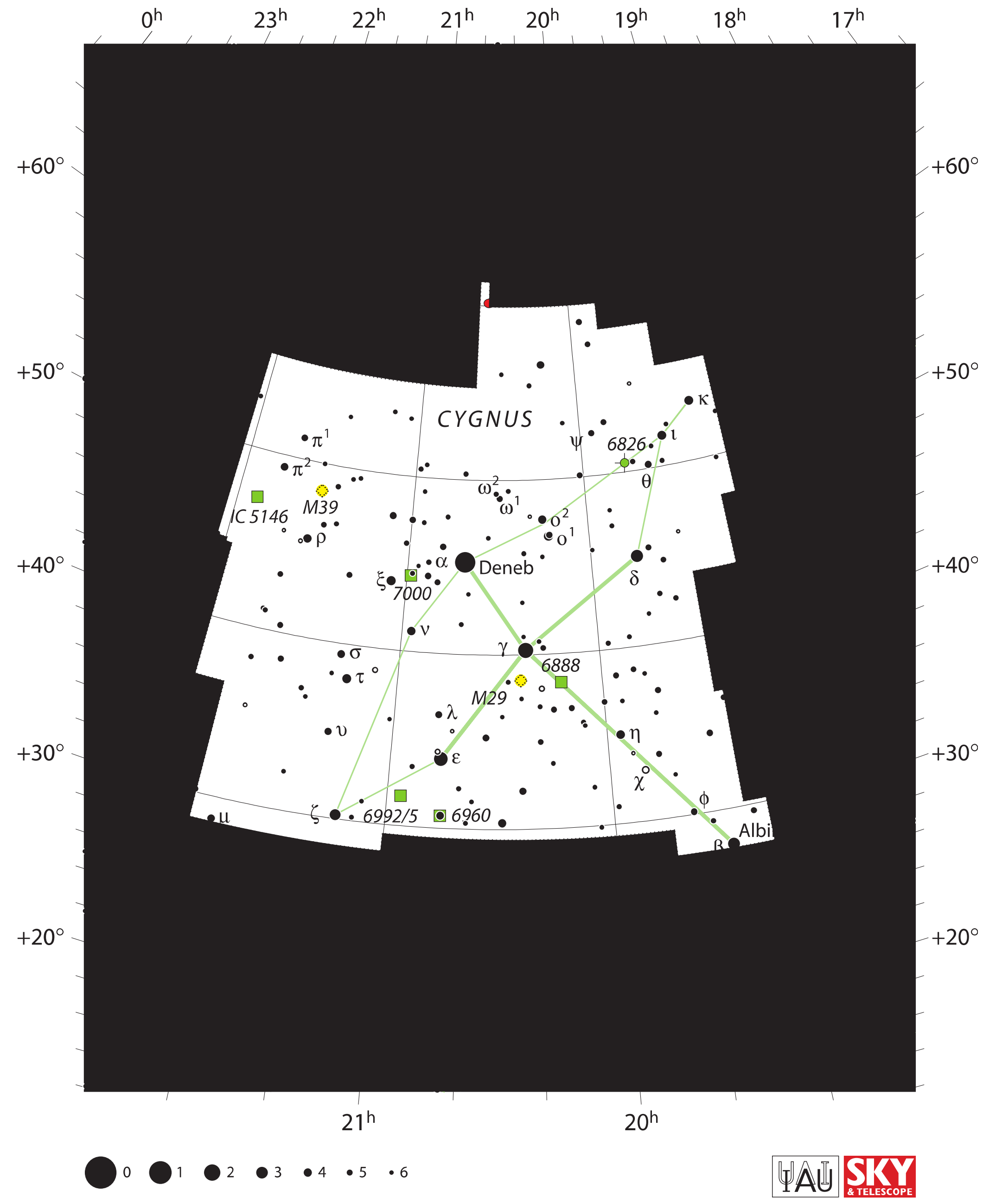
(c) Saturated, $y \gg 1$

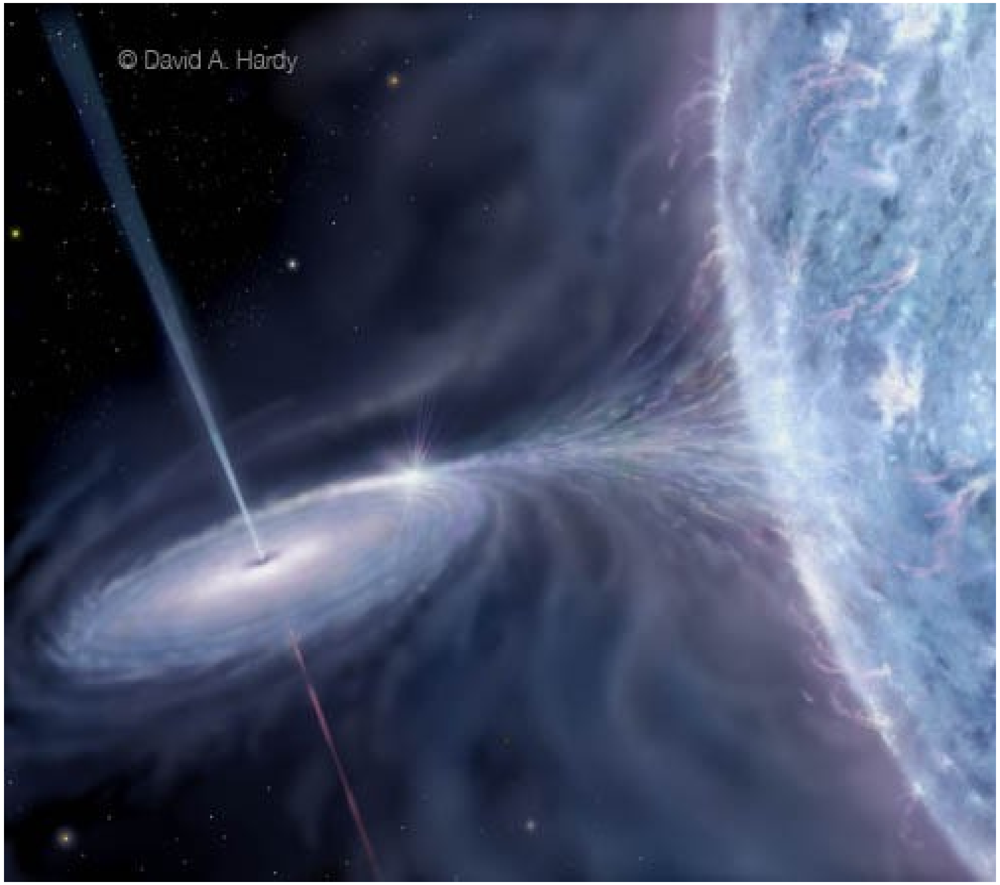




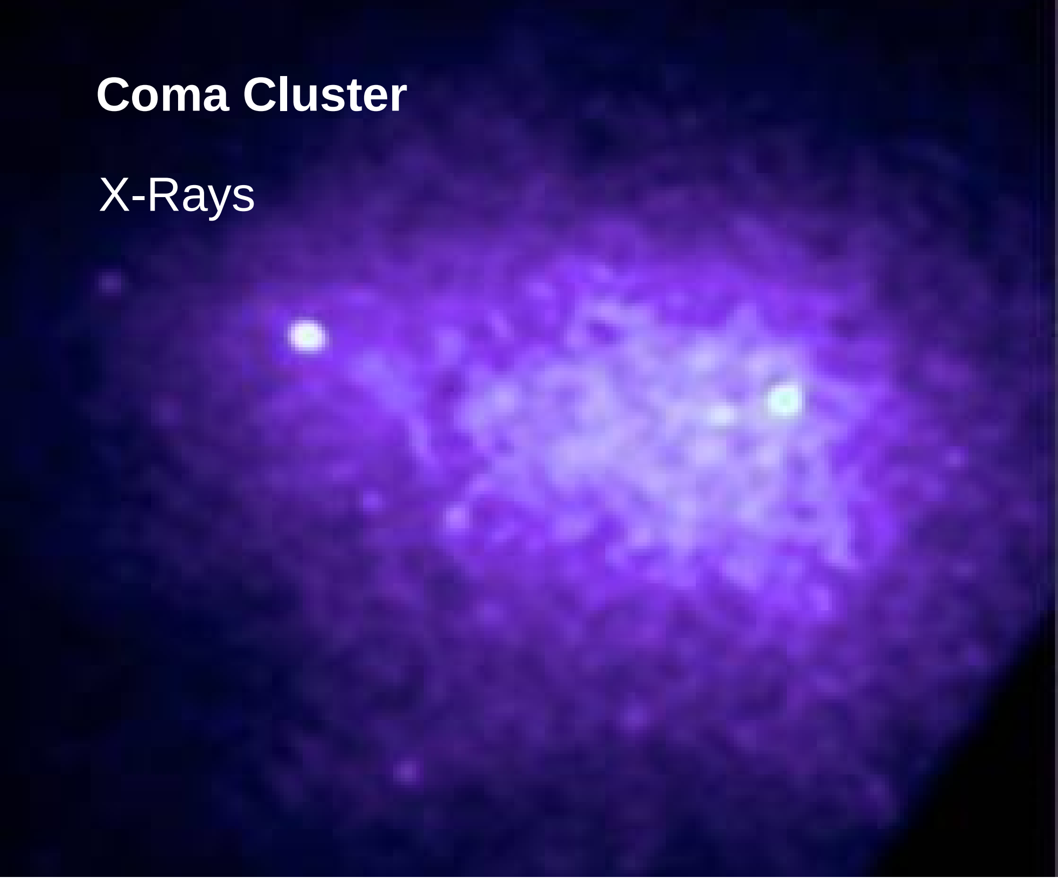
Thermonuclear bursts on neutron stars

Comptonization

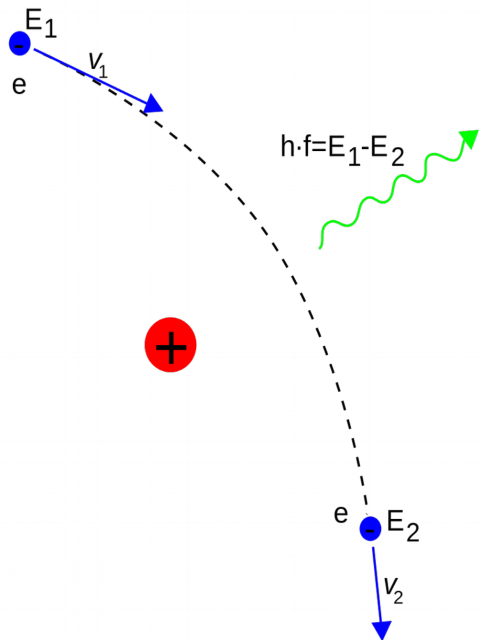




Apparent magnitude V ~ 9 mag. Easily visible with a small telescope/binocular!

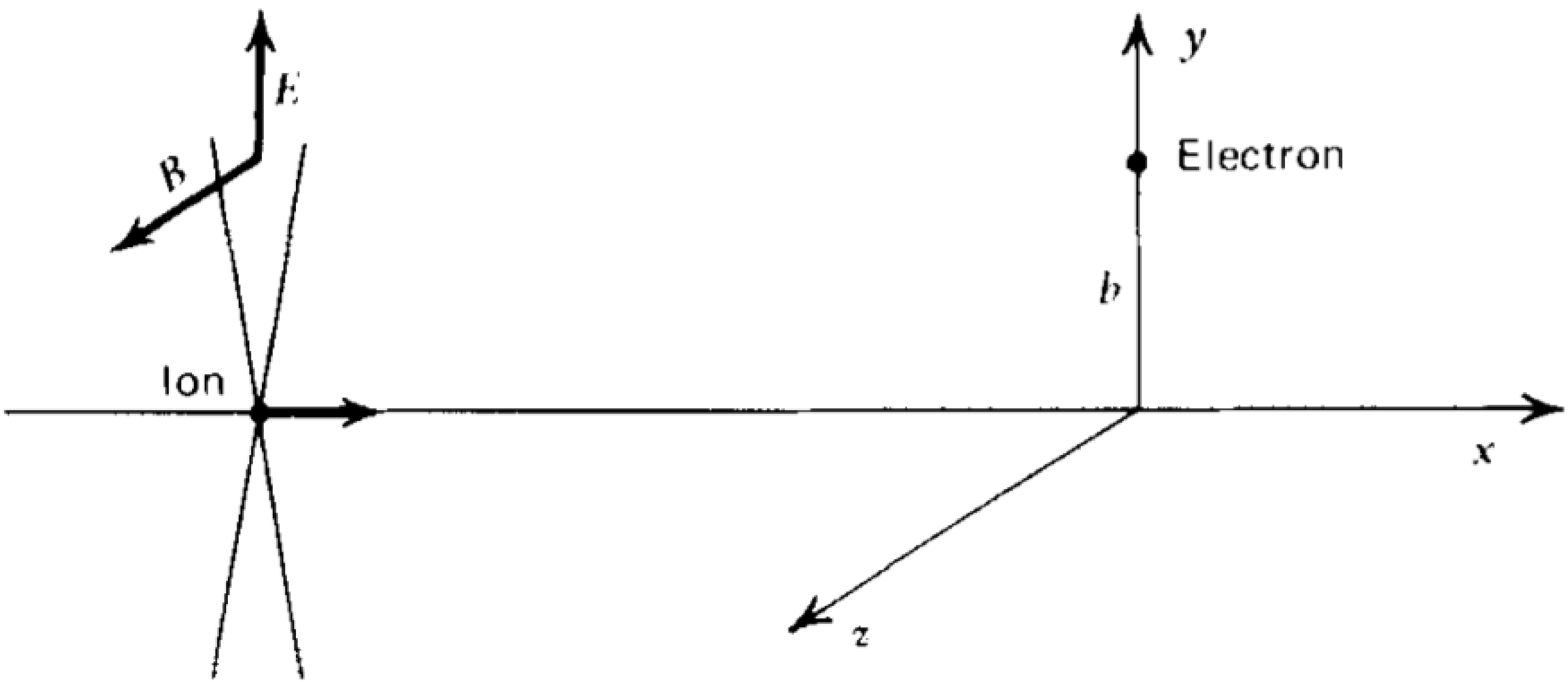


Bremsstrahlung
(“braking radiation”)



Relativistic Bremsstrahlung

- Take a distribution of relativistic electrons and non-rel. ions.
Move to the reference frame of the electron. The e- sees the ion approaching at relativistic speed. The electrostatic field of the ion is seen as a “pulse” of radiation with E and B orthogonal (electromagnetic pulse).
- Now take the radiation of this pulse and Compton scatter off the electron.
- Transform back to the lab frame (ion reference frame) and obtain the relativistic bremsstrahlung radiation. (See e.g., Section 5.4 on R&L).



Comptonization: Non-Thermal Distribution

Let's start with relativistic electrons with an isotropic non-thermal distribution:

$$N(\gamma) = K\gamma^{-p} = N(E)\,\frac{dE}{d\gamma}; \quad \gamma_{\min} < \gamma < \gamma_{\max}$$

Then for simplicity, let's take a monochromatic field of photons with frequency ν_0

From our previous discussion, we know the average Compton frequency, and we can find an expression for the Lorentz factor in this way:

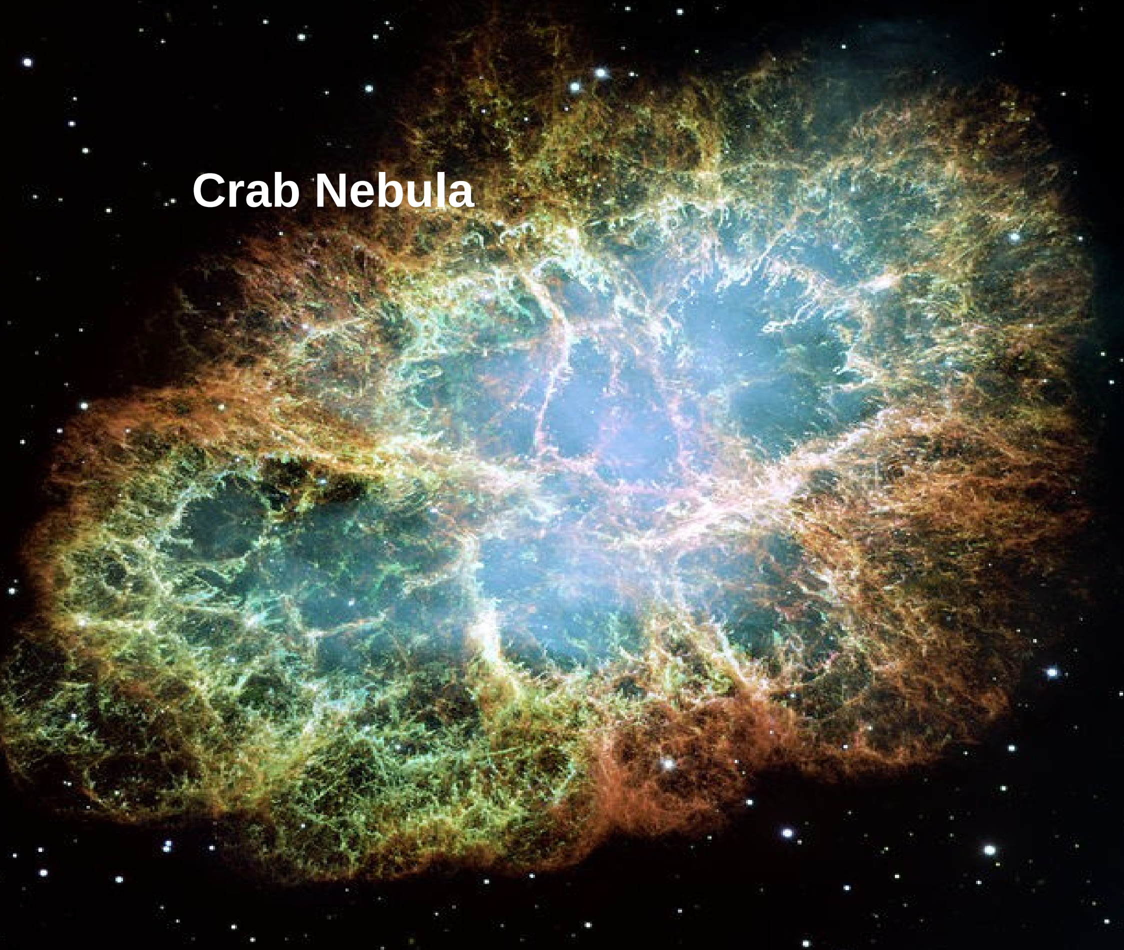
$$\nu_c = \frac{4}{3}\gamma^2\nu_0 \to \gamma = \left(\frac{3\nu_c}{4\nu_0}\right)^{1/2} \to \left|\frac{d\gamma}{d\nu}\right| = \frac{\nu_c^{-1/2}}{2}\left(\frac{3}{4\nu_o}\right)^{1/2}$$

Now we can derive an emission (erg/volume/time/frequency):

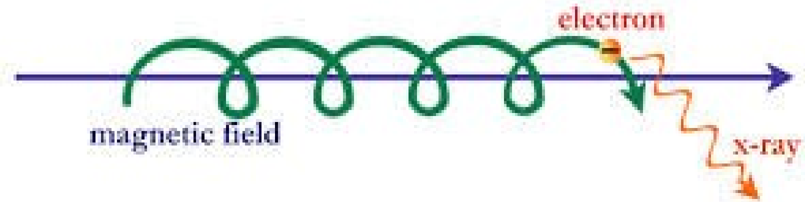
$$\varepsilon_\nu d\nu = P(\gamma)N(\gamma)d\gamma \rightarrow \varepsilon_\nu = P(\gamma)N(\gamma)\frac{d\gamma}{d\nu} = \frac{(4/3)^\alpha}{2}\sigma_T cK\frac{U_r}{\nu_0}\left(\frac{\nu_c}{\nu_0}\right)^{-\alpha}$$

where $\alpha = \dfrac{p-1}{2}$

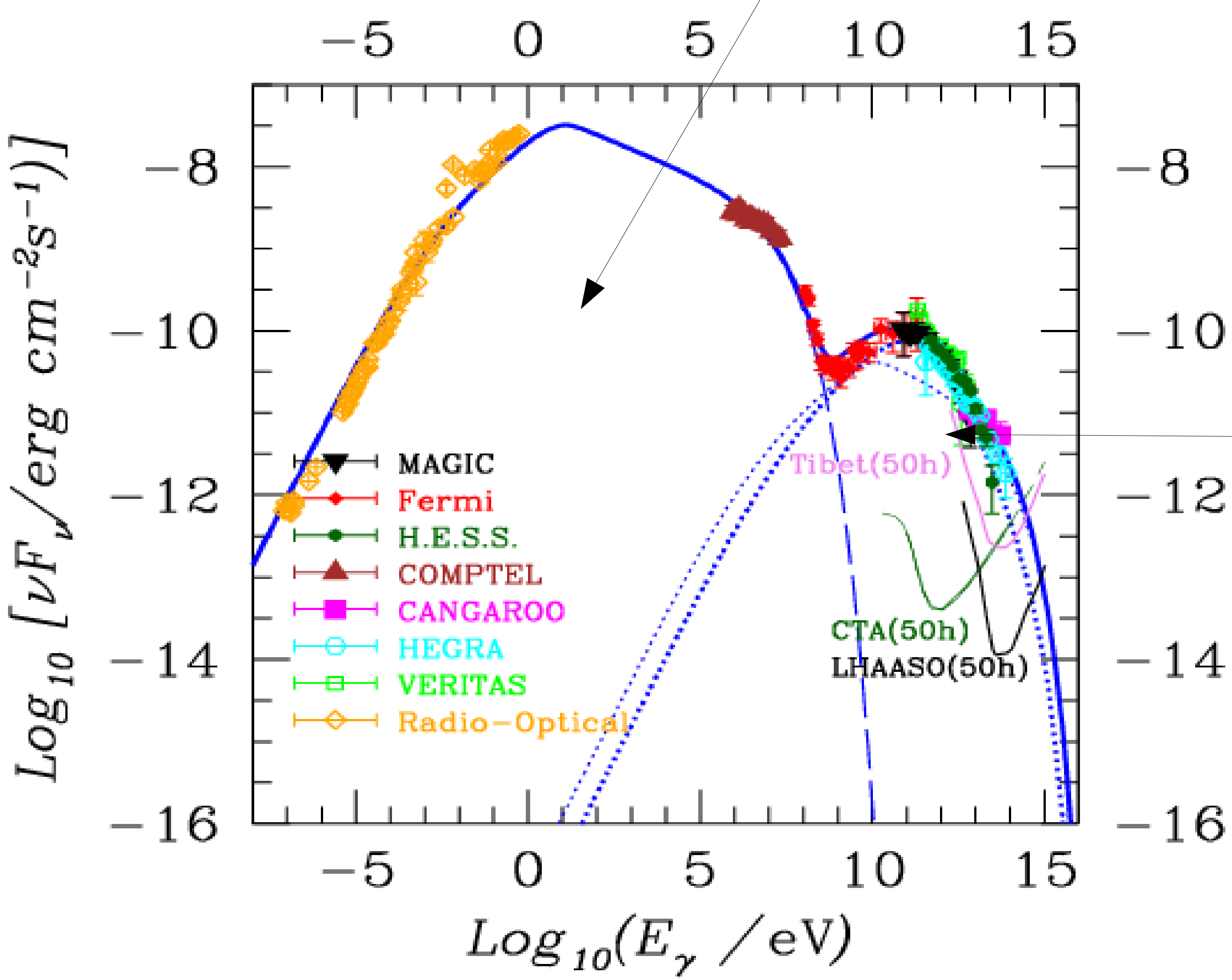
Note again the similarity with synchrotron radiation [power law p gives a power law with a slope (p-1)/2]



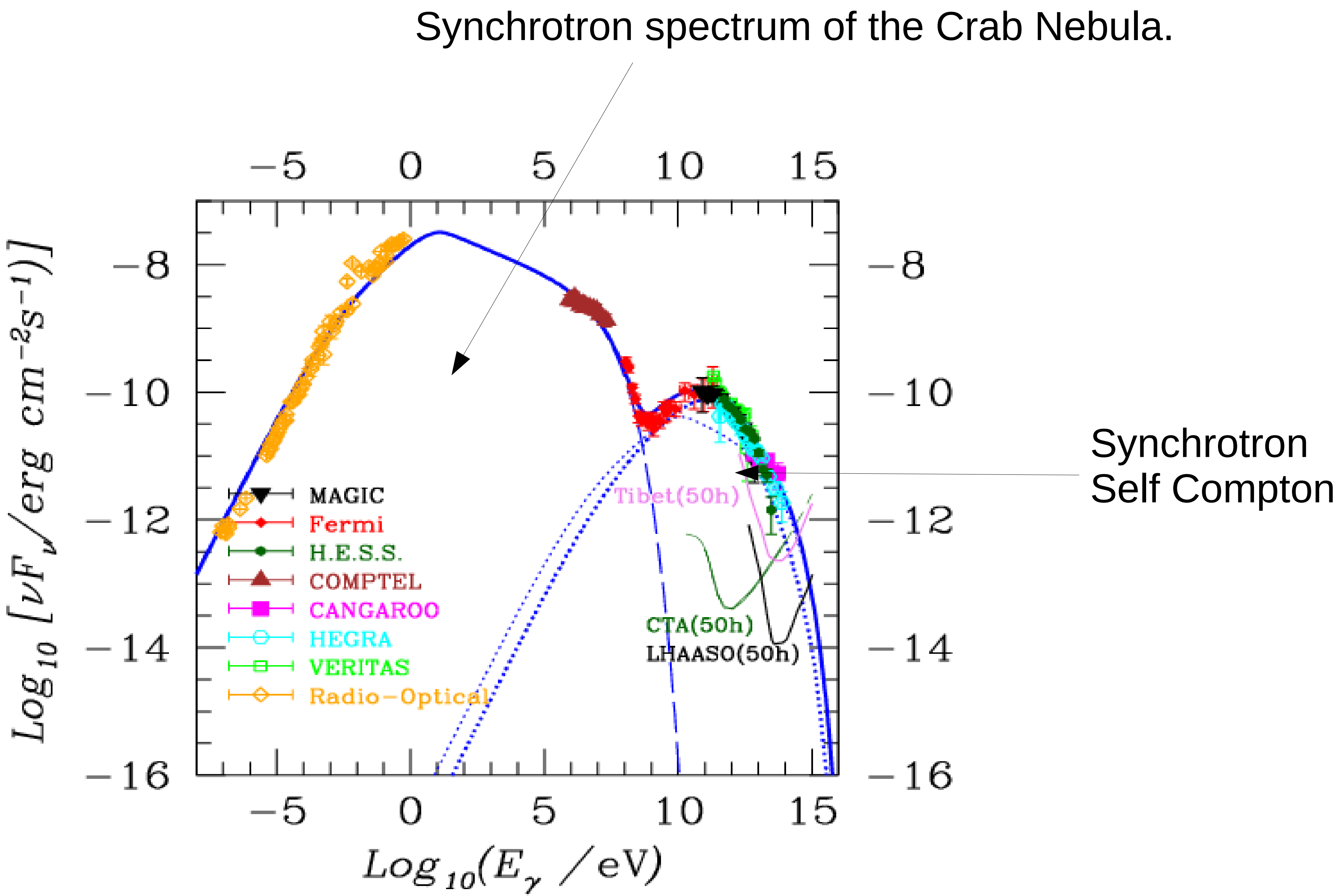

Synchrotron



Synchrotron spectrum of the Crab Nebula.



What is this bump here?



Synchrotron spectrum of the Crab Nebula.
Synchrotron
Self Compton
−5
0
5
10
15
−8
−10
−12
−14
−16
$Log_{10}\,[\nu F_\nu / erg\ cm^{-2} s^{-1})]$
$Log_{10}(E_\gamma\ / \mathrm{eV})$
MAGIC
Fermi
H.E.S.S.
COMPTEL
CANGAROO
HEGRA
VERITAS
Radio−Optical
Tibet(50h)
CTA(50h)
LHAASO(50h)