Lecturer

- Dr. Alessandro Patruno
- Science Park 904, C4-172
- Tel. 020 525 7472
- Research Field: *Accreting Neutron Stars, Pulsars, Stellar Binary Evolution, Intermediate Mass Black Holes*
- Teaching Material: Blackboard

Teaching Assistant

- Drs. Salome' Dibi-Rousselle
- Science Park 904, C4-273
- Tel. 020 525 7479
- Research Field: *Accreting X-ray binaries, Relativistic Jets*

Lecture Plan

- **L1:** Introduction to the Equation of State of Cold Catalized Matter
- **L2:** Electrostatic and beta-decay equilibrium corrections to the EoS of CCM.
- **L3:** White Dwarfs
- **L4:** Introduction to General Relativity
- **L5**: Spherically Symmetric Gravitational Field
- **L6:** Neutron Stars
- **L7:** Pulsars
- **L8:** Binary evolution and Accretion discs (part 1)
- **L9:** Accretion discs (part 2)
- **L10:** Observational techniques
- **L11:** Cooling of Neutron Stars
- **L12:** Black Holes 1
- **L13:** Black Holes 2
- **L14:** Special Session: preparation for written exam

Question: does this suite your expectations ?

How you will be evaluated

- Final written exam (compulsory, on June 17)
- Assignments (not compulsory, you need to hand at least 10 out of 13 homeworks, otherwise they do not count for final grade).
- Average homework grade is calculated on the top 10 scores

Grade of assignments is used ONLY if it is higher than the final exam, so to improve the final grade. It's weight is 50%

Some assignments can also be done in groups of two people. However, this will be specified on a case by case basis.

Text Book & Other Tools

- Shapiro & Teukolsky "Black Holes, White Dwarfs, and Neutron Stars" (**required**)
- Frank, King & Rane "Accretion Power in Astrophysics"
- Kippenhan & Wiegert "Stellar Structure and Evolution"
- Zel'dovic & Novikov "Relativistic Astrophysics, Vol. 1"

Computer demonstrations with use of Scientific Software

*Programming (**http://www.student.uva.nl/software**): Use of the Software "Mathematica"*

What we will do during the Lectures

- *Explanation of the Physics of Compact Stars*
- *Open Discussions*
- *Practical Demonstrations and experiments with possibly some instrumentation used to study Compact Objects*

What we will do *after* the Lectures

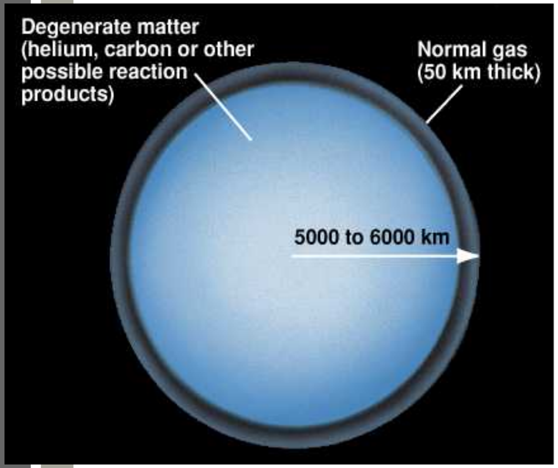
- *Use Blackboard for:*

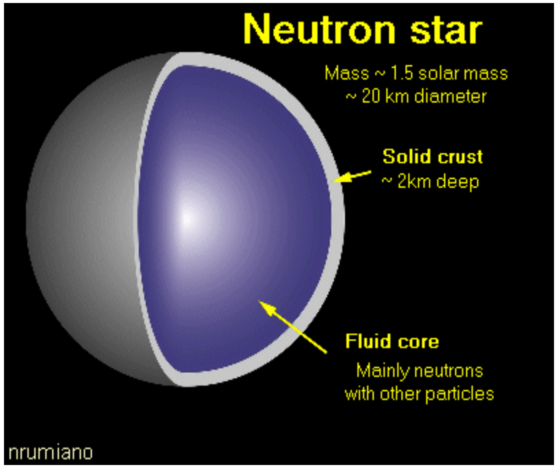
1. *Download Lecture Materials*
2. *Teaching Assignments*
3. *Start a discussion on any topic related with the course (check under "Course Material" --> "Group HEA 2011")*
4. *Announcements*
5. *Find useful links that you can use during (and after) the course*

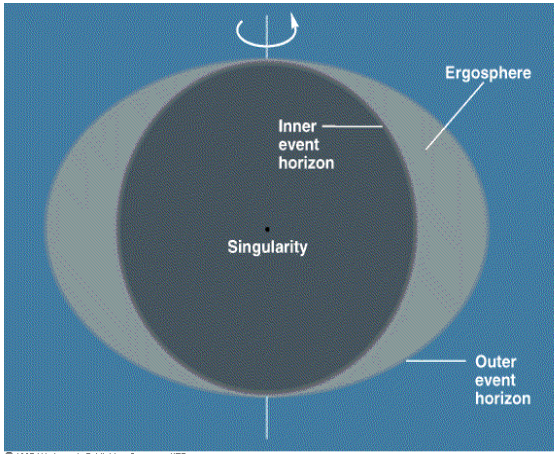
What you will do during the Werkolleges

- *Exercises and Examples*
- *Some integration to the main Lectures*
- *Programming (**http://www.student.uva.nl/software**): Use of the Software "Mathematica"*

Compact Objects as end product of *Stellar Evolution*

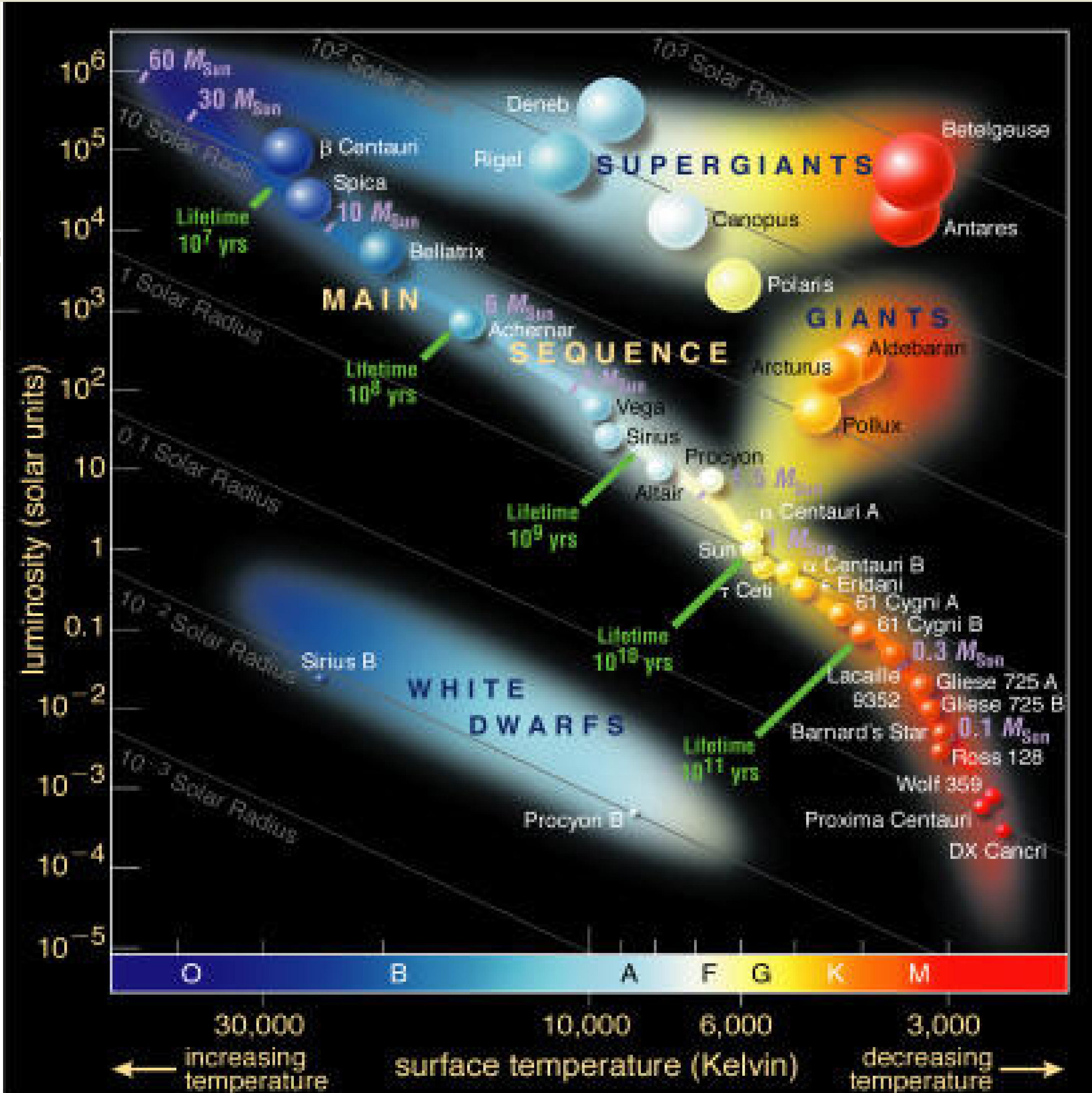






Lecture 1

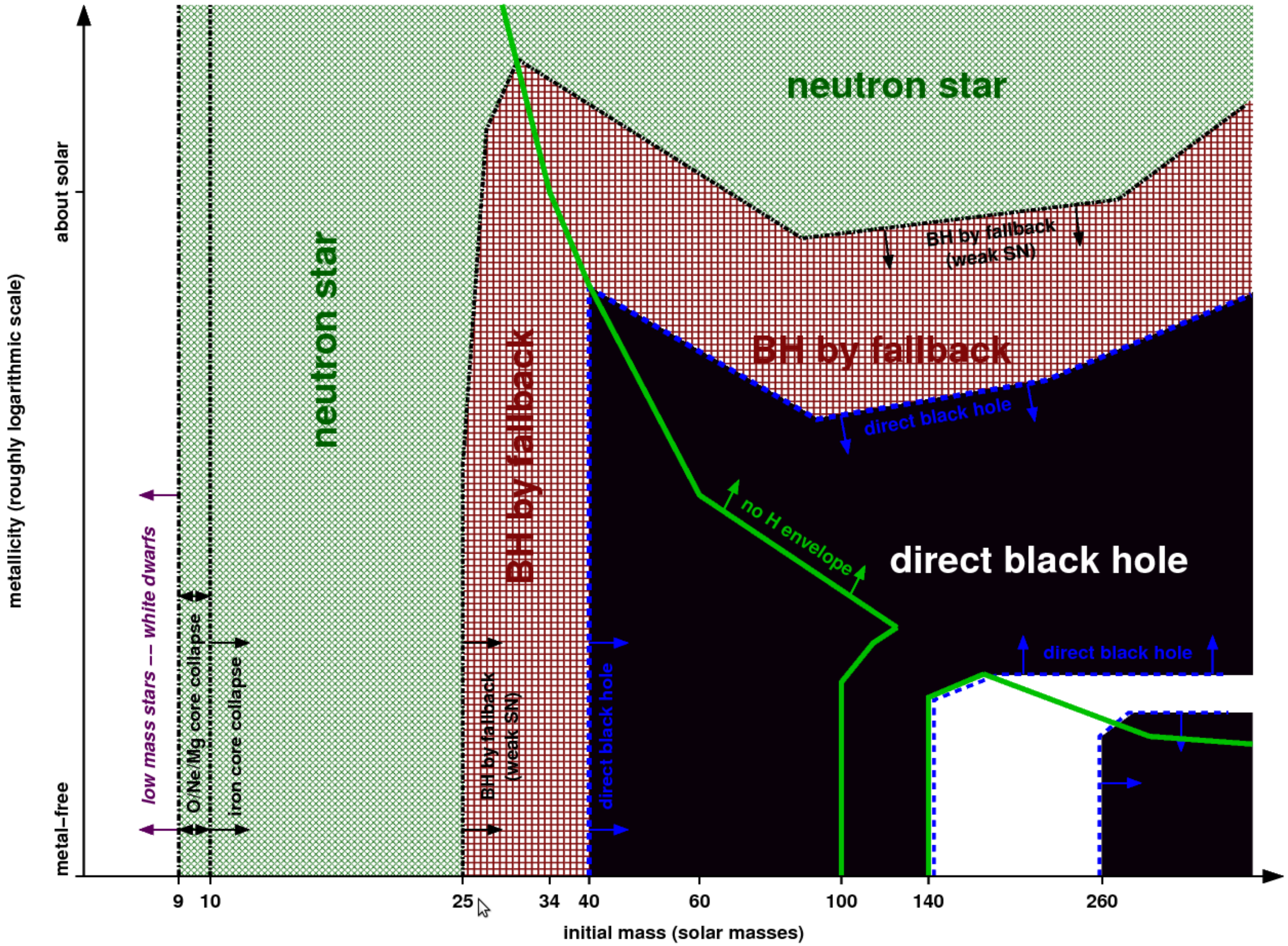
- Which forces stabilize White Dwarfs and Neutron Stars ?
- Degenerate Pressure and Fermi Dirac Distribution
- Equation of State at densities below 4x10^11 g/cm3 for an Ideal degenerate Fermi Gas



60 M_{Sun}
30 M_{Sun}
10 M_{Sun}
6 M_{Sun}
1.5 M_{Sun}
1 M_{Sun}
0.3 M_{Sun}
0.1 M_{Sun}
10^2 Solar Radii
10^3 Solar Radii
10 Solar Radii
1 Solar Radius
0.1 Solar Radius
10^{-2} Solar Radius
10^{-3} Solar Radius
Deneb
Rigel
Betelgeuse
SUPERGIANTS
Canopus
Antares
Polaris
GIANTS
Aldebaran
Arcturus
Pollux
β Centauri
Spica
Bellatrix
MAIN
SEQUENCE
Achernar
Vega
Sirius
Procyon
Altair
α Centauri A
Sun
α Centauri B
τ Ceti
ε Eridani
61 Cygni A
61 Cygni B
Lacaille 9352
Gliese 725 A
Gliese 725 B
Barnard's Star
Ross 128
Wolf 359
Proxima Centauri
DX Cancri
Lifetime 10^7 yrs
Lifetime 10^8 yrs
Lifetime 10^9 yrs
Lifetime 10^{10} yrs
Lifetime 10^{11} yrs
Sirius B
WHITE DWARFS
Procyon B
luminosity (solar units)
10^6
10^5
10^4
10^3
10^2
10
1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
O
B
A
F
G
K
M
30,000
10,000
6,000
3,000
increasing temperature
surface temperature (Kelvin)
decreasing temperature

How Massive *Single* Stars End their Lives

Heger et al. (2003), The Astrophysical Journal, Volume 591, Issue 1, pp. 288-300



Standard Model of

FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the unified theory of weak and electromagnetic interactions (electroweak). Gravity is included on this chart because it is one of the fundamental interactions even though not part of the "Standard Model."

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

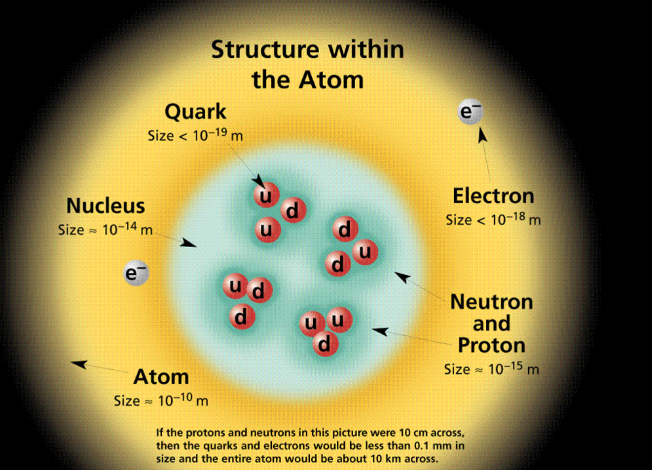
Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3

Spin is the intrinsic angular momentum of particles. Spin is given in units of ħ, which is the quantum unit of angular momentum, where $\hbar = h/2\pi = 6.58\times10^{-25}$ GeV s = 1.05×10^{-34} J s.

Electric charges are given in units of the proton's charge. In SI units the electric charge of the proton is 1.60×10^{-19} coulombs.

The **energy** unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in GeV/c^2 (remember $E = mc^2$), where 1 GeV = 10^9 eV = 1.60×10^{-10} joule. The mass of the proton is 0.938 GeV/c^2 = 1.67×10^{-27} kg.



If the protons and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

BOSONS

force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	–1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Color Charge

Each quark carries one of three types of "strong charge," also called "color charge." These charges have nothing to do with the colors of visible light. There are eight possib types of color charge for gluons. Just as elec cally-charged particles interact by exchanging photons, in strong interactions color-charged pa ticles interact by exchanging gluons. Leptons, photons, and ***W*** and ***Z*** bosons have no strong interactions and hence no color charge.

Quarks Confined in Mesons and Baryons

One cannot isolate quarks and gluons; they are confined in color-neutral particles called **hadrons**. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the en gy in the color-force field between them increases. This energy eventually is converted into ac tional quark-antiquark pairs (see figure below). The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge. Two types of hadrons have been observed in nature: **mesons** $q\bar{q}$ and **baryons** qqq.

Residual Strong Interaction

The strong binding of color-neutral protons and neutrons to form nuclei is due to residual strong interactions between their color-charged constituents. It is similar to the residual el trical interaction that binds electrically neutral atoms to form molecules. It can also be viewed as the exchange of mesons between the hadrons.

PROPERTIES OF THE INTERACTIONS

Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$

Baryons are fermionic hadrons.
There are about 120 types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	–1	1.672	3/2

Mesons $q\bar{q}$

Mesons are bosonic hadrons.
There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spi
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

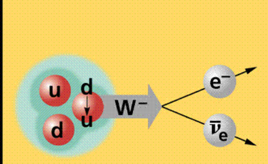
Matter and Antimatter

For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or – charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., Z^0, γ, and $\eta_c = c\bar{c}$, but not $K^0 = d\bar{s}$) are their own antiparticles.

Figures

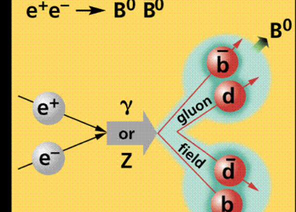
These diagrams are an artist's conception of physical processes. They are *not* exact and have *no* meaningful scale. Green shaded areas represent the cloud of gluons or the gluon field, and red lines the quark paths.

$n \rightarrow p\, e^-\, \bar{\nu}_e$



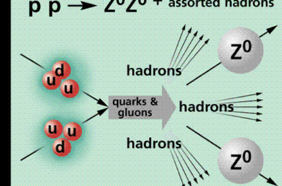
A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.

$e^+e^- \rightarrow B^0\, \bar{B}^0$



An electron and positron (antielectron) colliding at high energy can annihilate to produce B^0 and $\bar{B}^0$ mesons via a virtual Z boson or a virtual photon.

$p\, p \rightarrow Z^0 Z^0$ + assorted hadrons



Two protons colliding at high energy can produce various hadrons plus very high mass particles such as Z bosons. Events such as this one are rare but can yield vital clues to the structure of matter.

The Particle Adventure

Visit the award-winning web feature *The Particle Adventure* at **http://ParticleAdventure.org**

This chart has been made possible by the generous support of:
U.S. Department of Energy
U.S. National Science Foundation
Lawrence Berkeley National Laboratory
Stanford Linear Accelerator Center
American Physical Society, Division of Particles and Fields
BURLE INDUSTRIES, INC.

http://CPEPweb.org

Strength of Fundamental Forces

Force	Coupling Constant	Relative Strength
Strong	α_s	1
Electromagnetic	α	1/137
Weak	α_w	10^{-6}
Gravitational	α_G	10^{-40}

Fermi Dirac Statistic

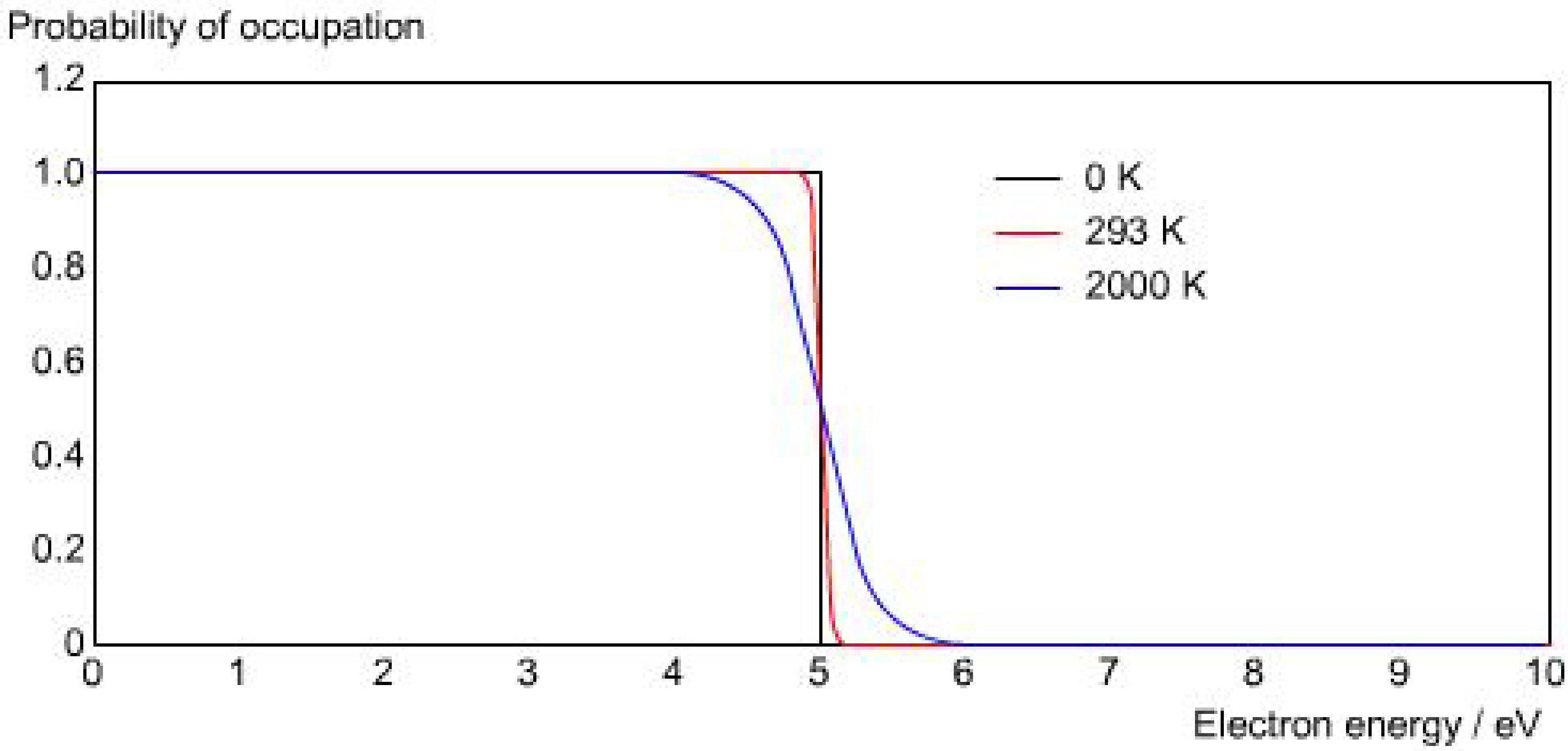
Hypotheses

1. Particles are indistinguishable

2. Fermions obey the Pauli Exclusion Principle

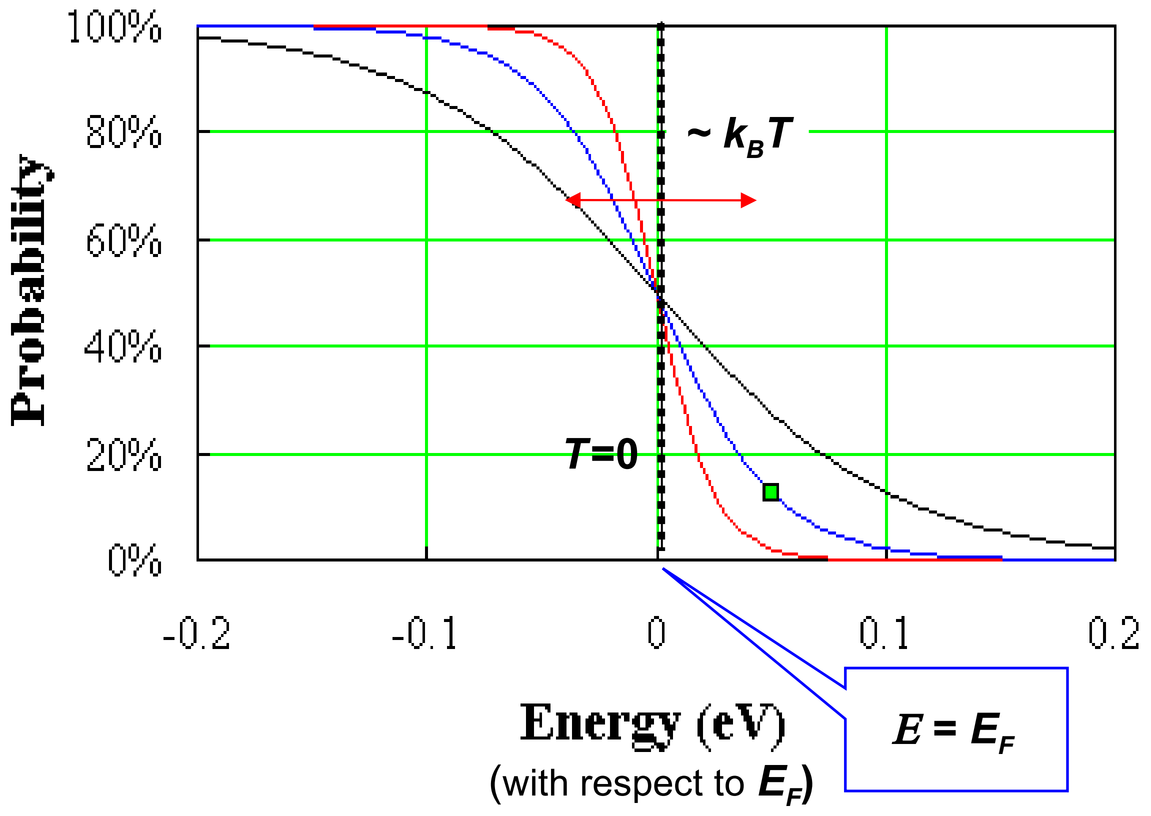
3. Total number of particles and total energy is fixed, and all the many arrangements for the particles within the allowed state are indistinguishable

Fermi Dirac Statistic



Fermi-Dirac distribution for several temperatures

Fermi Dirac Statistic



Summary of Lecture 1

- Gravity is the dominant force in compact stars, and it is balanced by degeneracy pressure provided by electrons and/or neutrons
- The degeneracy pressure is a consequence of the Pauli exclusion principle that applies to fermions
- The Equation of State is a relation between thermodynamic state variables of a system and describes the properties of matter inside WDs and NS
- The simplest EoS is the Chandrasekhar EoS of an Ideal Fermi Gas, with pressure that depends **only** on density (i.e., no T dependence)

Reading: Section 2.1 of S&T
Study: Lecture notes (see BlackBoard) + Section 2.2 & 2.3 S&T
Homeworks: Hand in on Friday, individual exercises (no groups)

Lecture Plan

- **L1:** Introduction to the Equation of State of Cold Catalized Matter
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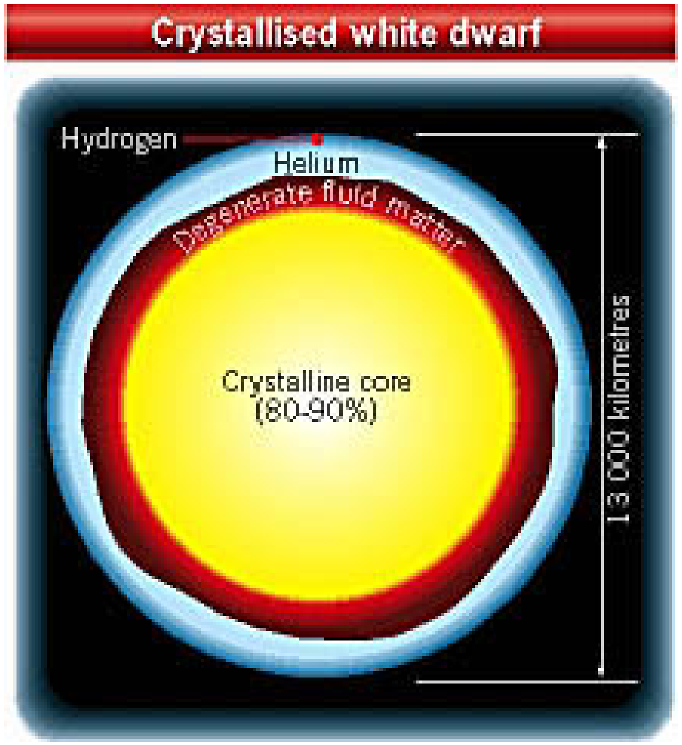
Study: Lecture notes (see BlackBoard) + Section 2.2 & 2.3 S&T
Homeworks: Hand in on Friday, individual exercises (no groups)

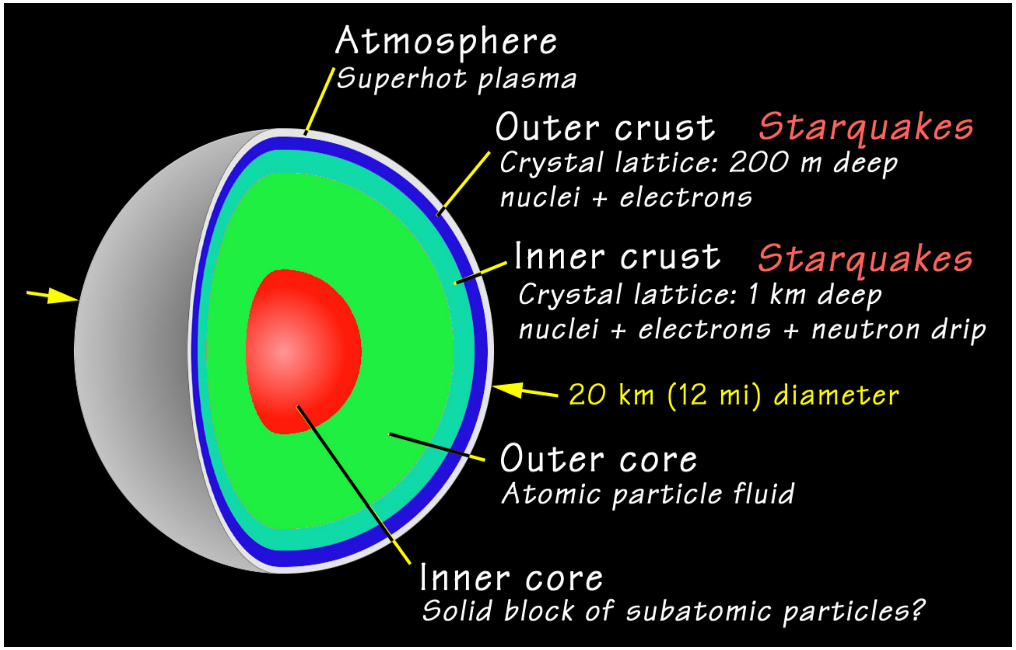
Lecture 2

- Electrostatic Correction to the Chandrasekhar EoS → Good approximation to describe WDs
- EoS of separated nuclei in beta-equilibrium with a relativistic electron gas → First order approximation for neutron star crust up to neutron drip
- Inverse beta equilibrium for an ideal n-p-e gas → First order approximation above neutron drip

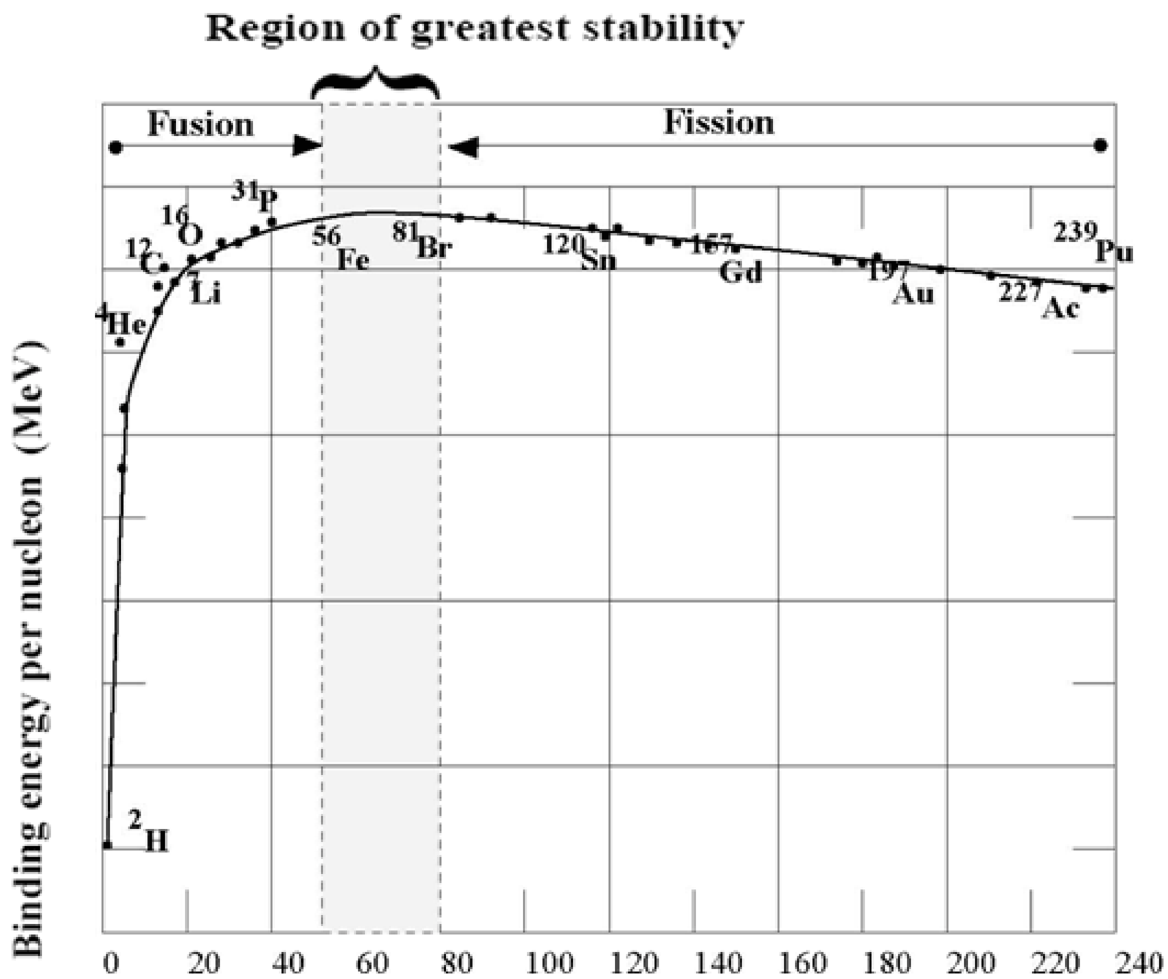
Study: Section 2.4 (up to page 32, included), 2.5 & 2.6 S&T
Reading: Section 2.7 S&T
Homeworks: Hand in on Tuesday, individual exercises (no groups)

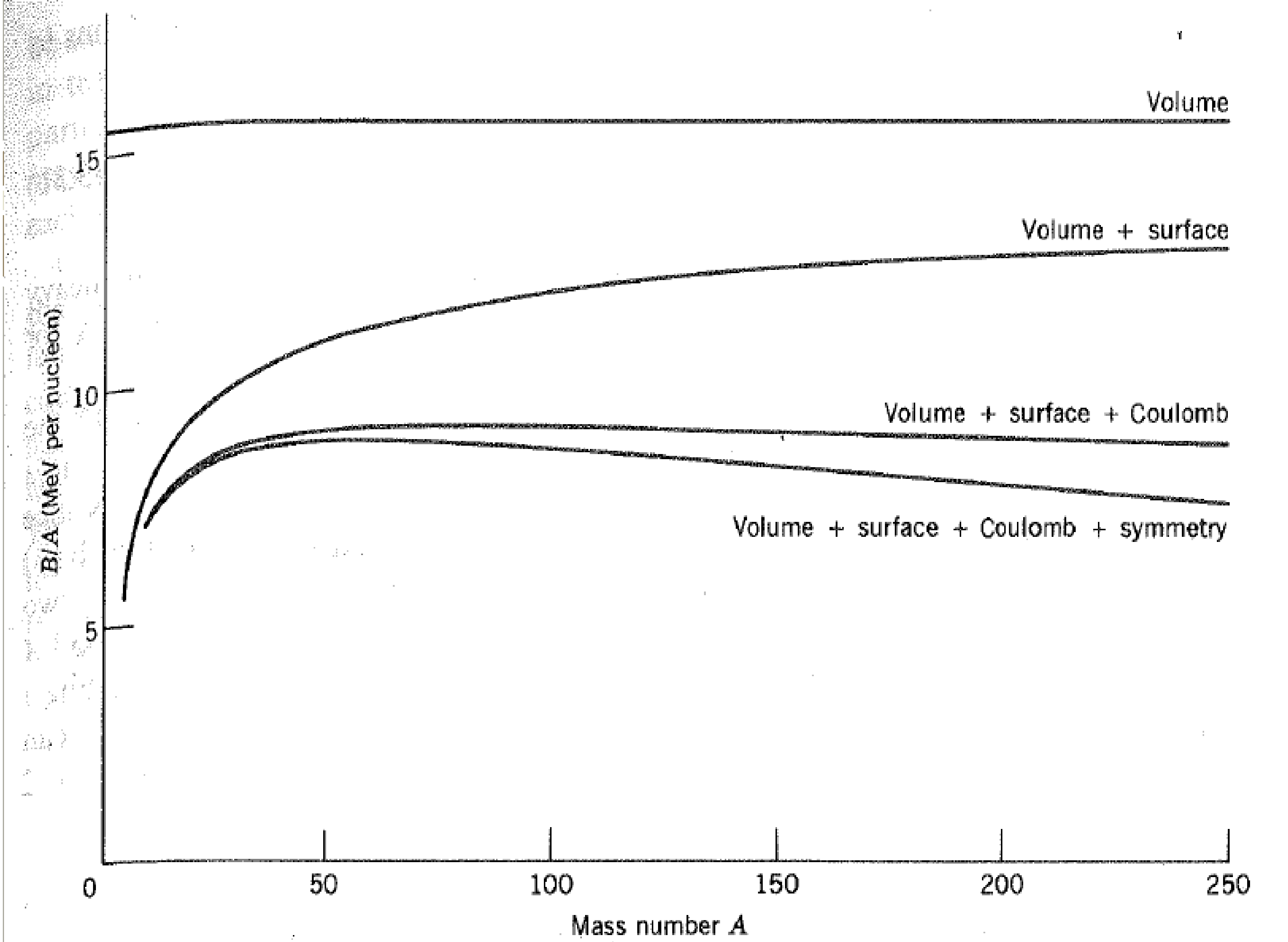


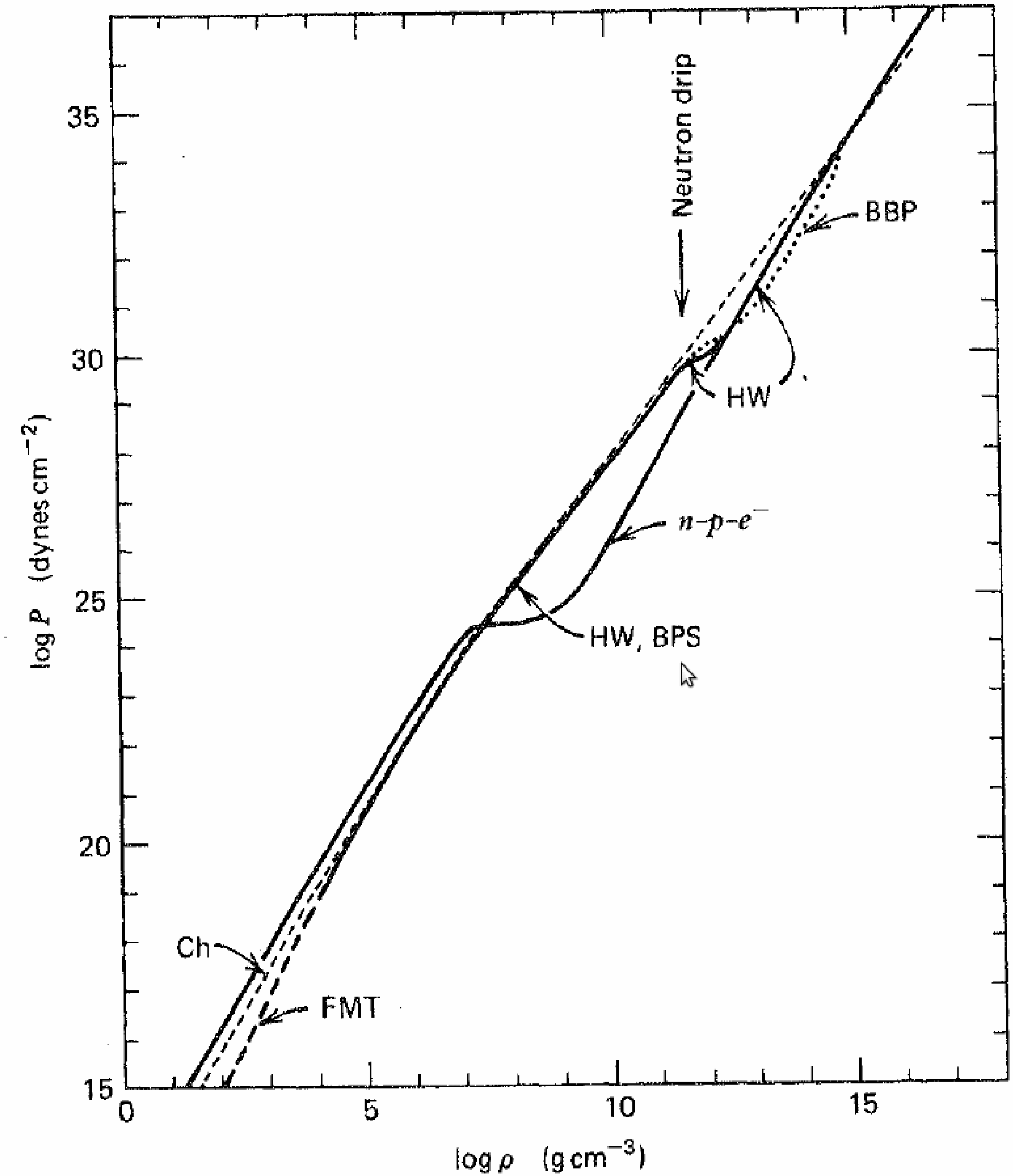
Neutron Star



Binding Energy of Nuclei




Volume
Volume + surface
Volume + surface + Coulomb
Volume + surface + Coulomb + symmetry
B/A (MeV per nucleon)
15
10
5
0
50
100
150
200
250
Mass number A



35
30
25
20
15
0
5
10
15
log P (dynes cm^{-2})
log ρ (g cm^{-3})
Neutron drip
BBP
HW
n–p–e^-
HW, BPS
Ch
FMT

Lecture Plan

- **L1:** Introduction to the Equation of State of Cold Catalized Matter
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Summary of Lecture 2

- Ideal Cold Fermi Gas EoS (Chandrasekhar EoS) can be improved by considering electrostatic corrections due to electron-electron and electron-ion interaction.
- Beta equilibrium needs also to be considered when calculating realistic EoS.
- Ideal Cold n-p-e Gas EoS can be calculated assuming beta equilibrium and zero total charge
- At large densities (rho>1E+11 g/cm3), the electron to proton to neutron ratio is 1:1:8 when considering beta equilibrium of the n-p-e gas.

Clarification on n-p-e gas

(1) Beta decay : $n \rightarrow p + e^- + \bar{\nu}_e$

(2) Inverse beta decay – (electron capture process): $p + e^- \rightarrow n + \nu_e$

(3) Inverse beta decay + (positron emission process): $\cancel{p \rightarrow n + e^+ + \nu_e}$ **FORBIDDEN**

Beta equilibrium is achieved when (1) and (2) balance

Q1: Is beta equilibrium possible at all densities ?
A1: No, because electrons need to have enough energy $Q=(m_n-m_p)c^{٢}=١.٢٩\ MeV$ to allow reaction (2)

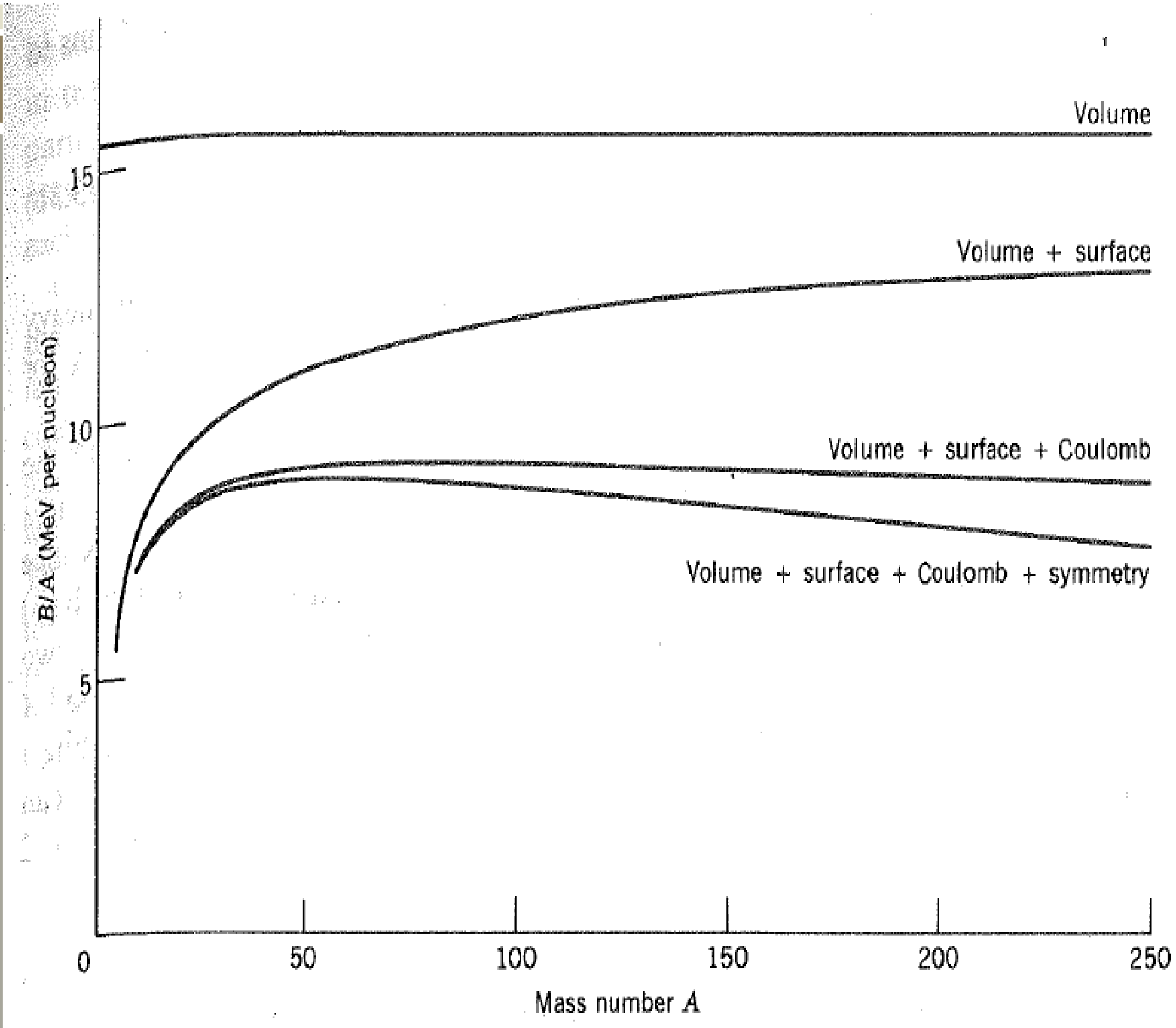
Minimum density to allow reaction (2) is $\rho > 1.2 \times 10^7\ g\ cm^{-3}$

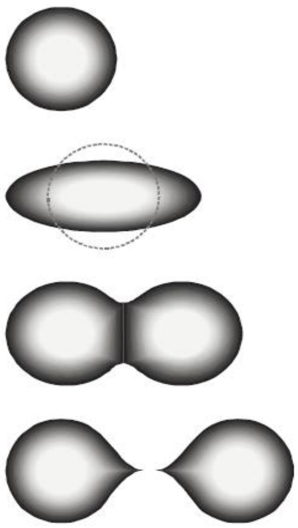
Q2: Is reaction (1) strongly suppressed at high densities ?
A2: Yes, because electrons produced in reaction (1) need to have more energy than the electron Fermi energy, otherwise they cannot exist (Pauli exclusion principle).

Minimum proton/neutron ratio is reached at density $\rho > 7.8 \times 10^{11}\ g\ cm^{-3}$

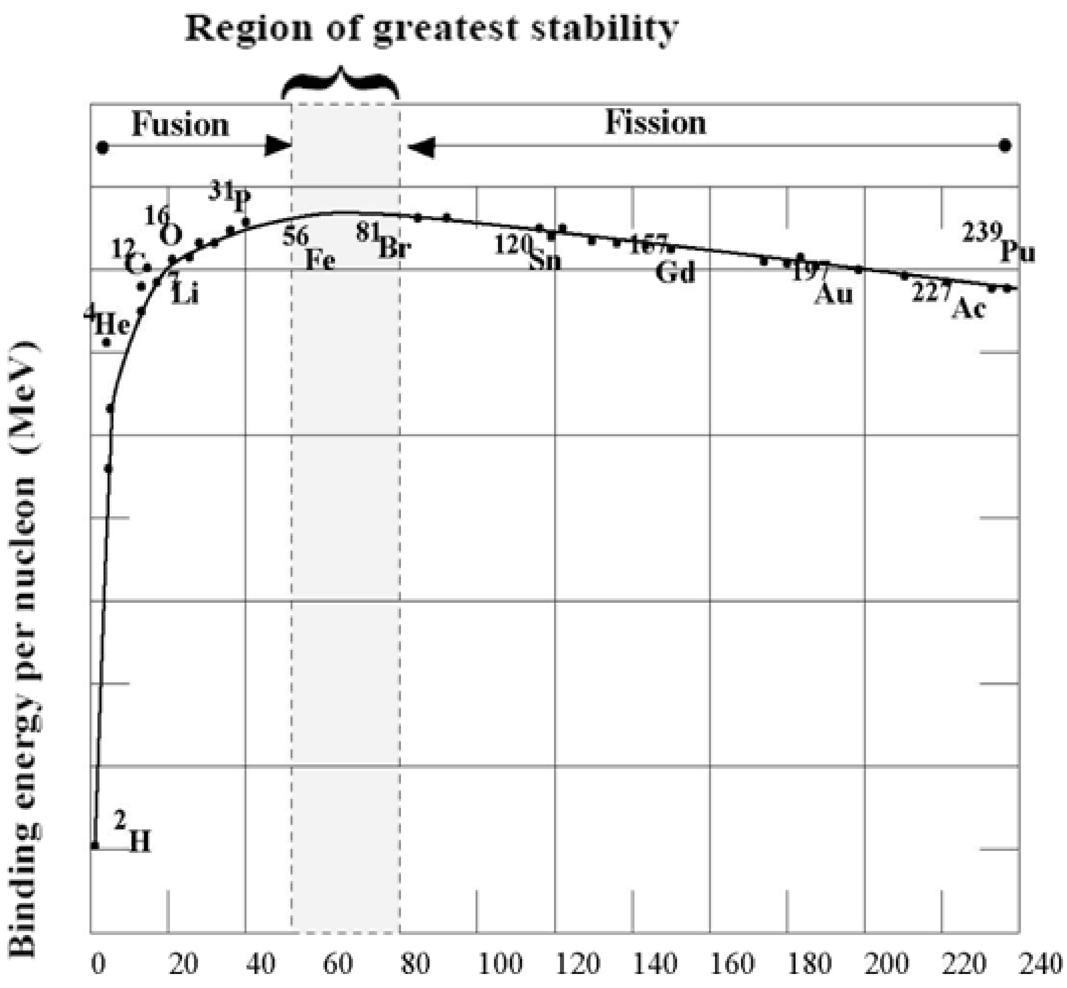
NOTE: If you change n-p-e gas with nuclei + electrons, the same situation applies, although the density threshold changes

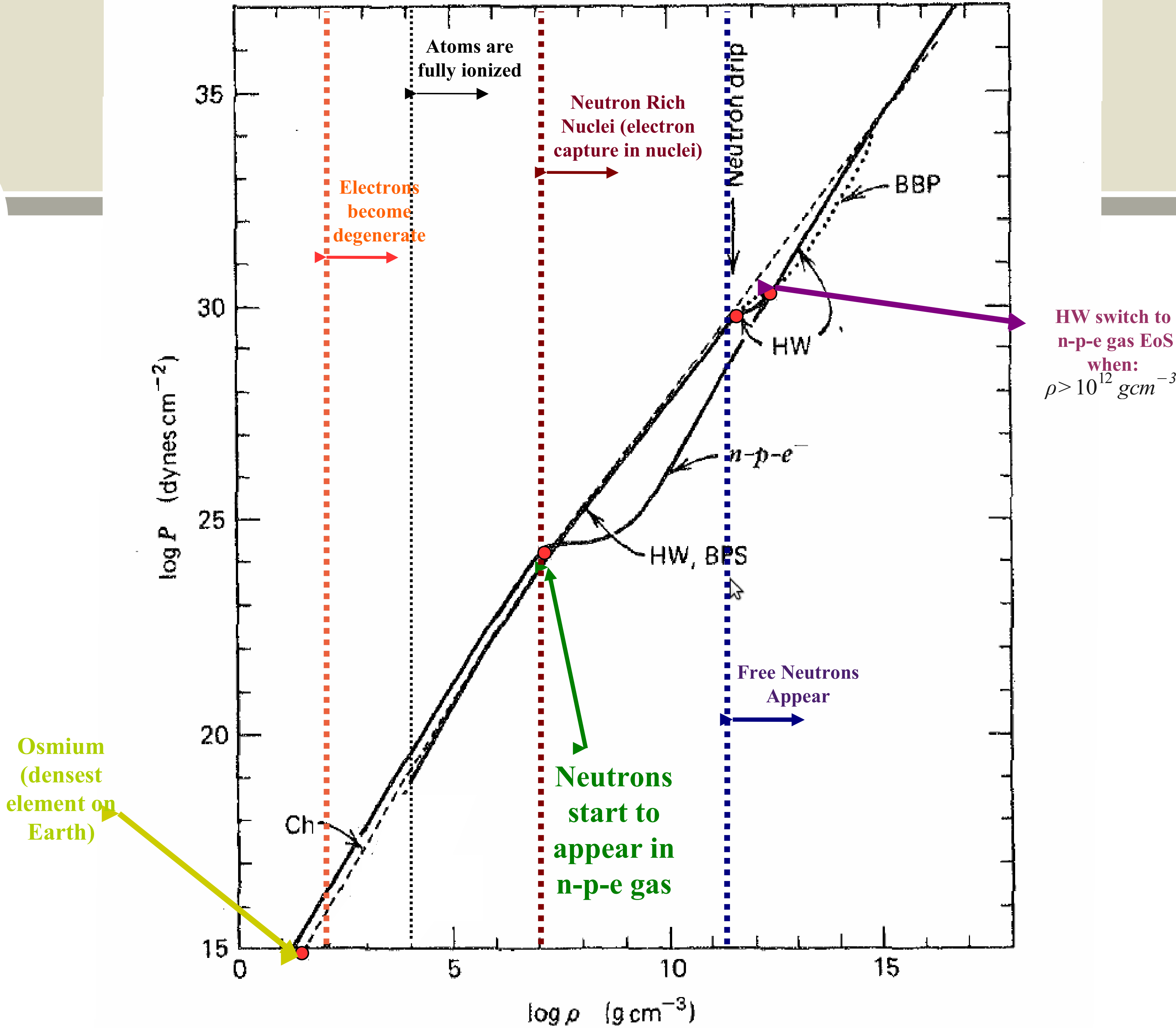
Liquid Drop Model





Binding Energy of Nuclei

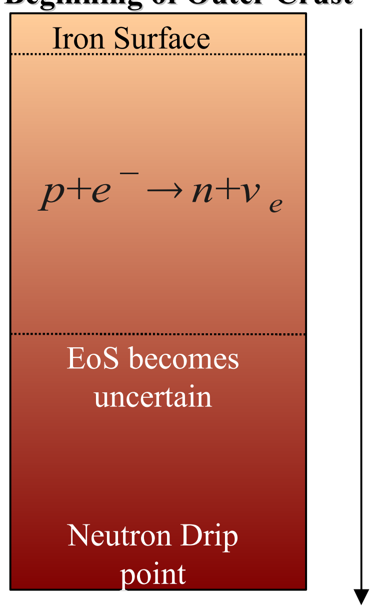




Atoms are fully ionized
Neutron Rich Nuclei (electron capture in nuclei)
Neutron drip
Electrons become degenerate
BBP
HW switch to n-p-e gas EoS when: $\rho > 10^{12}\ gcm^{-3}$
HW
n-p-e^-
HW, BPS
Free Neutrons Appear
Osmium (densest element on Earth)
Ch
Neutrons start to appear in n-p-e gas
log P (dynes cm^{-2})
log ρ (g cm^{-3})
35
30
25
20
15
0
5
10
15

Ground State Elements below Neutron Drip

Beginning of Outer Crust



End of Outer Crust

$\rho_{\max}$ [g cm^{-3}]	Element	Z	N	R_{cell} [fm]
8.02×10^{6}	^{56}Fe	26	30	1404.05
2.71×10^{8}	^{62}Ni	28	34	449.48
1.33×10^{9}	^{64}Ni	28	36	266.97
1.50×10^{9}	^{66}Ni	28	38	259.26
3.09×10^{9}	^{86}Kr	36	50	222.66
1.06×10^{10}	^{84}Se	34	50	146.56
2.79×10^{10}	^{82}Ge	32	50	105.23
6.07×10^{10}	^{80}Zn	30	50	80.58
8.46×10^{10}	^{82}Zn	30	52	72.77
9.67×10^{10}	^{128}Pd	46	82	80.77
1.47×10^{11}	^{126}Ru	44	82	69.81
2.11×10^{11}	^{124}Mo	42	82	61.71
2.89×10^{11}	^{122}Zr	40	82	55.22
3.97×10^{11}	^{120}Sr	38	82	49.37
4.27×10^{11}	^{118}Kr	36	82	47.92

Experimental values

Theoretical values

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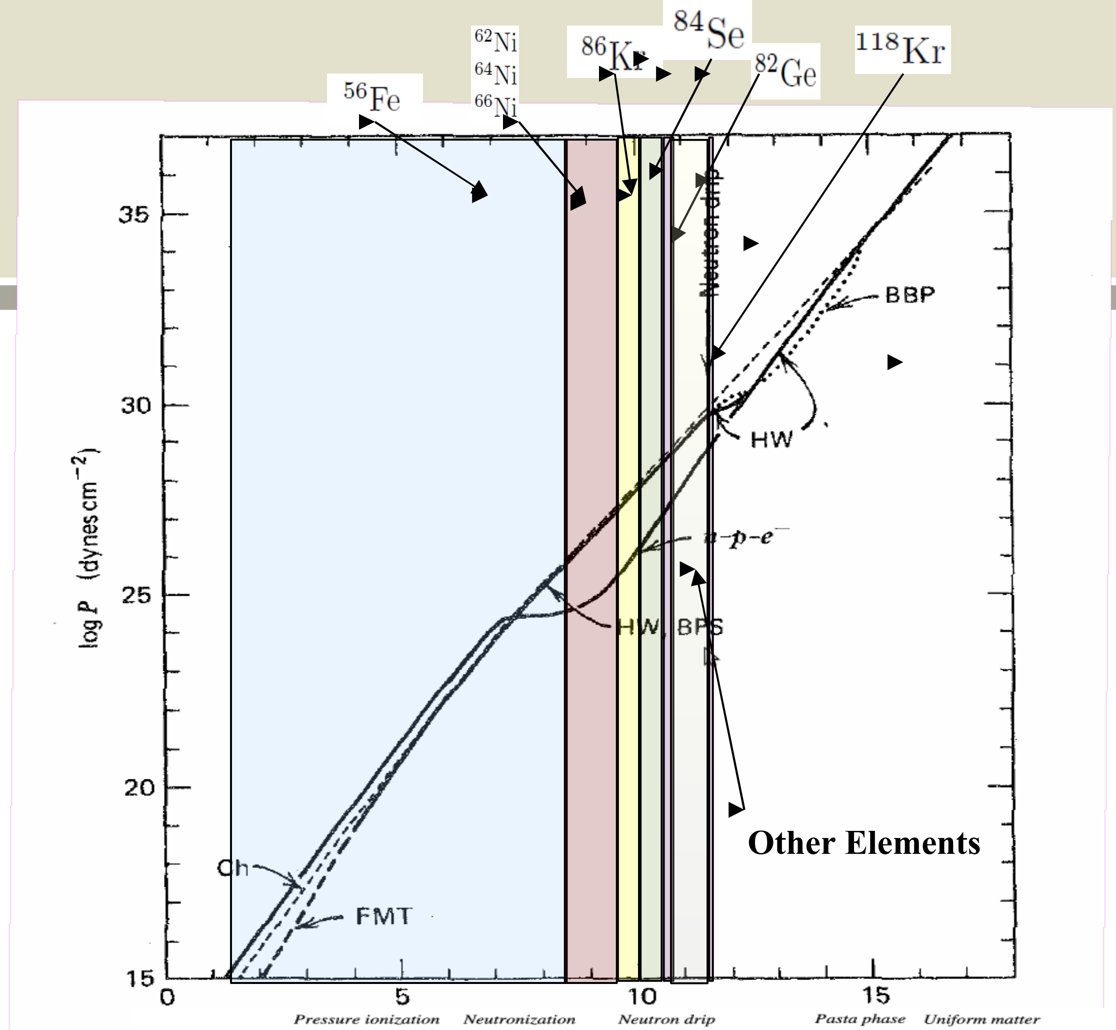
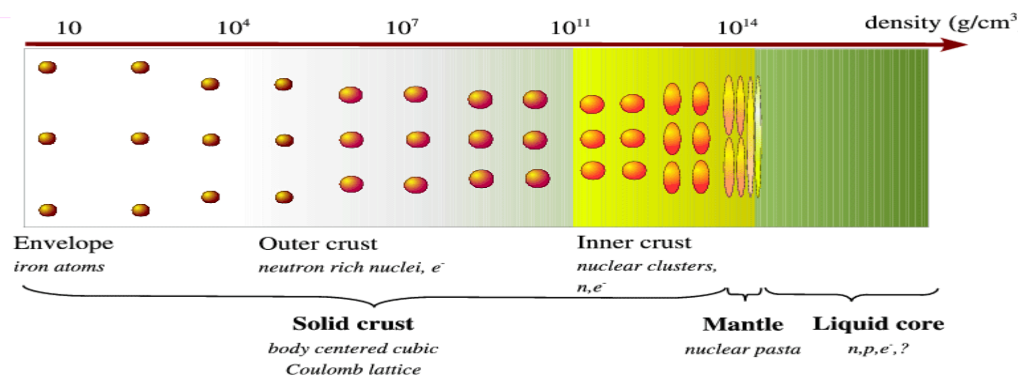
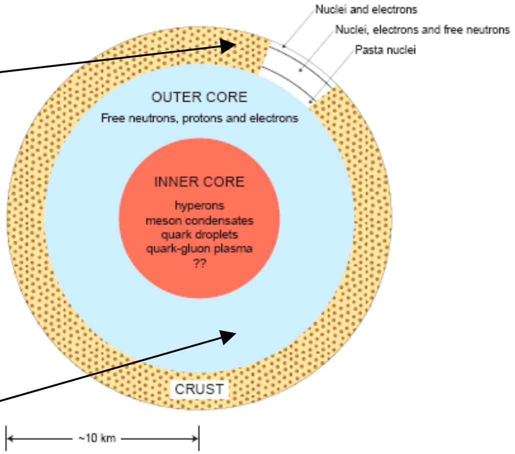
1.47×10^{11}	^{126}Ru	44	82	69.81
2.11×10^{11}	^{124}Mo	42	82	61.71
2.89×10^{11}	^{122}Zr	40	82	55.22
3.97×10^{11}	^{120}Sr	38	82	49.37
4.27×10^{11}	^{118}Kr	36	82	47.92





Summary of EoS of Cold Catalyzed Matter

EQUATION OF STATE	DENSITY RANGE (g/cm3)	COMPACT-OBJECT	Degree of Approximation
Ideal Cold Fermi Gas EoS (Chandrasekhar EoS) + Electrostatic Corrections	$0 < \rho < 10^4$ $0 < \rho < \rho_{drip}$	Part of Outer Crust in NS White Dwarfs	Good Good
Harrison-Wheeler EoS (Nuclei + Free Neutrons + Free Electrons in Beta Equilibrium)	$10^4 < \rho < 10^7$ $10^7 < \rho < 10^{12}$ $10^7 < \rho < \rho_{drip}$	NS Outer Crust Outer and Inner Crust + thin layer of Inner Core (until Nuclei completely dissolve) White Dwarfs	Good Good Poor (WD are not *yet* at beta equilibrium)
Ideal Cold n-p-e Gas	$10^7 < \rho < 10^{12}$ $10^{12} < \rho < 2\rho_{nucl}$	NS outer crust Outer core of NS	Poor Poor (in Lecture 6 we'll see why)

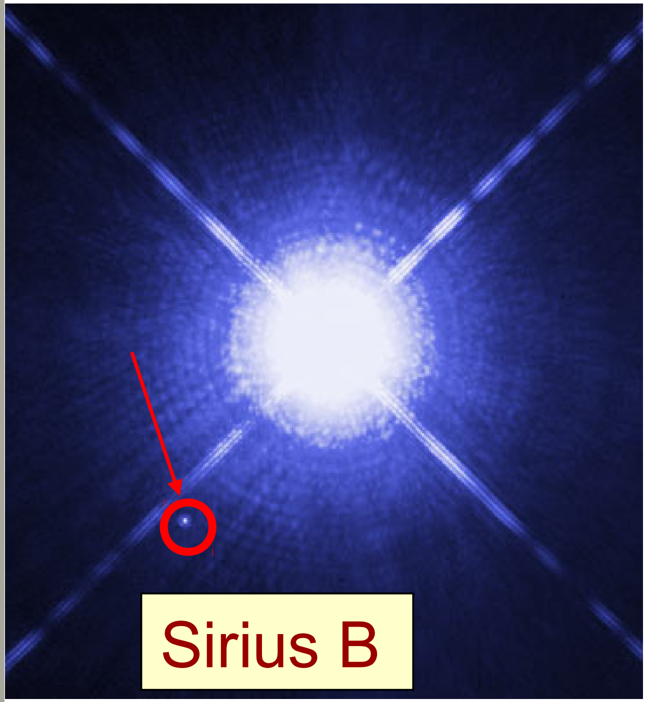


Lecture Plan

- **L1:** Introduction to the Equation of State of Cold Catalized Matter
- **L2:** Electrostatic corrections to the EoS of CCM and n-p-e gas
- **L3:** Harrison-Wheeler EoS.
- **L4: White dwarfs**
- **L5:** Chandrasekhar Limit. Introduction to General Relativity
- **L6**: Spherically Symmetric Gravitational Field
- **L7:** Neutron Stars
- **L8:** Pulsars
- **L9:** Binary evolution and Accretion discs
- **L10:** Observational techniques
- **L11:** Cooling of Neutron Stars
- **L12:** Black Holes 1
- **L13:** Black Holes 2
- **L14:** Special Session: preparation for written exam

White Dwarf Discovery

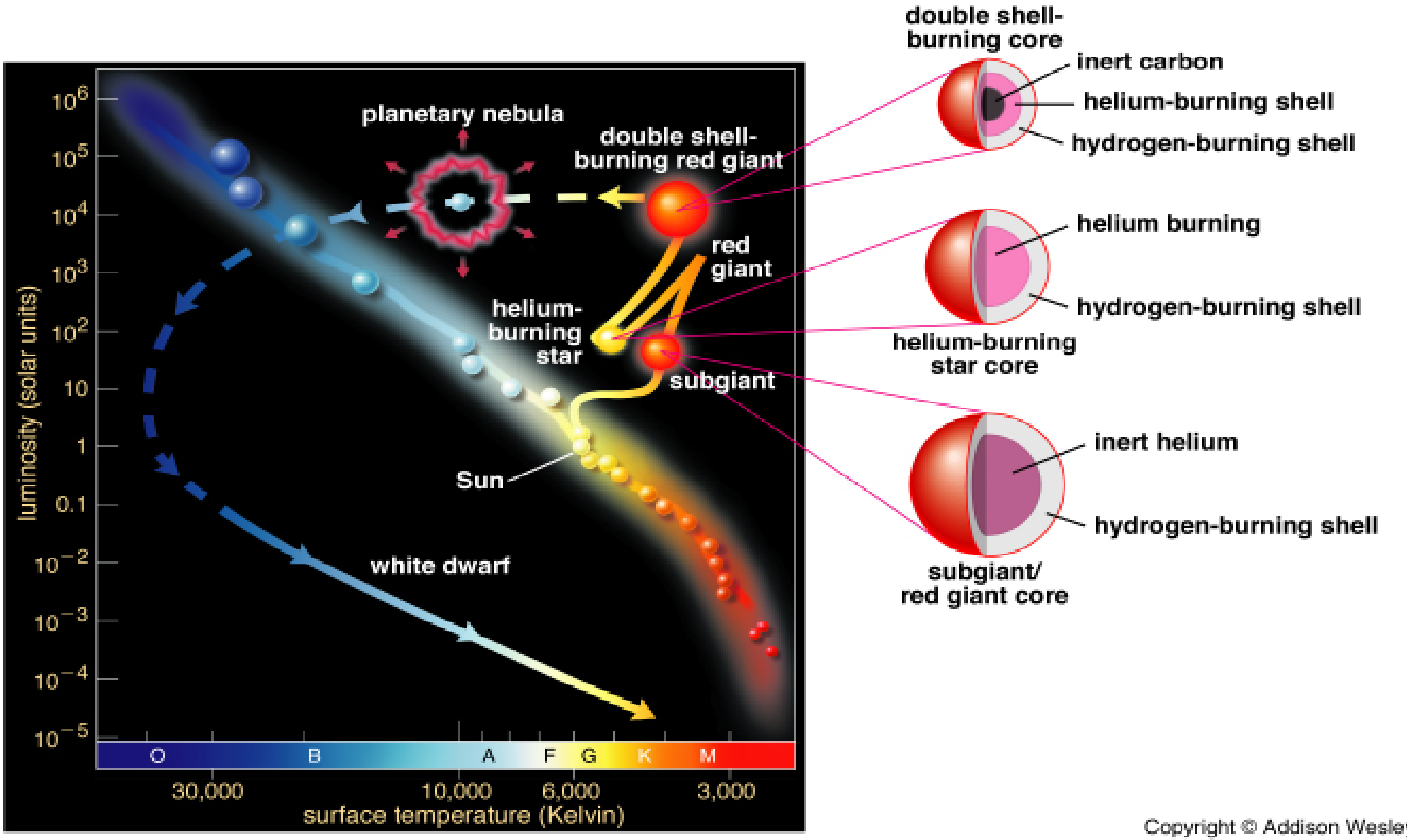
- First indirect evidence of a WD in Sirius B: 1834 (F.W. Bessel)
- First direct detection of the WD Sirius B: 1862 (A.G. Clark)
- First Measure of WD optical spectrum (M and R) in Sirius B: 1925 (W.S. Adams)
- First Isolated White Dwarf (Van Maanen Star): 1917 (van Maanen)



"I adhere to the conviction that the star Sirius is a binary system consisting of a visible and an invisible star. There's no reason to suppose that luminosity is an essential quality of cosmic bodies. Visibility of countless stars is no argument against the invisibility of countless others."

From a letter of F. Bessel to A. von Humboldt in 1836

Formation of White Dwarfs



Hydrostatic Equilibrium

with Hydrostatic Equilibrium + EoS you can build a model for your star (and calculate Mass and Radius)

$$m(r) = \int_0^{r'} \rho\, 4\pi r^2\, dr \rightarrow \frac{dm(r)}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = \frac{-G\, m(r)\, \rho}{r^2}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G \rho$$

+

Equation of State

Sructure of a star (e.g., M-R relation)

Lane Emden Equation

2nd order diff. equation

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right)+\theta^n=0$$

$$\varrho=\varrho_c\,\theta^n$$

$$r=a\,\xi \qquad a=\sqrt{\frac{(n+1)K\varrho_c^{1/n-1}}{4\pi G}}\,.$$

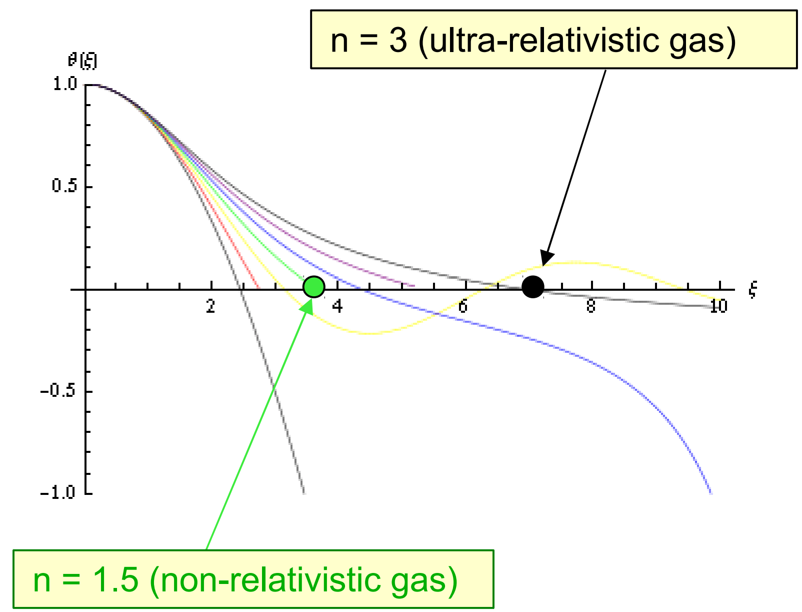
$$\Gamma=1+\frac{1}{n} \rightarrow \Gamma=\frac{5}{3}\,(\textit{nonrelativistic gas})$$

$$\Gamma=1+\frac{1}{n} \rightarrow \Gamma=\frac{4}{3}\,(\textit{ultrarelativistic gas})$$

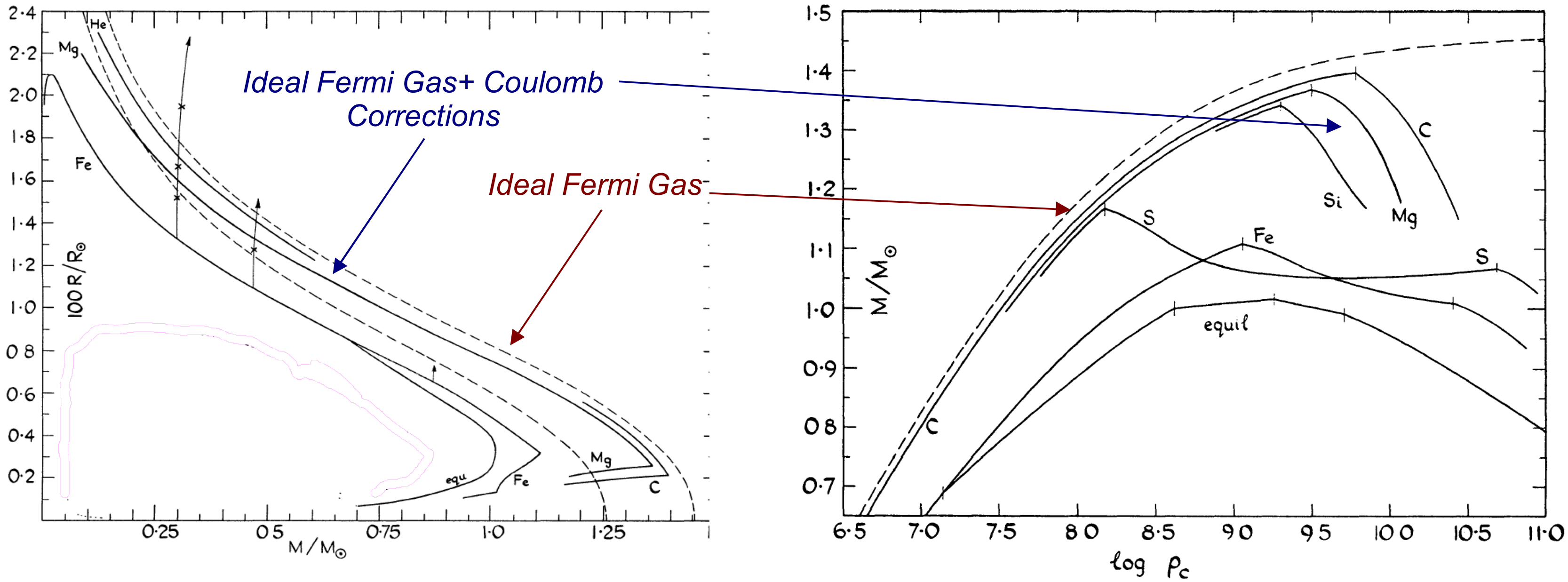
Boundary Conditions

$$\varrho_c=\varrho(0) \longrightarrow \theta(0)=1$$

$$\frac{dP}{dr}=0 \longrightarrow \theta'(0)=0$$



Solutions and Coulomb corrections



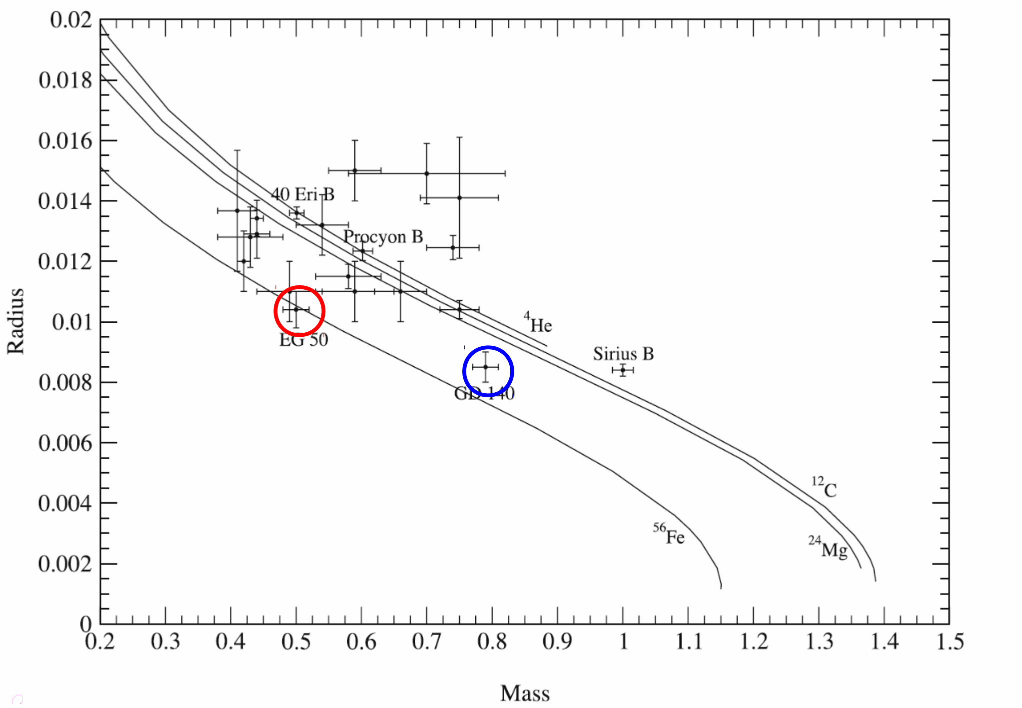
Hamada, E.E. Salpeter: Models for zero-temperature stars, ApJ 134, 683 (1961)

M-R relation for White Dwarfs

$$4\pi^2 A^3 = GP^2(M_1+M_2) \longrightarrow \text{Mass}$$

$$flux \propto \frac{R^2}{D^2} \longrightarrow \text{Radius}$$

The inverse problem is more common: observe Mass and Radius and infer EoS



Title: Testing the White Dwarf Mass-Radius Relation with HIPPARCOS

Authors: Provencal, J. L.; Shipman, H. L.; Hog, Erik; Thejll, P.

Publication: Astrophysical Journal v.494, p.759

Publication Date: 02/1998

ρ_{max} [g cm^{-3}]	Element	Z	N	R_{cell} [fm]
8.02×10^6	^{56}Fe	26	30	1404.05
2.71×10^8	^{62}Ni	28	34	449.48
1.33×10^9	^{64}Ni	28	36	266.97
1.50×10^9	^{66}Ni	28	38	259.26
3.09×10^9	^{86}Kr	36	50	222.66
1.06×10^{10}	^{84}Se	34	50	146.56
2.79×10^{10}	^{82}Ge	32	50	105.23
6.07×10^{10}	^{80}Zn	30	50	80.58
8.46×10^{10}	^{82}Zn	30	52	72.77
9.67×10^{10}	^{128}Pd	46	82	80.77
1.47×10^{11}	^{126}Ru	44	82	69.81
2.11×10^{11}	^{124}Mo	42	82	61.71
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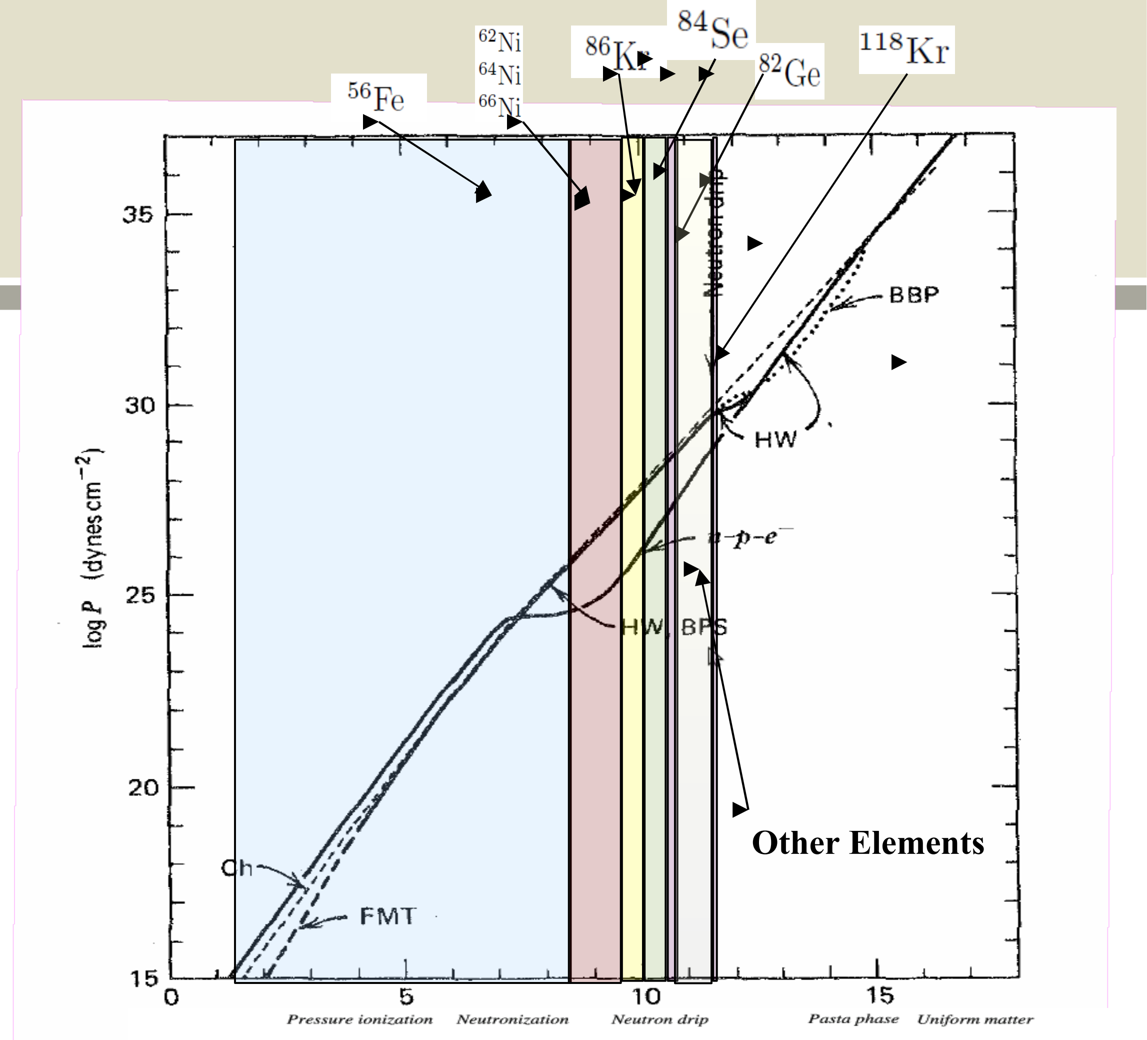

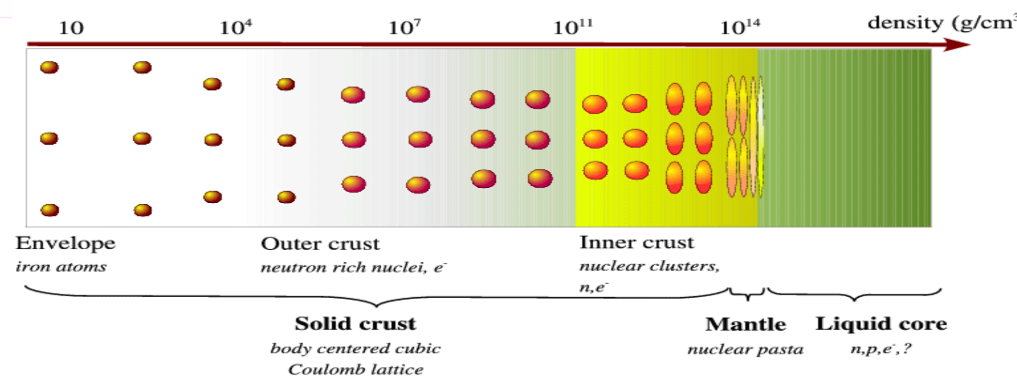

Lecture Plan

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Summary of Lecture 4

- White Dwarfs can be quite accurately described with a polytropic equation of state

- By coupling the polytropic EoS with the hydrostatic equilibrium equation, we can build the structure of a white dwarf (M-R relation)

- The high density limit gives a maximum mass which does not depend on the central density ! (Chandrasekhar limit).

- The Chandrasekhar limit is the maximum mass a WD can have, which is a consequence of the faster growth of the gravitational pull when compared to the growth of the degenerate electron gas pressure

- Effect of Coulomb and beta equilibrium corrections on M-R and composition of WDs

What you need to study so far

STUDY:
Lecture notes (L1)
Shapiro & Teukolsky: 2.2, 2.3, 2.4 (up to end of pp. 32), 2.5, 2.6, 3.3, 3.4, 3.5

READING:
Shapiro & Teukolsky: 2.1, 2.7, 3.1

Lecture 5

- Can we follow exactly the same identical procedure outlined in Lecture 4 to build a neutron star model ?

No, because beside the EoS, we cannot use the hydrostatic equilibrium in Newtonian form as calculated for White Dwarfs

$$\frac{mv^2}{2}=\frac{GMm}{R}\rightarrow v^2=\frac{2\mathrm{GM}}{R}$$

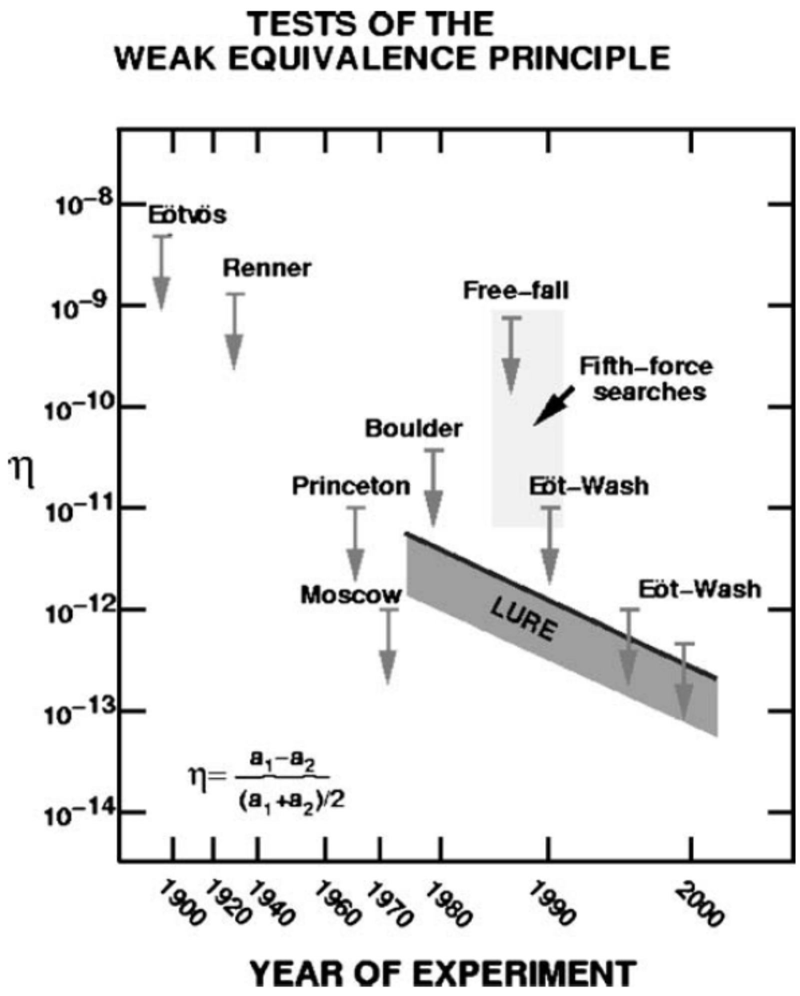
$$v=c\rightarrow R_s=\frac{2\mathrm{GM}}{c^2}$$

When R is of the same order of magnitude of Rs then General Relativity has to be used to calculate the hydrostatic equilibrium

$$\frac{R_{WD}}{R_s}\approx 3000 \qquad \frac{R_{NS}}{R_s}\approx 3 \qquad \frac{R_{BH}}{R_s}<1$$

For Neutron Stars we need GR !

Equivalence Principle



*"At every space-time point in an arbitrary gravitational field, a **local** inertial (Lorentz) frame can be chosen so that the laws of physics take on the form they have in Special Relativity"*

Equivalence of gravitational mass and inertial mass has been verified with extreme precision !

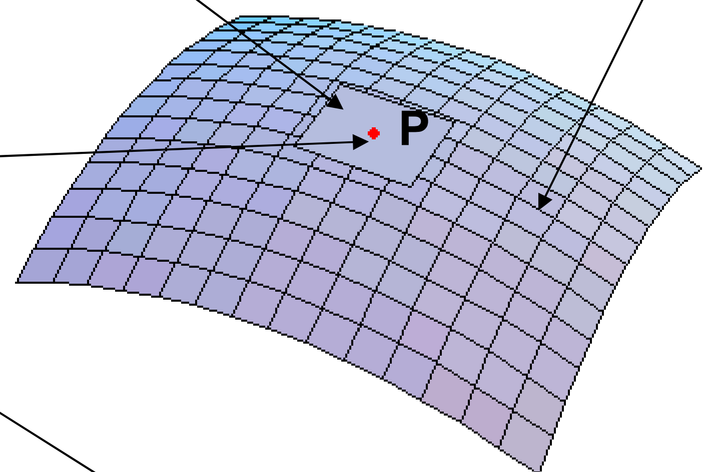
Meaning of EP

Local Inertial Frame (Lorentz frame)

$$ds^2 = \left[\eta_{\alpha\beta} + O\left(x^2\right)\right] dx^\alpha dx^\beta$$

Curved Space-time

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$



Event "**P**" chosen as origin of coordinates

$$ds^2 = \left[\eta_{\alpha\beta} + O\left(x^2\right)\right] dx^\alpha dx^\beta$$

Implies that no effect of gravity can be detected in the local inertial frame (freely falling observer)

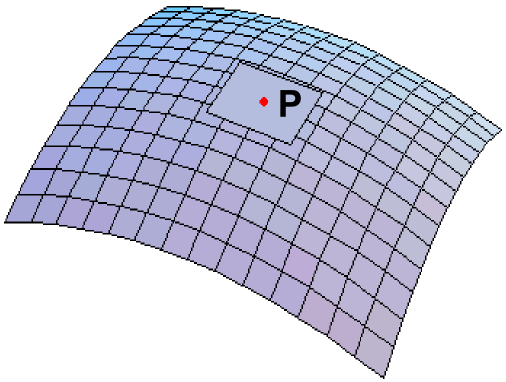
$$g_{\alpha\beta,\gamma} = 0$$

Sets the length scale over which departures from SR are noticed

Meaning of EP

Local Inertial Frame (Lorentz frame)

$$ds^2 = \left[\eta_{\alpha\beta} + O\left(x^2\right)\right] dx^\alpha dx^\beta$$



Note: **Any** observer can choose a reference frame such that $g_{\alpha\beta} = \eta_{\alpha\beta}$ at any arbitrary point in space. However, only a *local inertial observer* finds that

$$g_{\alpha\beta} = \eta_{\alpha\beta} + O\left(x^2\right)$$

The meaning of this is the following: a local inertial observer is a free falling observer, i.e., an observer with zero acceleration.
The condition above implies: $g_{\alpha\beta,\gamma} = 0$
In other words, departures from the description of Special Relativity will be noticed on a scale set by the second derivative of $g_{\alpha\beta}$

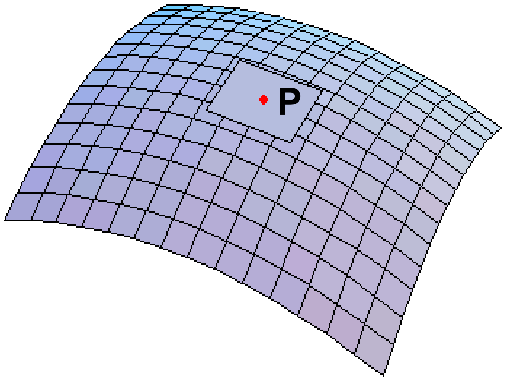
$g_{\alpha\beta} = \eta_{\alpha\beta} \longrightarrow$ Lorentz Metric at P

$g_{\alpha\beta,\gamma} = 0 \longrightarrow$ Metric as Lorentz as possible **near** P

Physical Interpretation of EP

Local Inertial Frame (Lorentz frame)

$$ds^2 = \left[\eta_{\alpha\beta} + O\left(x^2\right)\right] dx^\alpha dx^\beta$$



1) $g_{\alpha\beta}$ represents the effect of gravitational fields.
2) A freely falling observer can perform the local transformation

$$g_{\alpha\beta} = \eta_{\alpha\beta} + O\left(x^2\right)$$

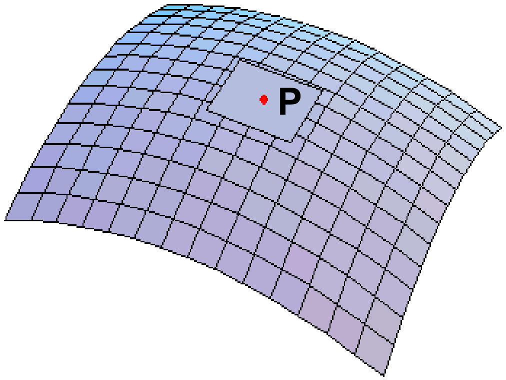
1) + 2) imply: ***it is NOT possible for an observer freely falling in a uniform gravitational field to detect the gravitational field itself***

$$F = m_I \cdot a \qquad\qquad F = \frac{m_g \cdot GM}{R^2}$$

Physical Interpretation of EP

Local Inertial Frame (Lorentz frame)

$$ds^2 = \left[\eta_{\alpha\beta} + O\left(x^2\right)\right] dx^\alpha dx^\beta$$



EP tells how to write laws of physics in a form which is valid in a general coordinate system, i.e., when the metric has the form:

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

For example, in SR, you can write conservation of energy-momentum with the relation:

$$\nabla_\alpha \cdot T^{\alpha\beta} = 0$$

In SR $\nabla_a = \frac{\partial}{\partial x_a}$, however in GR you can't use just this relation,

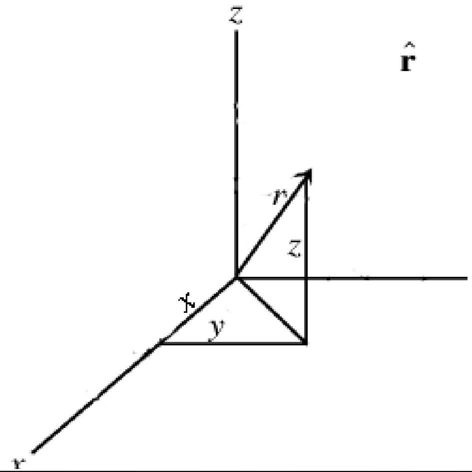
because ***the basis vectors are not constant in space*** !

SR vs. GR: a simple analogy

Rectangular coordinates:

$$\nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

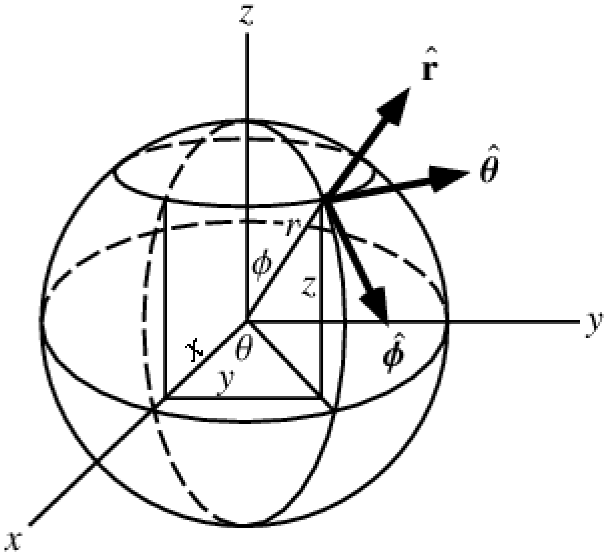
$$\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z} \longrightarrow \nabla_a = \frac{\partial}{\partial x_a}$$



Spherical coordinates:

$$\nabla \cdot A = \frac{\partial}{\partial r}A^r + \frac{2}{r}A^r + \frac{1}{r\sin\phi}\frac{\partial}{\partial\theta}A^\theta + \frac{1}{r}\frac{\partial}{\partial\phi}A^\phi + \frac{\cot\phi}{r}A^\phi,$$

$$\nabla \neq \frac{\partial}{\partial r}\hat{r} + \frac{\partial}{\partial\phi}\hat{\phi} + \frac{\partial}{\partial\theta}\hat{\theta} \longrightarrow \nabla_a \neq \frac{\partial}{\partial x_a}$$



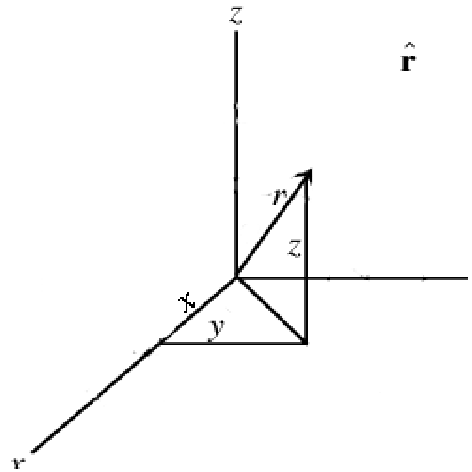
Difference between SR and GR

SPECIAL RELATIVITY

The Minkowski metric implies a flat geometry.
In SR you can always reduce your metric to a Minkowski metric (Lorentz coordinate system).
Therefore the laws of physics written in the form:

$$\nabla_\alpha \cdot T^{\alpha\beta} = 0$$

are valid in any Lorentz coordinate system.



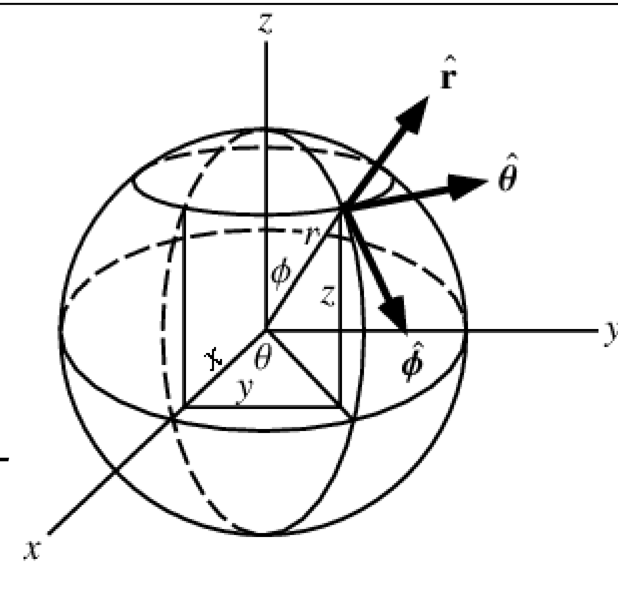
GENERAL RELATIVITY

In GR you cannot always reduce your metric to a Minkowski metric (Lorentz coordinate system).
Therefore the laws of physics written in the form:

$$\nabla_\alpha \cdot T^{\alpha\beta} = 0$$

are NOT valid in any general coordinate system if $\nabla_a = \frac{\partial}{\partial x_a}$.

Why ? *Because your basis vectors are not constant in space-time, in other words:* $g_{\alpha\beta}$ *is not constant and there is NO coordinate transformation to eliminate its coordinate derivatives everywhere*

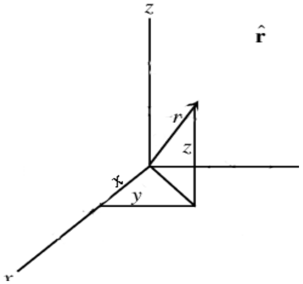


Difference between SR and GR

SPECIAL RELATIVITY

Coordinate Derivative:

$$\nabla_a A^b = \frac{\partial A^b}{\partial x^a} = A_{a,b}$$



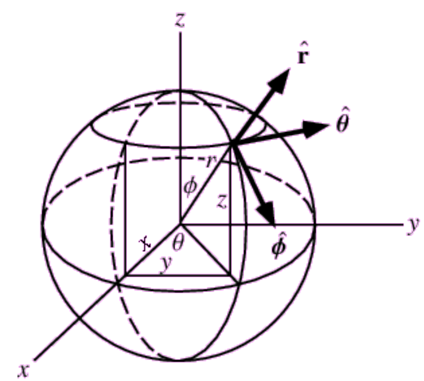
GENERAL RELATIVITY

Covariant Derivative:

$$\nabla_a A^b = \frac{\partial A^b}{\partial x^a} - \Gamma^c_{ab} A_c = A_{a,b} - \Gamma^c_{ab} A_c = A_{a;b}$$

Christoffel Symbols

$$\Gamma^c_{ab} = \frac{1}{2} g^{ad} \left(g_{ab,c} + g_{ac,b} + g_{cb,a} \right)$$



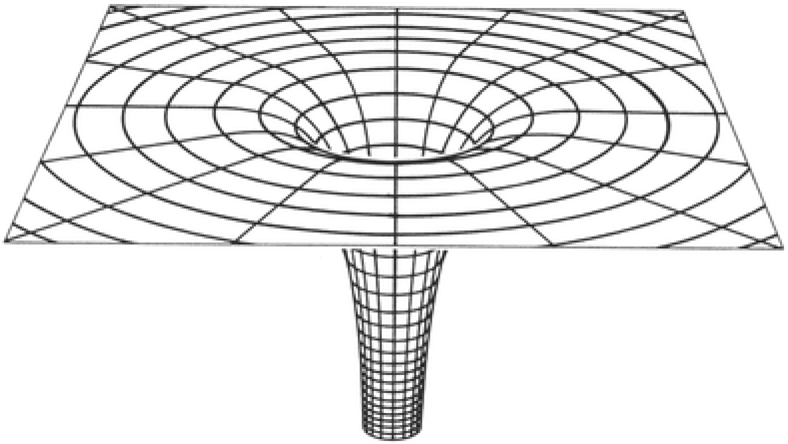
Represents the effect of gravitational fields !

$\Gamma^c_{ab} = 0$ *in a Local Lorentz Frame*

Difference between SR and GR

How to represent intervals in GR to calculate physical quantities relevant to compact objects ?

We will see the special case of a Spherically Symmetric Metric: the Schwarzschild solution, that applies quite accurately to many neutron stars and black holes.



Before doing this, we need to understand how does a test particle move in GR with respect to what we know from SR ?

Lecture Plan

- **L1:** Introduction to the Equation of State of Cold Catalized Matter
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Summary of Lecture 5

- General Relativity is the theory of gravitation that interprets the effect of a gravitational field as deformations of space-time

- Special Relativity cannot be used to describe the laws of physics in gravitational fields in a global reference frame, but only in *local inertial frames*

- The motion of a test particle in space-time follows curves of extremal length called *geodesics*

- In Special Relativity, geodesics are four-dimensional straight lines

- In General Relativity, geodesics are four-dimensional straight lines only in local inertial frames, otherwise they are described by an equation that includes the effect of space-time warping (Christoffel symbols)

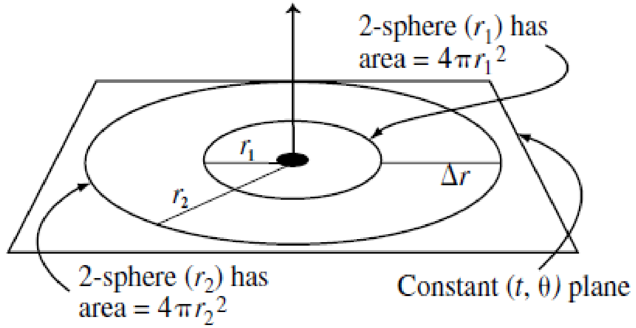
What you need to study so far

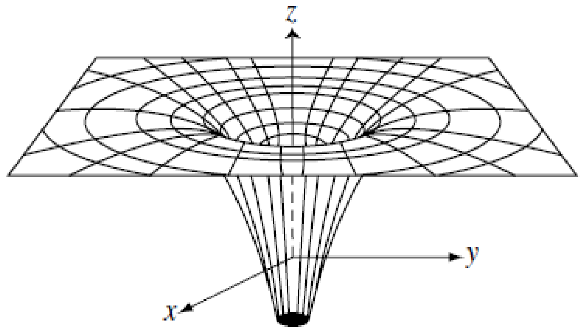
STUDY:
Lecture notes 1 (L1)
Shapiro & Teukolsky: 2.2, 2.3, 2.4 (up to end of pp. 32), 2.5, 2.6, 3.3, 3.4, 3.5, 5.1, 5.2

READING:
Shapiro & Teukolsky: 2.1, 2.7, 3.1

Schwarzschild Metric

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

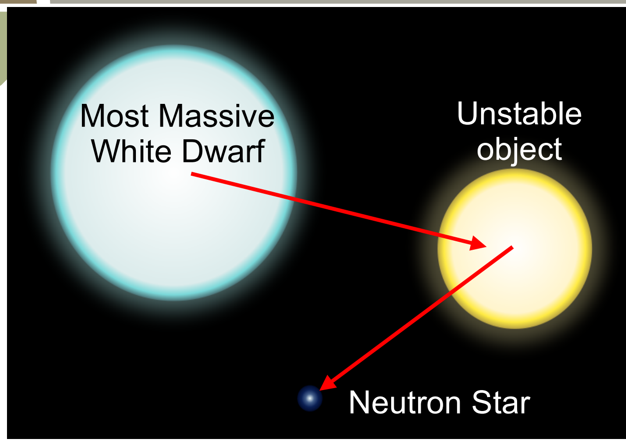


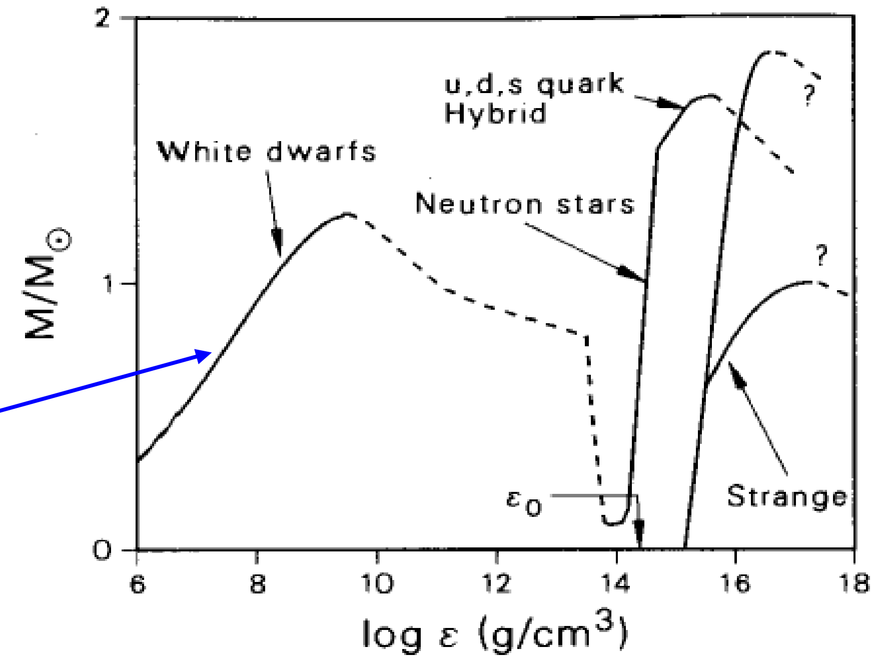


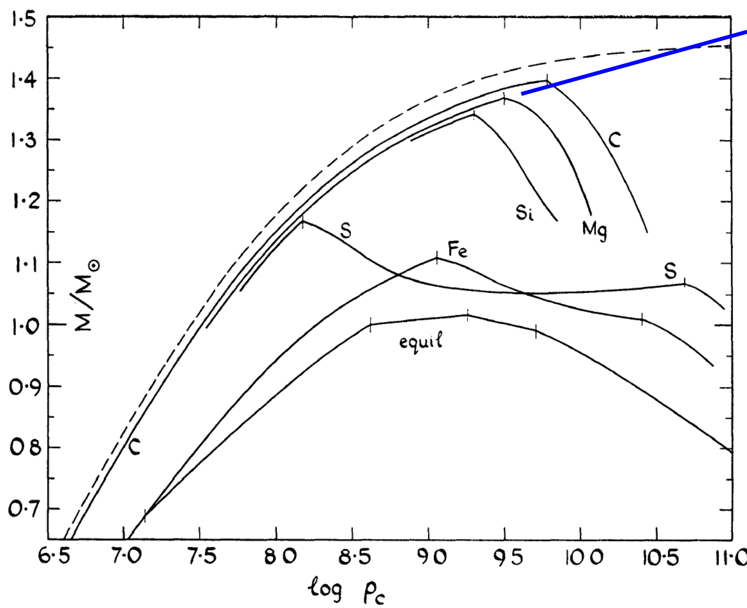
TOV Equation

$$\frac{dP(r)}{dr} = -\frac{G}{r^2} \underbrace{\left[\rho(r) + \frac{P(r)}{c^2}\right]}_{\text{Newtonian Hydrostatic Equilibrium}} \overbrace{\underbrace{\left[M(r) + 4\pi r^3 \frac{P(r)}{c^2}\right]}_{\text{Special Relativistic correction of order (v/c)^2}} \underbrace{\left[1 - \frac{2GM(r)}{c^2 r}\right]^{-1}}_{\text{General Relativistic correction}}}^{\text{Positive Defined Quantities}}$$

Equilibrium Configurations







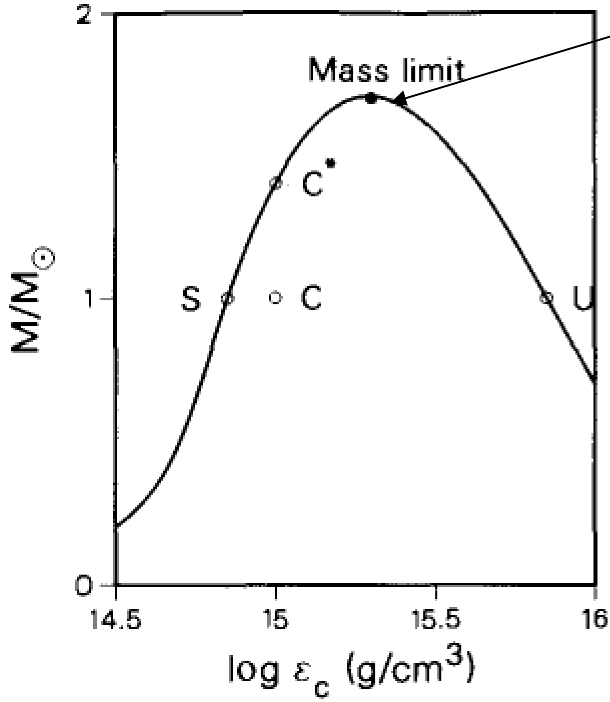
See Lecture 4

$\Gamma = \frac{5}{3}$ $\rho \propto M^2$ $R \propto M^{-1/3}$	$\Gamma = \frac{4}{3}$ $M = const$

Equilibrium Configurations

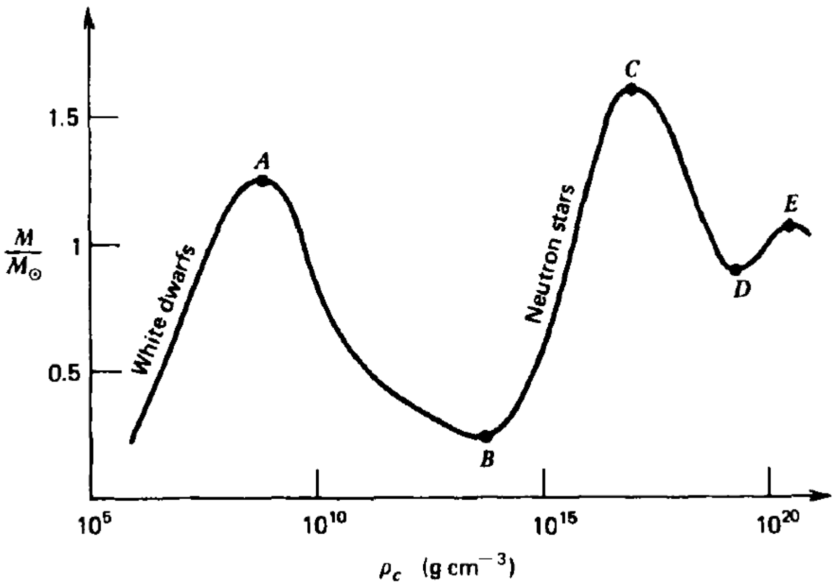
Critical Point

$$\frac{\partial M}{\partial \rho_c} = 0$$

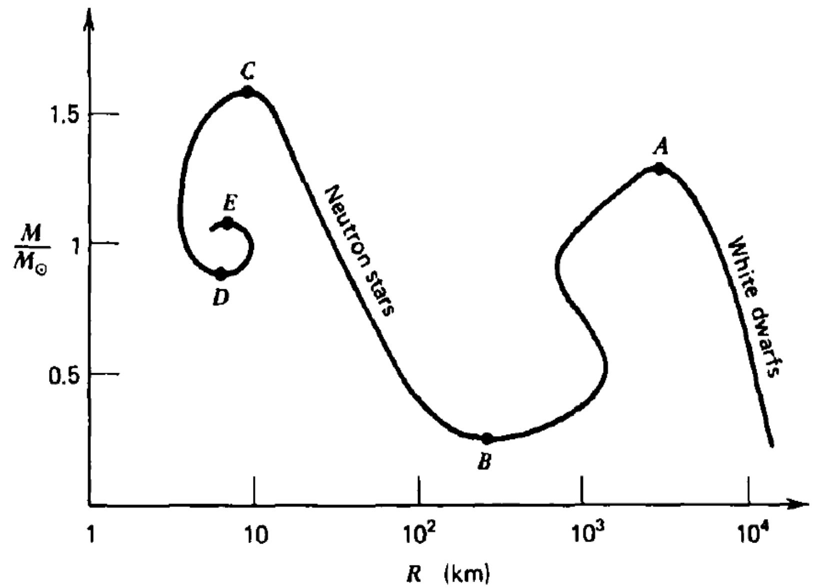


Necessary condition for equilibrium: $\frac{\partial M}{\partial \rho_c} > 0$

Equilibrium Configurations

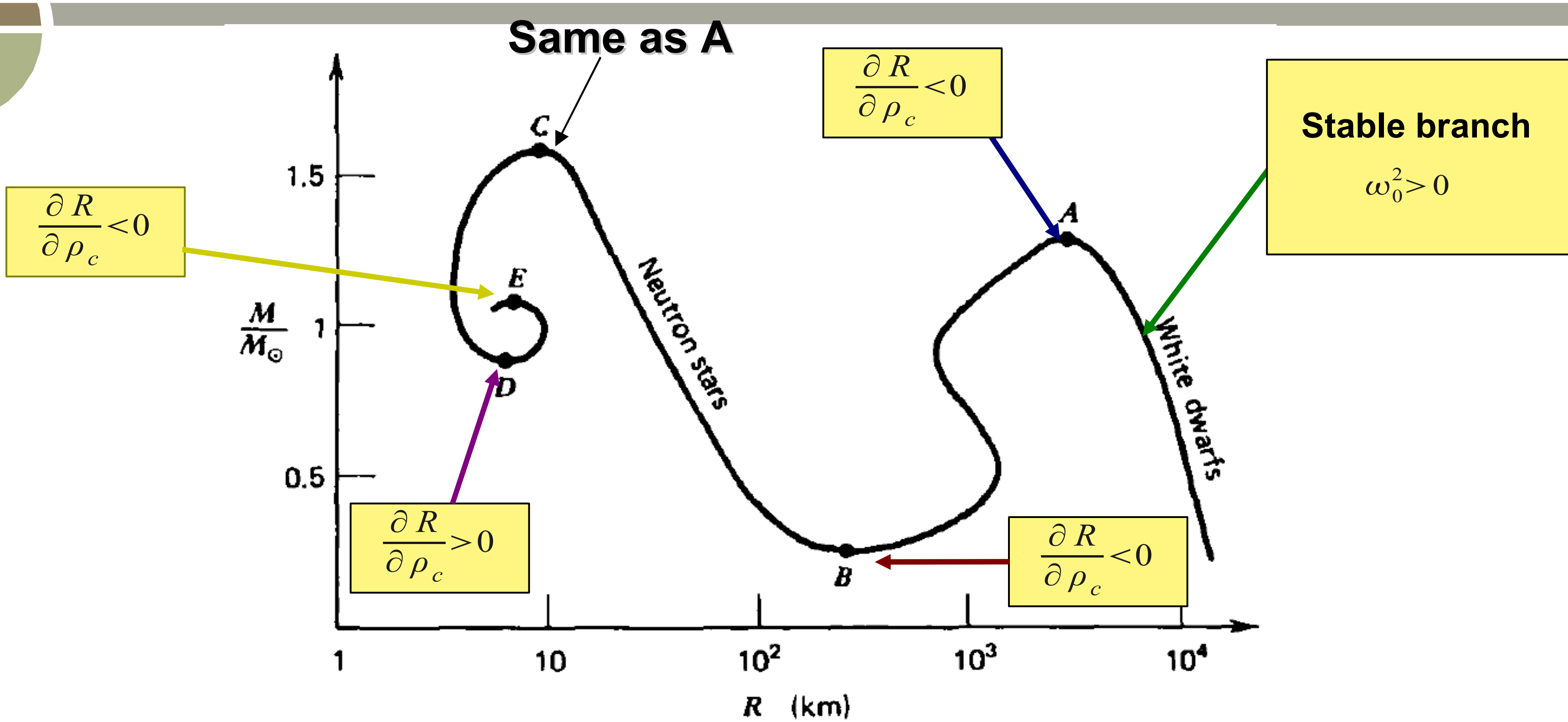


Mass - Density



Mass - Radius

Equilibrium Configurations



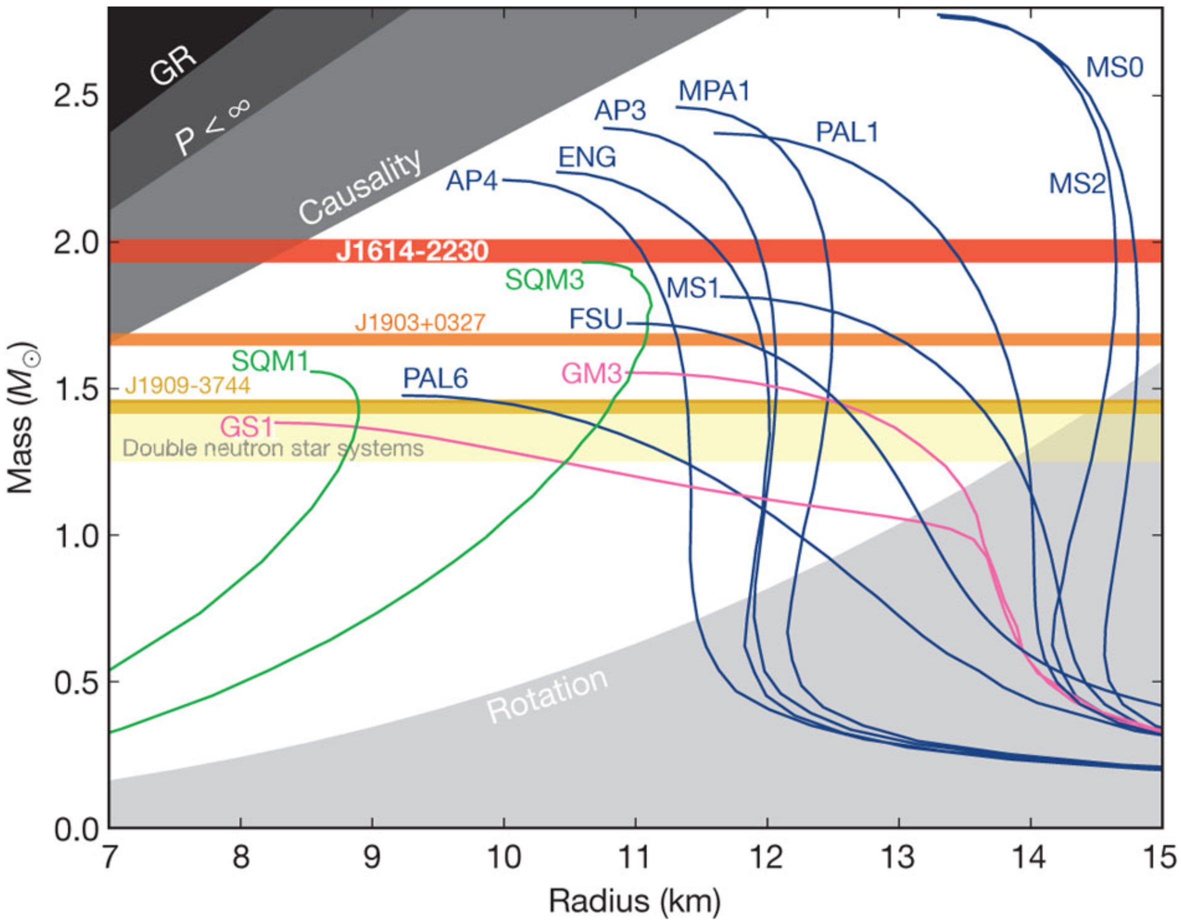
A Since $\omega_0^2 > 0$ just on the right of A, and since $\omega_0^2 < \omega_1^2 < \omega_2^2$ it must be $\omega_0^2 < 0$

B Since $\omega_1^2 > 0$ because never happened that $\frac{\partial R}{\partial \rho_c} > 0$, and since $\omega_0^2 < \omega_1^2 < \omega_2^2$ it must be $\omega_0^2 > 0$

D Odd mode changes stability, since $\omega_0^2 < \omega_1^2 < \omega_2^2$ it must be $\omega_1^2 < 0$

E Even mode changes stability. Since $\omega_1^2 < 0$, it must be $\omega_2^2 < 0$

M-R relation for Neutron Stars



A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest, T. Pennucci, S. M. Ransom
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Nature, Volume:467, pp:1081-1083. 28 Oct. 2010

Lecture Plan

- **L1:** Introduction to the Equation of State of Cold Catalized Matter
- **L2:** Electrostatic corrections to the EoS of CCM and n-p-e gas
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- **L12:** Black Holes 1
- **L13:** Black Holes 2
- **L14:** Special Session: preparation for written exam

Summary of Lecture 6

- The Schwarzschild metric is the only vacuum solution of a static spherically symmetric gravitational field

- Proper time and proper radial distance depend on the mass of the gravitational field source (M) and on the radial distance (r).

- The proper circumference does not depend on M and/or r, and its value is identical to the Minkowski space-time

- Gravitational Redshift is a consequence of proper time dependence on M and r

- Tolman-Oppenhemer-Volkoff hydrostatic equation is a consequence, among others, of proper distance dependence on M and r

- Stability of a fluid equilibrium configuration depends on stability of radial mode of oscillation

What you need to study so far

STUDY:

Shapiro & Teukolsky (S&T): Lecture Notes 1, 2.2, 2.3, 2.4 (up to end of pp. 32), 2.5, 2.6, 3.3, 3.4, 3.5, 5.1, 5.2, 5.3, 5.5, 5.6, 5.7, Lecture Notes 2

READING:
S&T: 2.1, 2.7, 3.2

<u>**IN THIS LECTURE:**</u> S&T 8.2, Lecture Notes 3, S&T 8.5, 8.6, 9.3

TOV Equation

Newtonian Hydrostatic Equilibrium

Special Relativistic correction of order (v/c)^2

General Relativistic correction

$$\frac{dP(r)}{dr} = -\frac{G}{r^2}\left[\rho(r) + \frac{P(r)}{c^2}\right]\underbrace{\left[M(r) + 4\pi r^3\frac{P(r)}{c^2}\right]\left[1 - \frac{2GM(r)}{c^2 r}\right]^{-1}}_{\text{Positive Defined Quantities}}$$

$$\frac{dP}{dr} = \frac{-Gm(r)\rho}{r^2}$$

Note that the correction factors are all positive definite. It is as if Newtonian gravity becomes stronger for any value of r.

TOV Equation

Relativity strengthens gravity !

↓

GR destabilizes stars: collapse is easier

$$E = E_{Newt.} + \Delta E_{GR} < E_{Newt.}$$

↓

Maximum Mass of NS (and WD) smaller than in Newtonian case

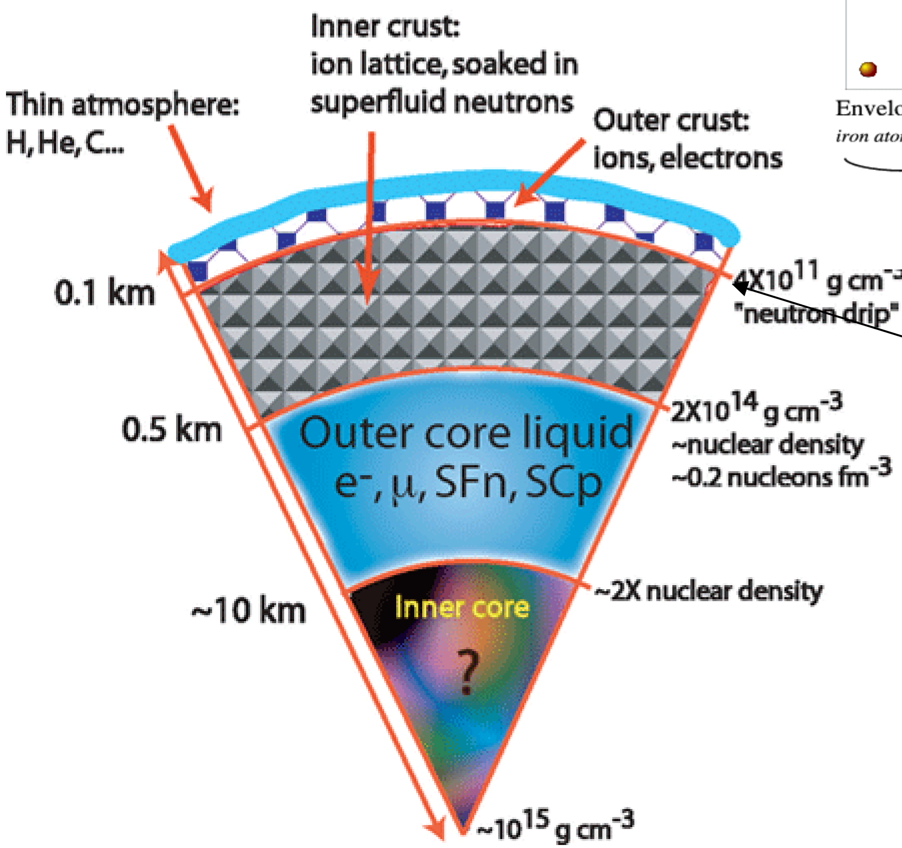
Oppenheimer and Volkoff assumed a polytropic equation of pure neutron gas

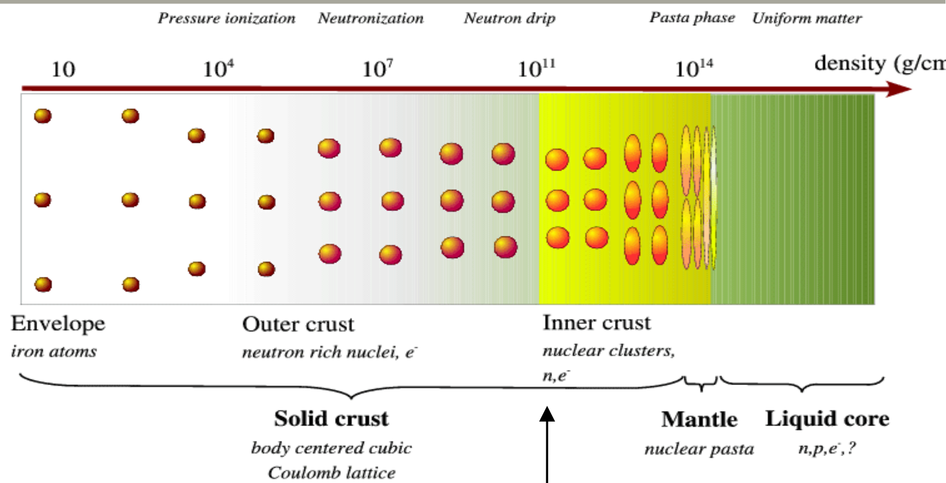
They got: $M_{max} = 0.7 \ M_{sun} \quad R = 9.6 \ km \quad \rho_c = 5 \times 10^{15} \ g \ cm^{-3}$

Maximum mass too small ! Observed values are around 1.2-2.0 Solar masses !

Pure ideal neutron gas too simple to be realistic !

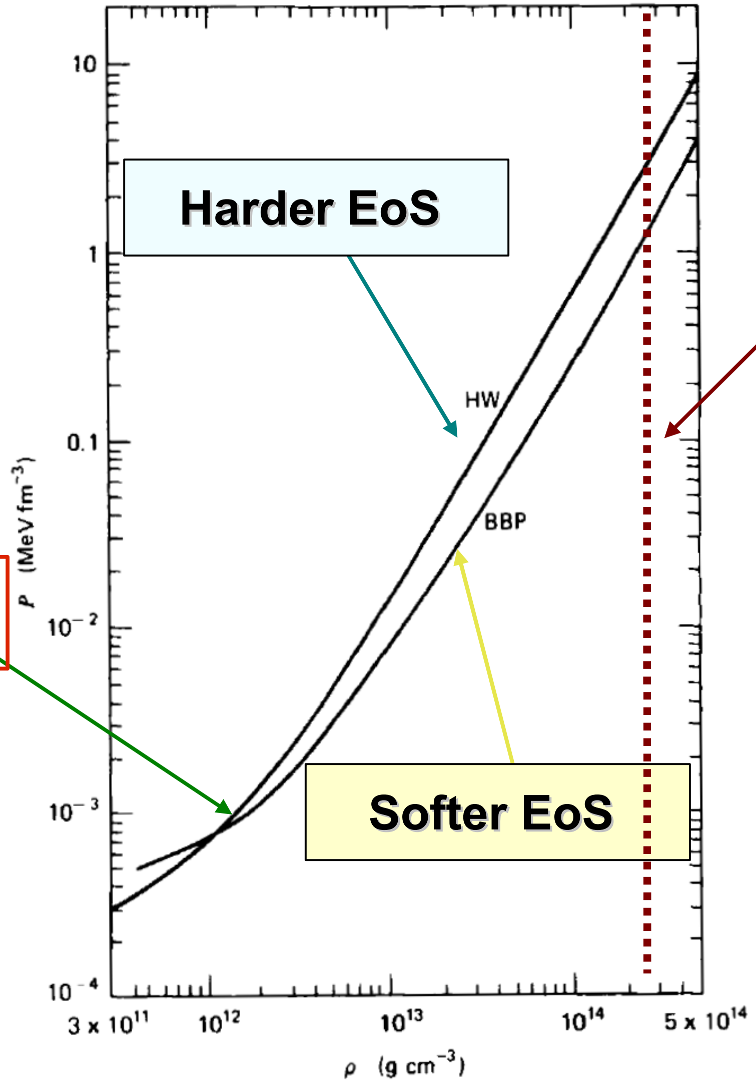
Neutron Star Structure





So far we are here

EoS of the Inner Crust

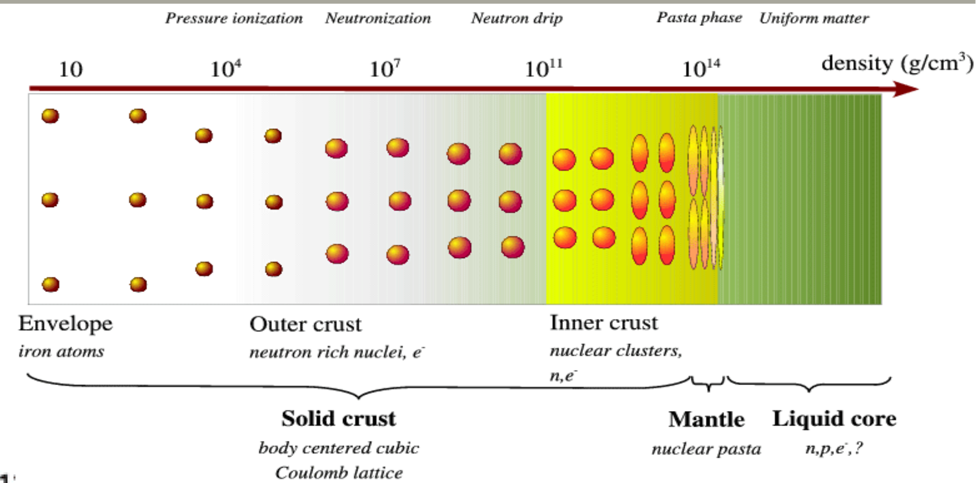


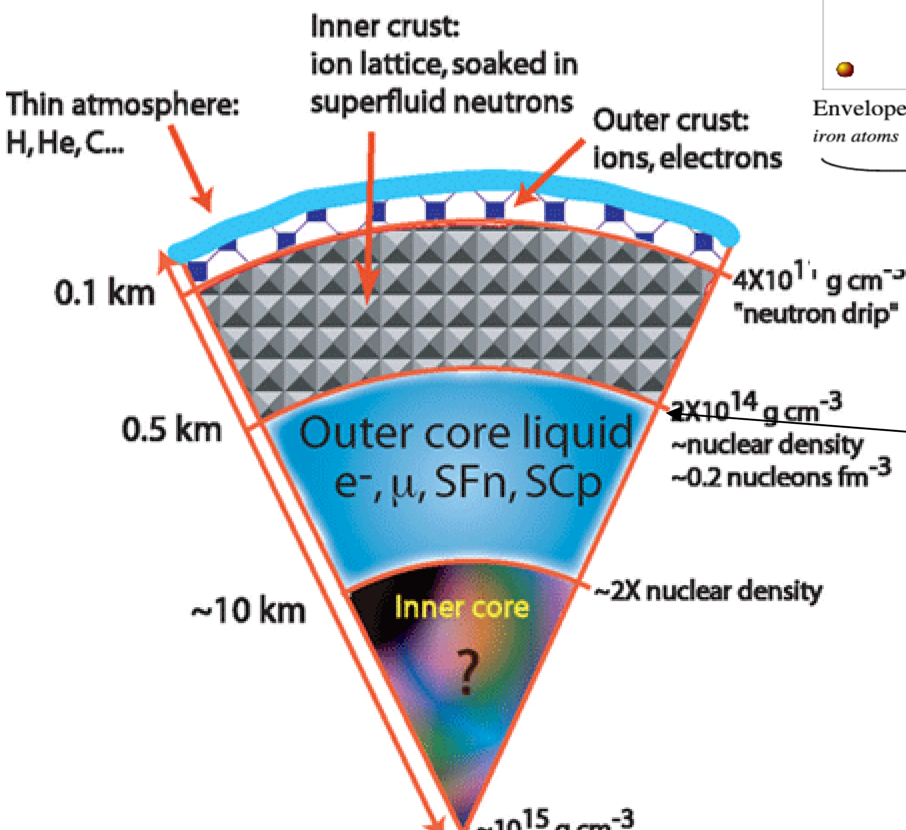
Nuclei survive up to here (slighly ***less*** than nuclear density). Different from n-p-e gas !

HW uses a pure n-p-e gas above ~10^12 g/cm3

Figure 8.1 S&T

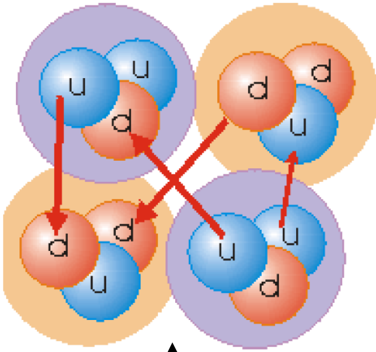
Neutron Star Structure

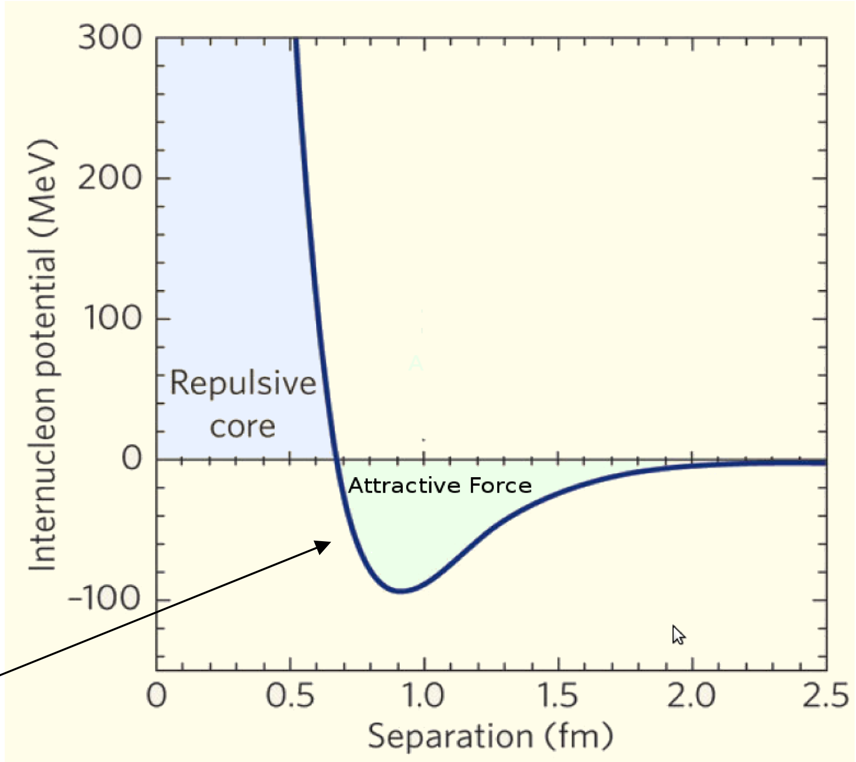




Now we are here

Nucleon-Nucleon Interaction





"Residual" strong force

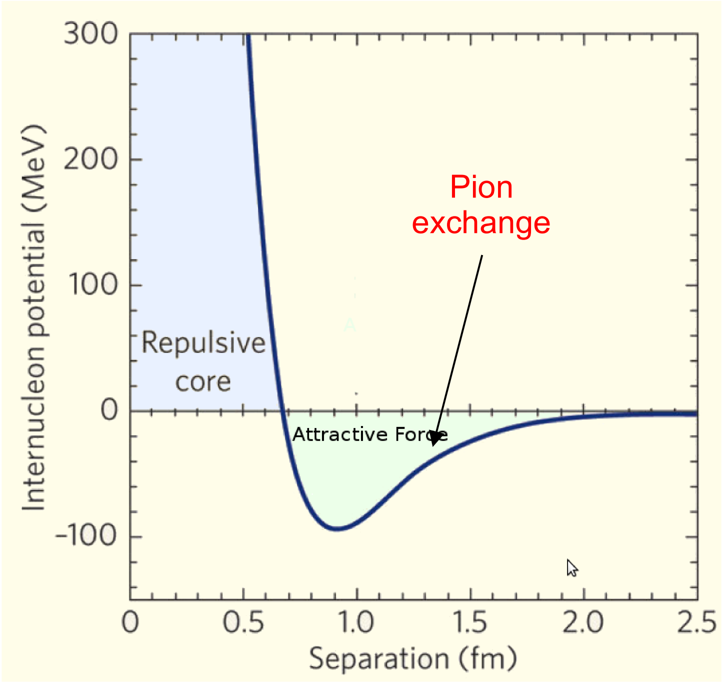
The Yukawa Potential

$$V(r)=\pm g^2\frac{e^{-\mu r^2}}{r}$$

Yukawa Potential

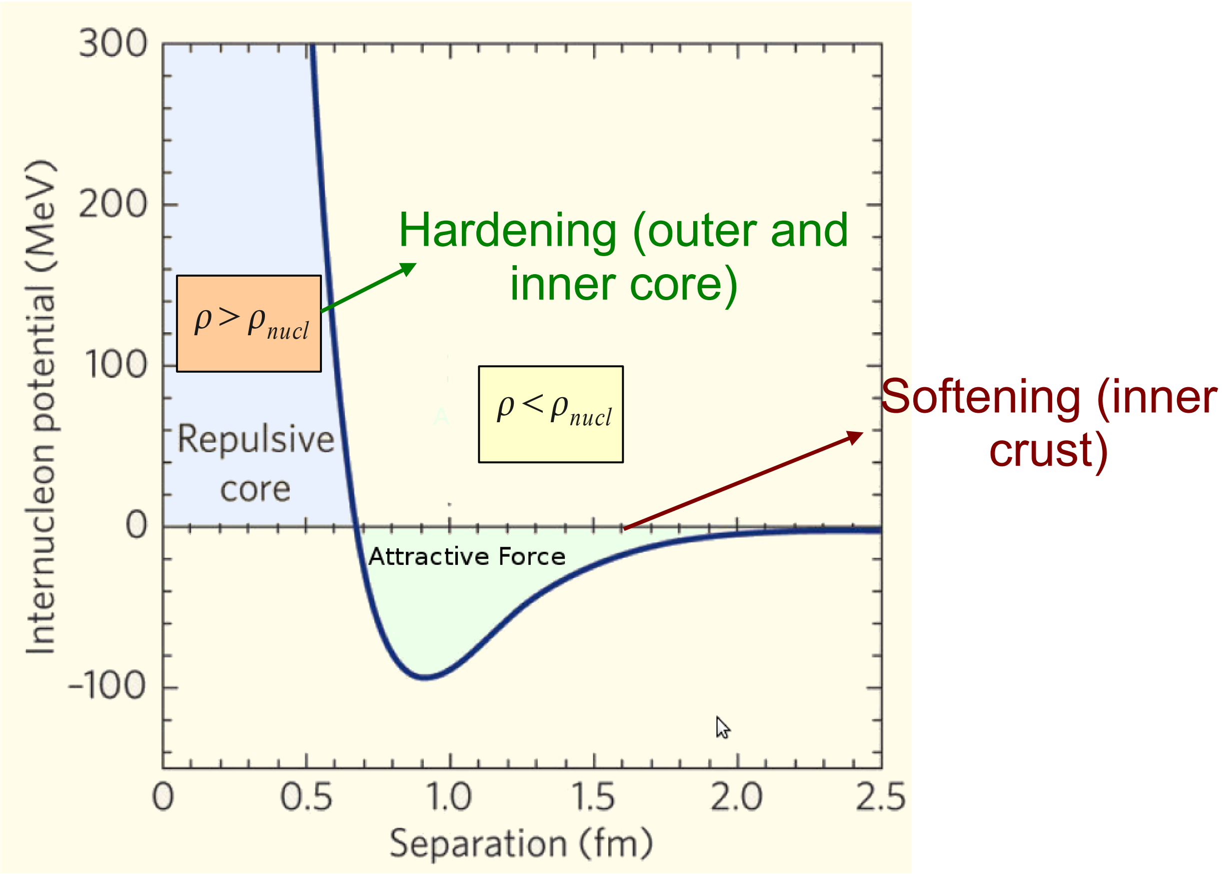
$$\mu=\frac{m_x c}{\hbar}=\frac{1}{\lambda_c}$$ → Nuclear force has a range of ~1.4 fm, so the field quanta has mass ~140 Mev, i.e., pions

g → "Charge" → $\alpha_s=\frac{g^2}{(\hbar c)}$



Inverse of the Compton wavelength of the field quanta → When particle is massless, e.g., photons, the potential has an infinite range $\xrightarrow{\text{Coulomb potential}}$ $V(r)=\pm g^2\frac{1}{r}$

The Yukawa Potential

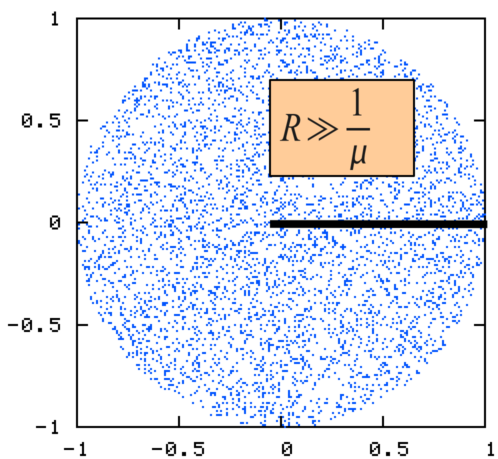


EoS with a Yukawa Potential

1. Take a uniform sphere of particles

2. Interaction energy in a volume V is:

$$E_V = \frac{1}{2}\sum_{i \neq j} V_{i,j} = \pm\frac{1}{2} n^2 g^2 \int\int \frac{e^{-\mu r_{ij}}}{r_{ij}} dV_i dV_j$$



3. Since R >> 1/mu, integrate to infinity:

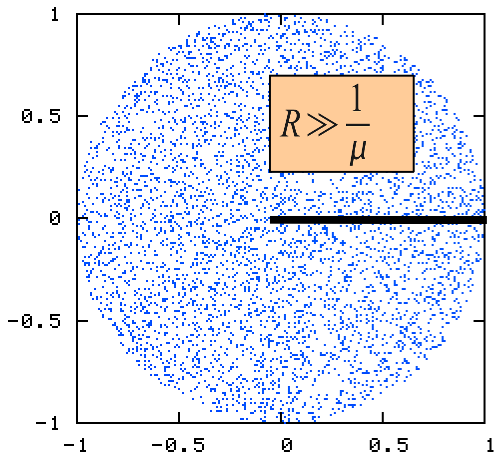
$$\int_0^\infty \frac{e^{-\mu r}}{r} 4\pi r\, dr = \frac{4\pi}{\mu^2} \longrightarrow E_V = \pm\frac{1}{2} n^2 g^2 \frac{4\pi}{\mu^2} V$$

Total energy density is: $\boxed{\epsilon = \epsilon_{kin} \pm \frac{2\pi n^2 g^2}{\mu^2}} \longrightarrow$

Add kinetic term because particles have also momentum !

The kinetic energy can be calculated from the ideal Fermi gas relations in relativistic and non-relativistic approximation (see Section 2.3 in Shapiro & Teukolsky).

EoS with a Yukawa Potential



4. To calculate the EoS , take the usual expression (cf. Eq. 2.1.7 in S&T):

$$P = n^2 \frac{d}{dn}\left(\frac{\epsilon}{n}\right) \longrightarrow P = P_{kin} \pm \frac{2\pi n^2 g^2}{\mu^2}$$

with $P_{kin} = K n^{\Gamma}$

Conclusion: When density is smaller/larger nucler density, Pressure is softened/hardened by the NN interaction (attractive/repulsive below/above nuclear density).

Is this it ?

Realistic NN-Potential

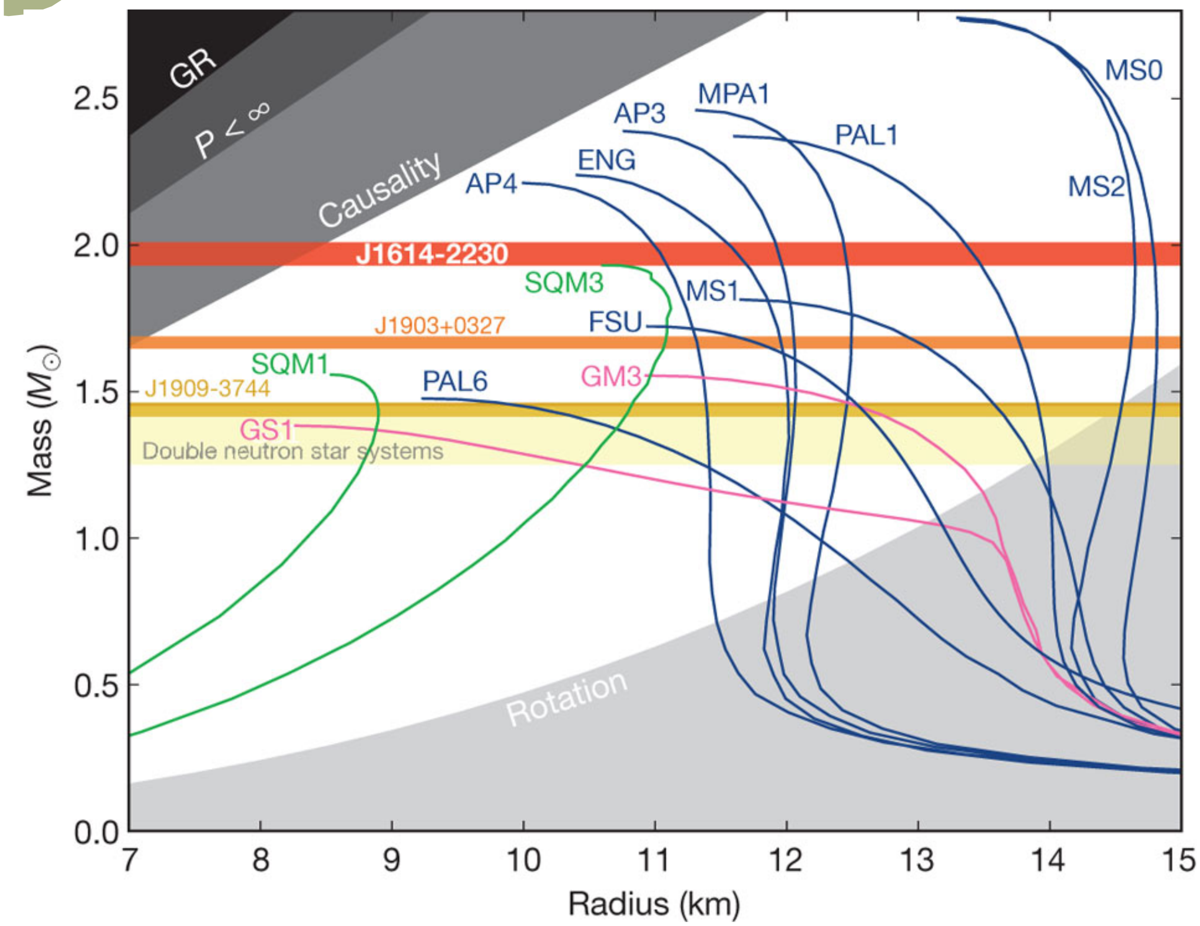
Yukawa potential is a crude first approximation of the nucleon-nucleon interaction.

A realistic nucleon-nucleon potential does not have simply a radial part, but also extra components related with the spin and isospin of the particles

$$\hat{v}_{ij} = \sum_{u=1}^{18} v_u(r_{ij}) \hat{O}_{ij}^{u} \,,$$

$$\begin{aligned}
\hat{O}_{ij}^{u=1,\ldots,14} \quad = \quad & 1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \,, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \,, (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \,, \hat{S}_{ij} \,, \hat{S}_{ij}(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \\
& \hat{\boldsymbol{L}} \cdot \hat{\boldsymbol{S}} \,, \hat{\boldsymbol{L}} \cdot \hat{\boldsymbol{S}}(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \,, \hat{L}^2 \,, \hat{L}^2(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \,, \hat{L}^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \,, \\
& \hat{L}^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \,, (\hat{\boldsymbol{L}} \cdot \hat{\boldsymbol{S}})^2 \,, (\hat{\boldsymbol{L}} \cdot \hat{\boldsymbol{S}})^2(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \,.
\end{aligned}$$

M-R relation for Neutron Stars



A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest, T. Pennucci, S. M. Ransom
M. S. E. Roberts & J. W. T. Hessels

Nature, Volume:467, pp:1081-1083. 28 Oct. 2010

Does Inner Core really exist ?
"Minimal model"
n-p-e-mu gas survives

Is nature so simple ?

Lecture Plan

- **L1:** Introduction to the Equation of State of Cold Catalized Matter
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Summary of Lecture 6

- TOV equations tell you that GR has the effect of destabilizing WDs and NS.

- The NS inner crust can be described with a mixture of nuclei + free neutrons and free electron (Baym-Bethe-Pethick EoS)

- Nuclei survive almost up to the outer core, when the density is comparable to nuclear density

- To describe the outer core we need to take into account the Nucelon-Nucleon interaction, which is extremely compicated

- Yukawa Potential --> zeroth-order approximation for nuclear interaction

- Inner core is the most dense region, with an unknown composition and different possibilities that depend on poorly understood nuclear physics.

- Models have to reproduce observed nuclear parameters K, W and S (compressibility, bulk modulus, symmetry coefficient)

What you need to study so far

STUDY:

- Shapiro & Teukolsky (S&T): 2.2, 2.3, 2.4 (up to the end of pp. 32), 2.5, 2.6, 3.3, 3.4, 3.5, 5.1, 5.2, 5.3, 5.5, 5.6, 5.7, 8.1, 8.2, 8.5, 8.6, 9.3
- Lecture Notes 1, Lecture Notes 2, Lecture Notes 3 (all but Section 3-4 "Minimum NS mass", "Maximum neutron star mass").

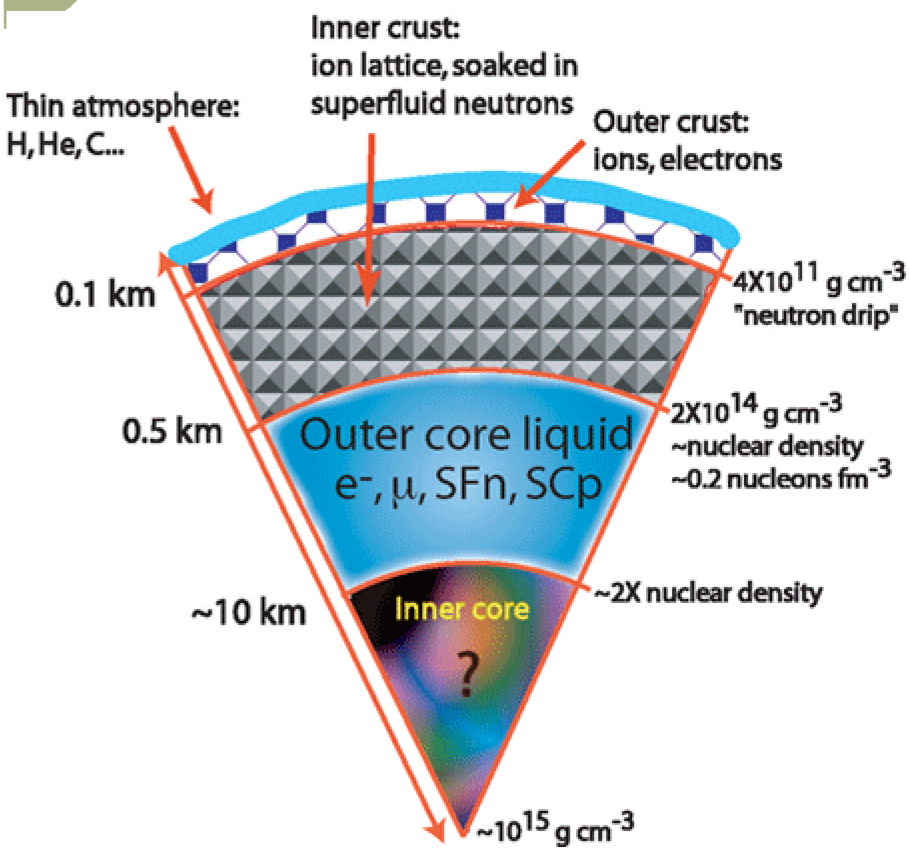
READING:

- S&T: 2.1, 2.7, 3.2
- Section 3-4 of Lecture Notes 3 ("Minimum NS mass" and "Maximum NS mass").

<u>IN THIS LECTURE:</u>

- S&T 10.4, 10.5, 10.9, 10.10
- Lecture Notes 4

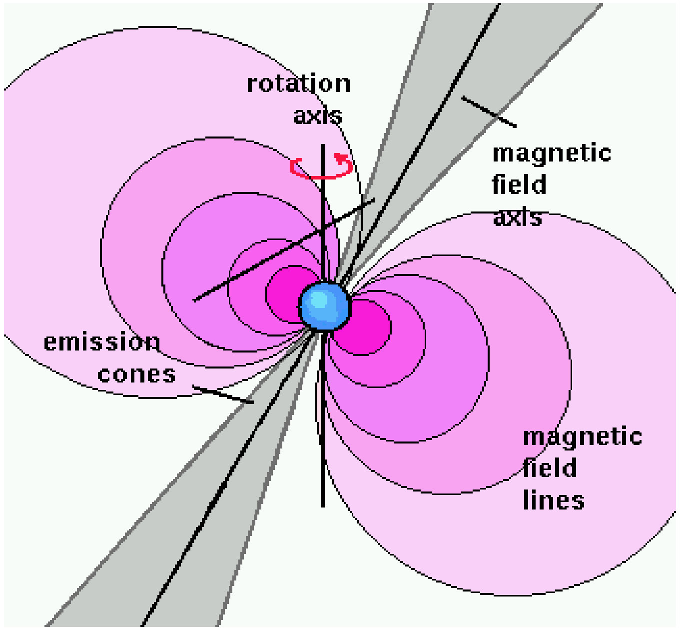
Neutron Star Structure



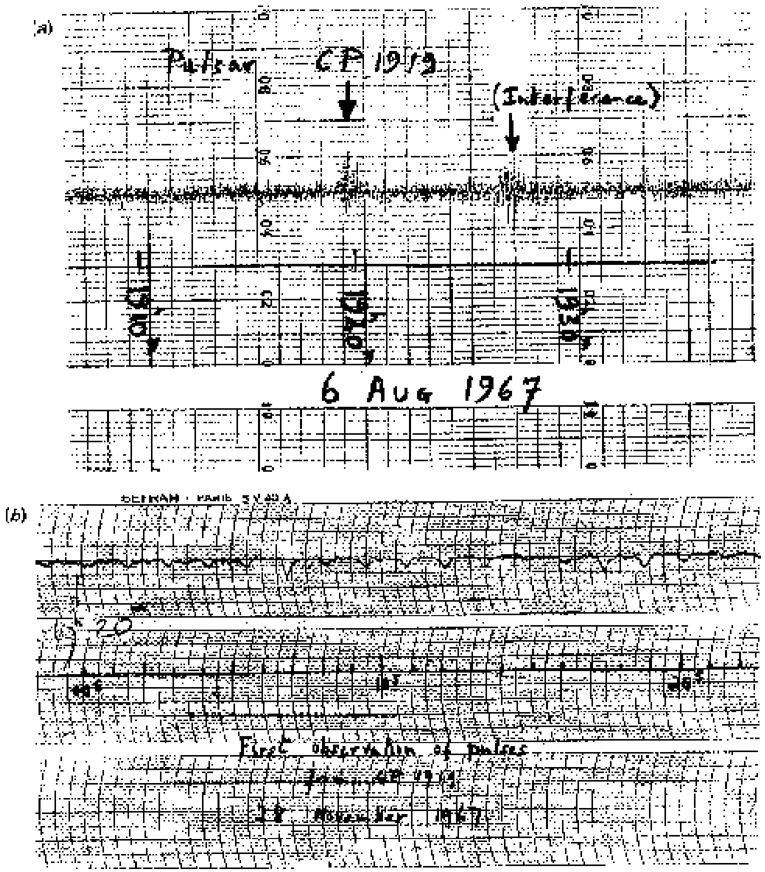
What is a superfluid ?

What is a superconductor ?

Is there a way to detect these components ?



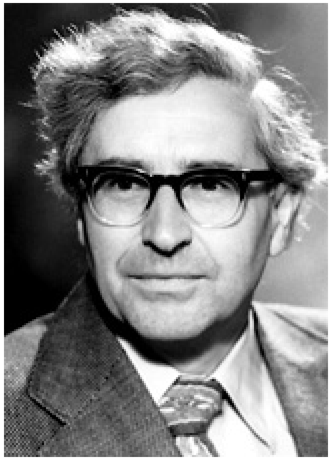
Pulsars



- Discovered in 1967 by J. Bell (and her Ph.D. supervisor A. Hewish)

- First evidence that Neutron Stars do really exist

- Initially mistakenly interpreted as "Little Green Men"



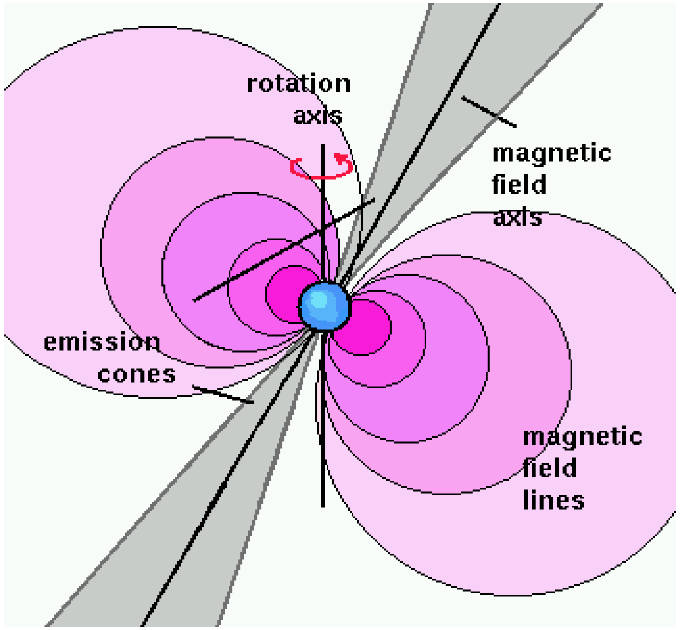
Pulsars

Neutron Stars are born with ultra-strong magnetic field (probably the highest in the Universe!)

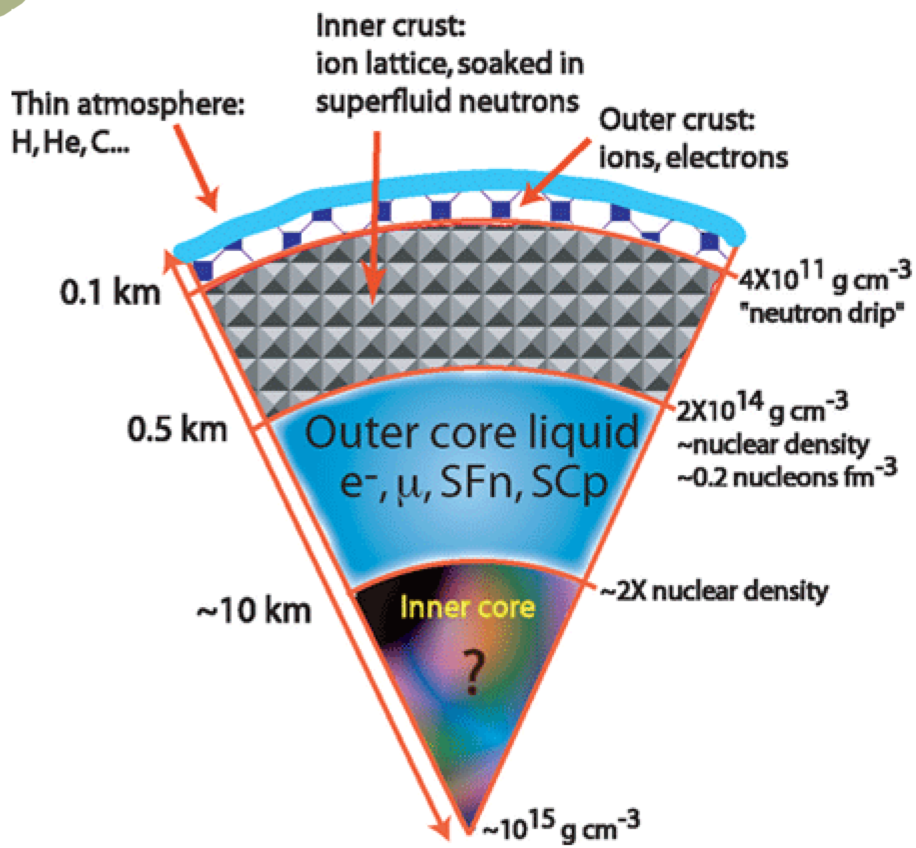
1) Oblique Rotator Model → Magneto Dipole Radiation

2) Dispersion Measure → Pulsar Distance

3) Pulsar Glitches → Hadron Superfluidity



Superfluidity



Superfluid n-n and p-p forms when thermal energy is smaller than the latent heat for the phase transition to a paired state:

$$kT < \Delta$$

$$\Delta \sim 1-2\,MeV \approx 10^{10}\,K$$

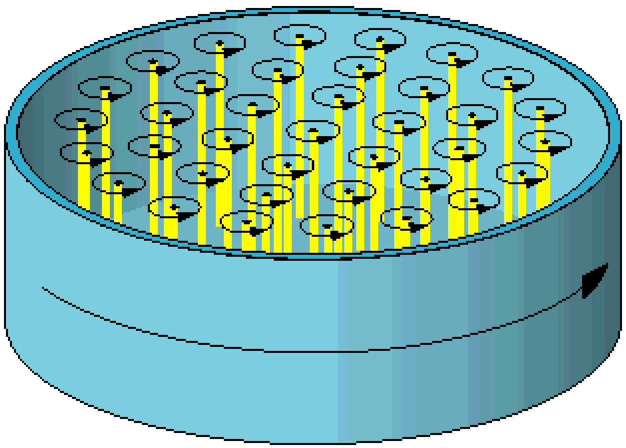
Internal NS temperature:

$$T < 10^{9}\,K$$

Note: Pairing energy << Fermi Energy so the effect on the EoS is minimal !

Superfluidity

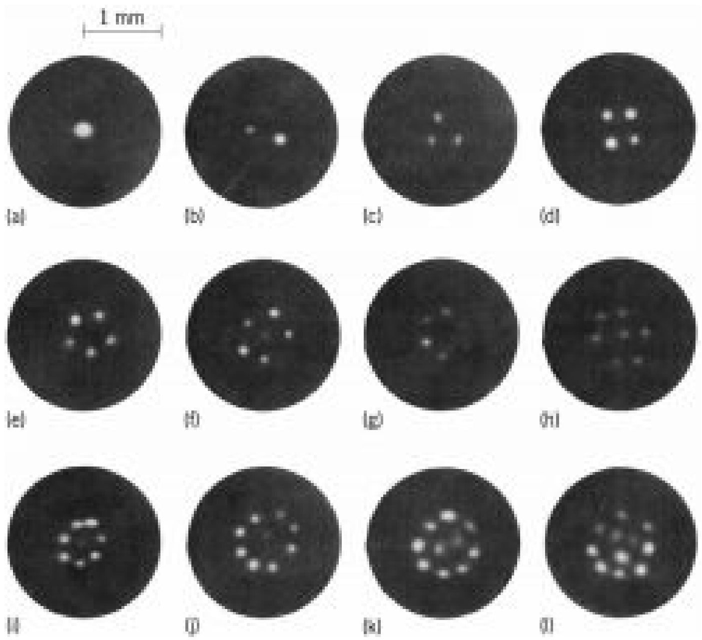
Superfluids have zero viscosity



$\nabla \times \boldsymbol{v} = 0$ Microscopically (irrotational flow)

$\nabla \times \langle \boldsymbol{v} \rangle = 2\Omega$ Macroscopically

$k = \oint \boldsymbol{v} \cdot \boldsymbol{dr} = \dfrac{h}{2\mathrm{m}_n}$ Quantized circulation



Number of quantized vortices per unit area

$$n_v = \frac{4\,\Omega\, m_n}{h}$$

Superconductors vs. Perfect Conductor

Definition: *A superconductor is a material with a definite phase transition and critical temperature Tc, below which the resistivity is zero, and who exhibits the Meissner effect*

$$\boldsymbol{j} = \sigma\,\boldsymbol{\varepsilon} = \frac{1}{\rho}\boldsymbol{\varepsilon} \longrightarrow \boldsymbol{\varepsilon} = \rho\,\boldsymbol{j} = 0$$

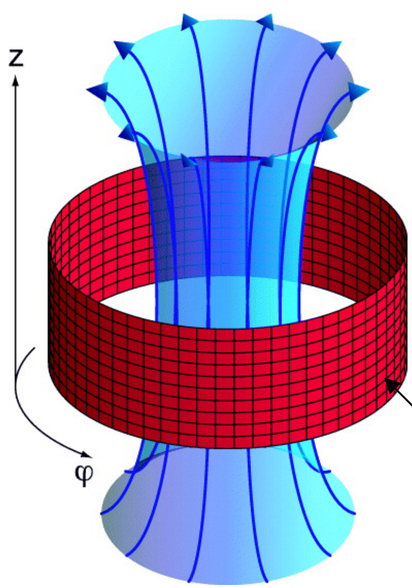
Ohm's Law

$$\phi = \int \boldsymbol{B} \cdot \boldsymbol{ds} \longrightarrow \nabla \times \varepsilon = -\frac{\partial \boldsymbol{B}}{\partial t}$$

Faraday's Law

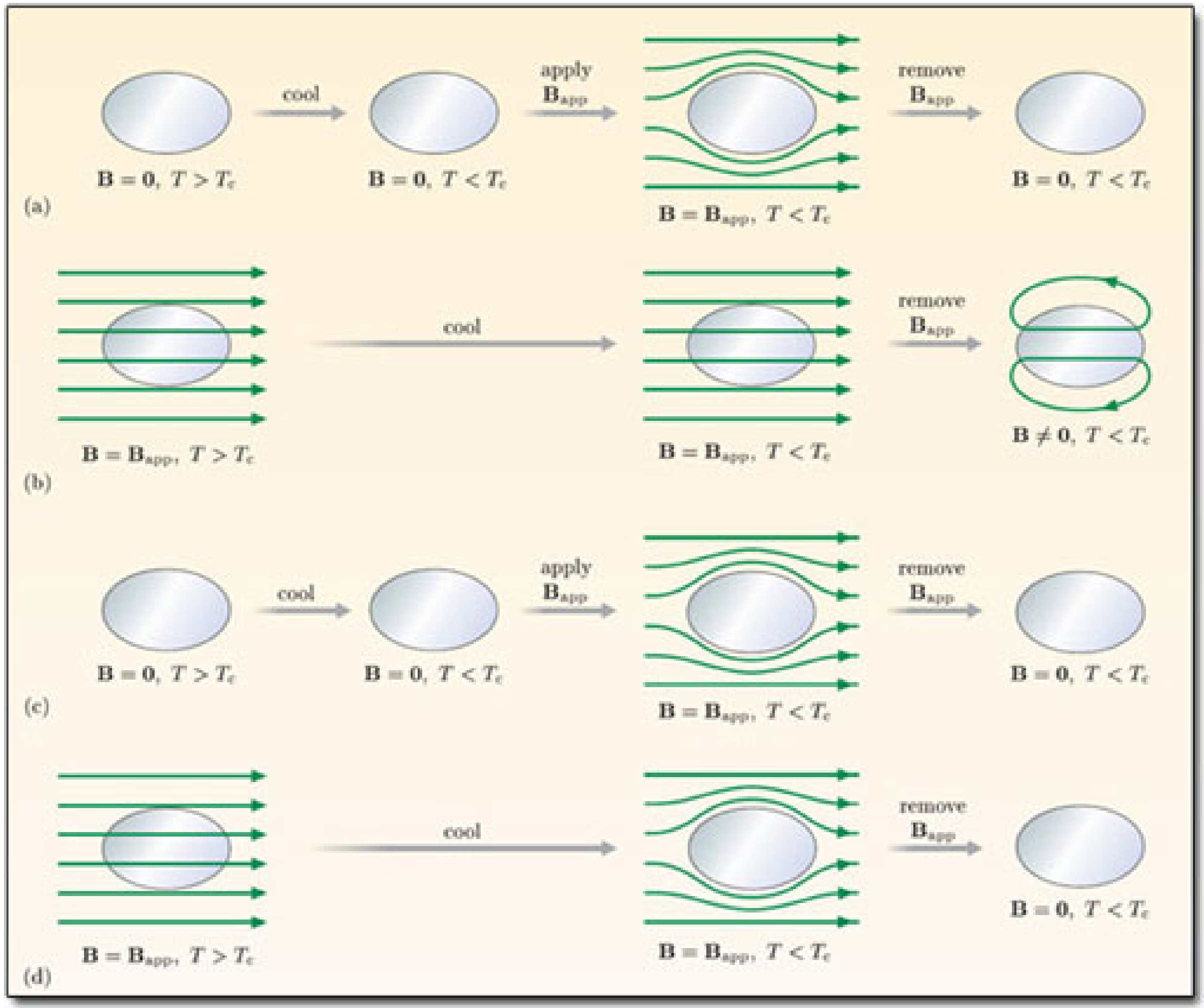
Stoke's theorem

$$\int (\nabla \times \varepsilon)\,\boldsymbol{ds} = \oint \varepsilon\,\boldsymbol{dr} = -\frac{d\phi}{dt} = 0$$

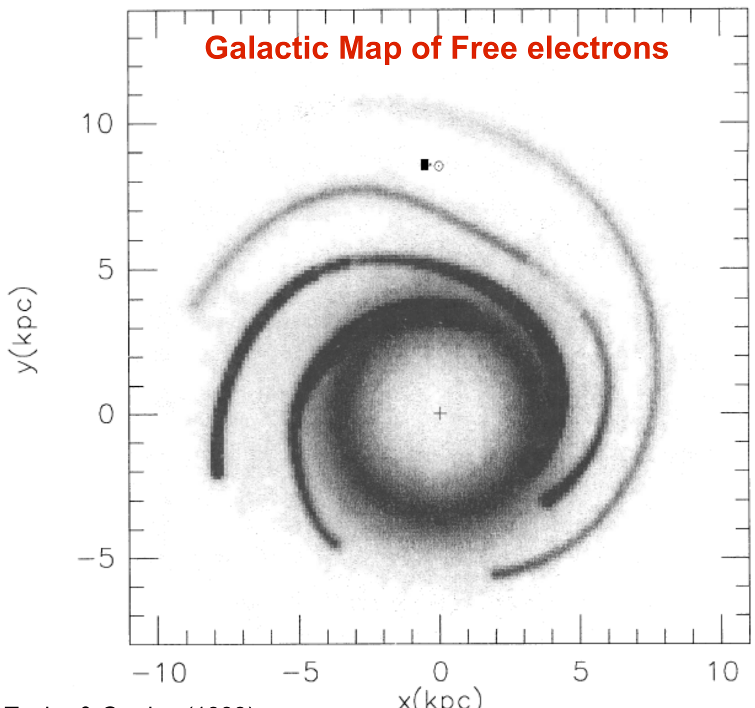


Persistent Current Flowing in the superconductive ring

Meissner Effect

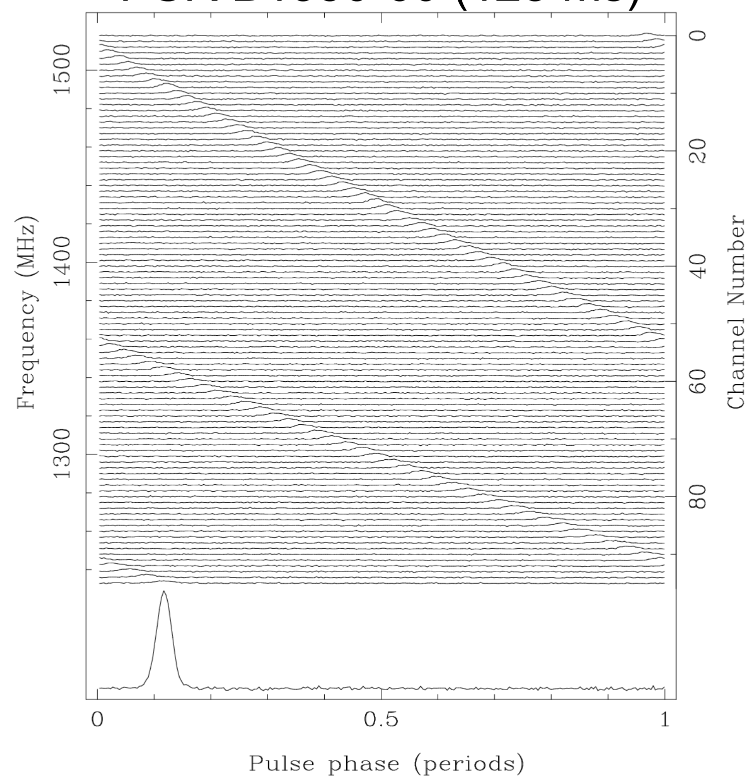


Dispersion Measure



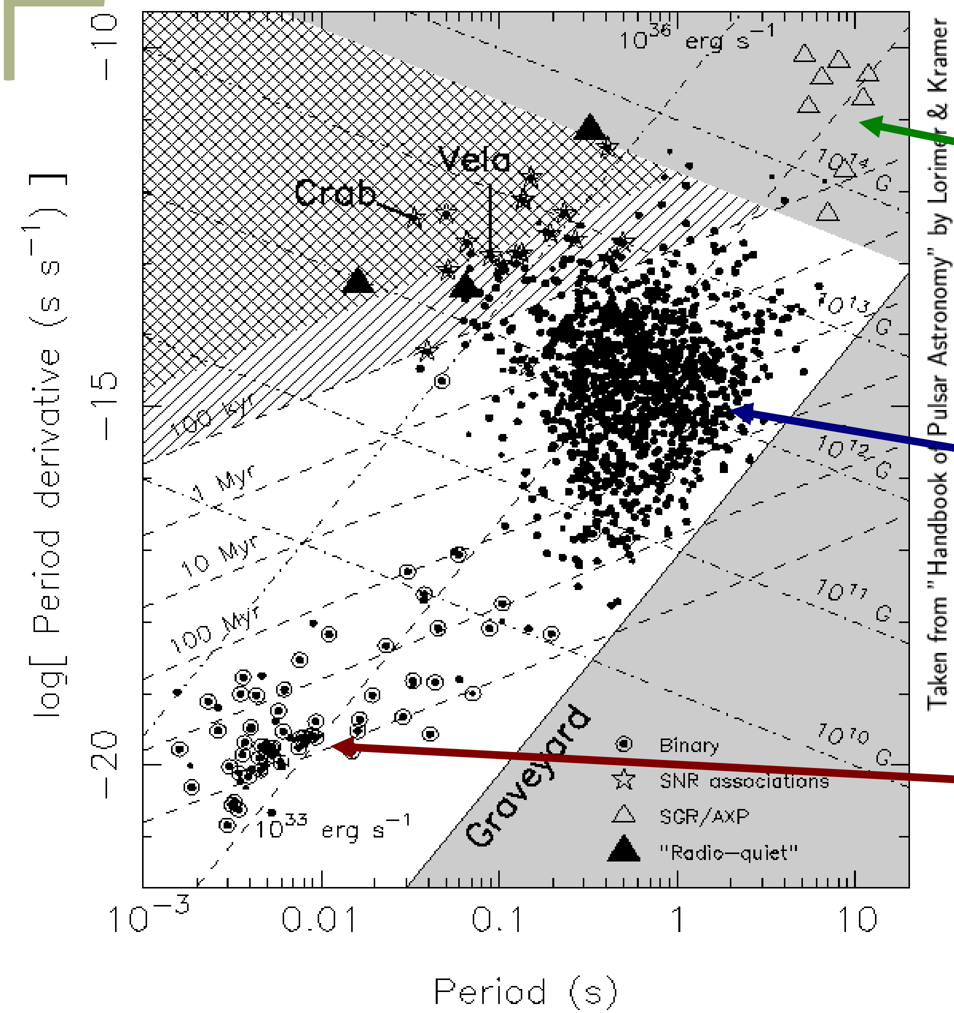
Taylor & Cordes (1993)
(See also Cordes & Lazio 2002)

PSR B1356-60 (128 ms)



$$DM \equiv \int_0^L n_e \, dl \equiv \langle n_e \rangle L \longrightarrow t_{arr}(\omega) = \int_0^L \frac{dl}{v_g} \approx \frac{1}{c} \int_0^L \left(1 + \frac{\omega_p}{2\omega} \right) dl = \frac{L}{c} + \frac{2\pi e^2}{mc\,\omega^2} DM$$

P-Pdot diagram



Magnetars

Young NS
Extremely high B field
Spin ~ 5-10 s

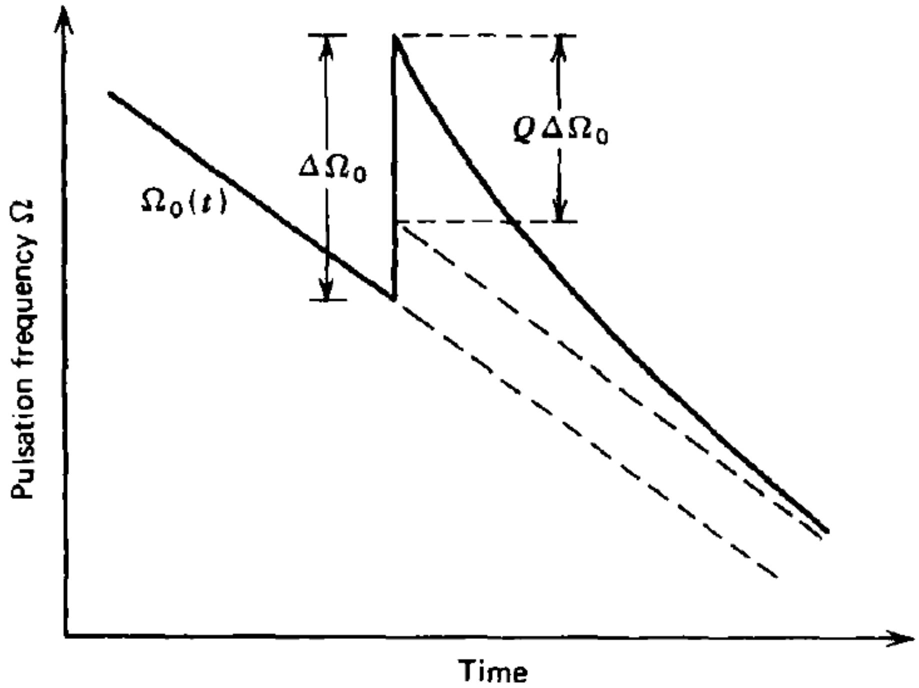
Radio Pulsars

Young NS
High B field
Spin ~ 0.01-5 s

Millisecond Pulsars

Very Old NS
Low B field
Spin ~ 1-10 ms

Pulsar Glitches: the two component Model



$$\Omega(t) = \Omega_0(t) + \Delta\Omega_0\left[Qe^{-t/\tau} + 1 - Q\right]$$

Lecture Plan

- **L1:** Introduction to the Equation of State of Cold Catalized Matter
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- L7: Neutron Stars
- L8: Neutron Stars & Pulsars
- **L9: Pulsars and Accretion discs**
- **L10:** Accretion *(?)*
- **L11**: Black Holes
- **L12:** Observational techniques
- **L13:** Cooling of Neutron Stars (?)
- **L14:** Special Session: preparation for written exam

Summary of Lecture 7

- Pulsars are rapidly spinning magnetized neutron stars that lose energy via magneto dipole radiation

- The dispersion measure is an excellent probe to measure pulsar distances

- Superfluids and Superconductors are important components of a Neutron Star

- Superfluid neutrons in the outer core cannot spin rigidly, but they need to form vortex lines

- Superconductive protons form fluxoids, which are similar to vortex lines of magnetic flux

- Glitches are rotational anomalies of pulsars, that probably involve the superfluid component

What you need to study so far

STUDY:

- Shapiro & Teukolsky (S&T): 2.2, 2.3, 2.4 (up to the end of pp. 32), 2.5, 2.6, 3.3, 3.4, 3.5, 5.1, 5.2, 5.3, 5.5, 5.6, 5.7, 8.1, 8.2, 8.5, 8.6, 9.3, 10.4, 10.5, 10.9, 10.10
- Lecture Notes 1, Lecture Notes 2, Lecture Notes 3 (all but Section 3-4 “Minimum NS mass”, “Maximum neutron star mass”), Lecture Notes 4.

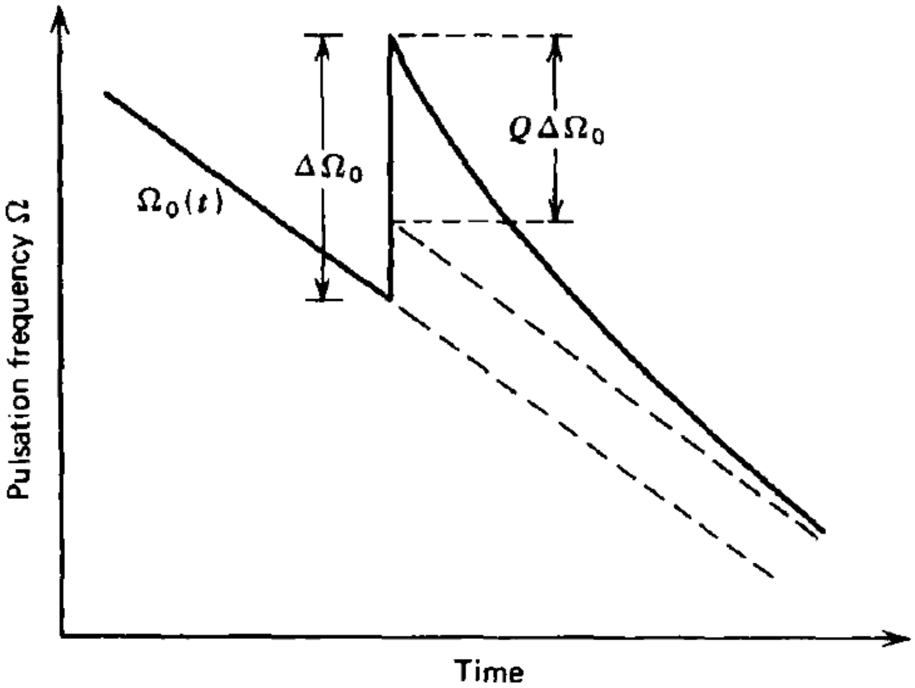
READING:

- S&T: 2.1, 2.7, 3.2
- Section 3-4 of Lecture Notes 3 (“Minimum NS mass” and “Maximum NS mass”).

<u>IN THIS LECTURE:</u>

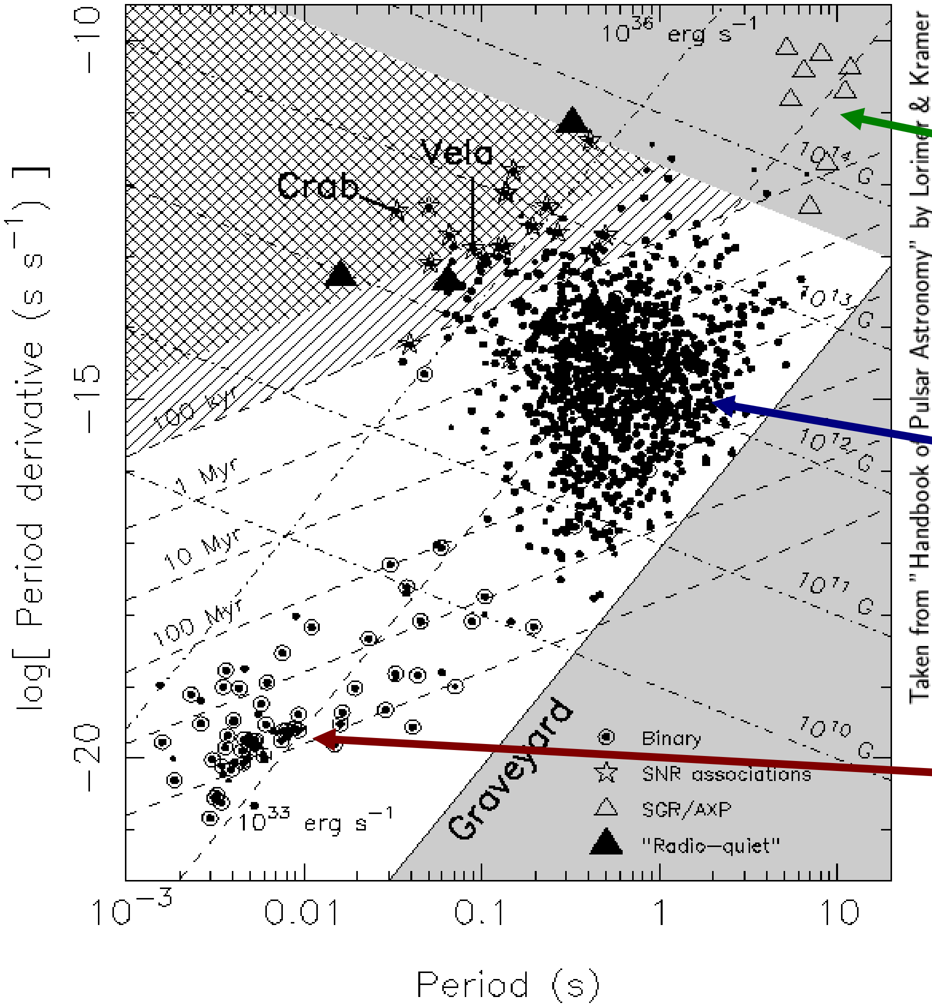
- S&T: Complete 10.5 and 10.10, 13.7.
- Lecture Notes 5
- Reading: 13.1

Pulsar Glitches: the two component Model



$$\Omega(t) = \Omega_0(t) + \Delta\,\Omega_0\left[Qe^{-t/\tau} + 1 - Q\right]$$

P-Pdot diagram



Magnetars

Young NS
Extremely high B field
Spin ~ 5-10 s

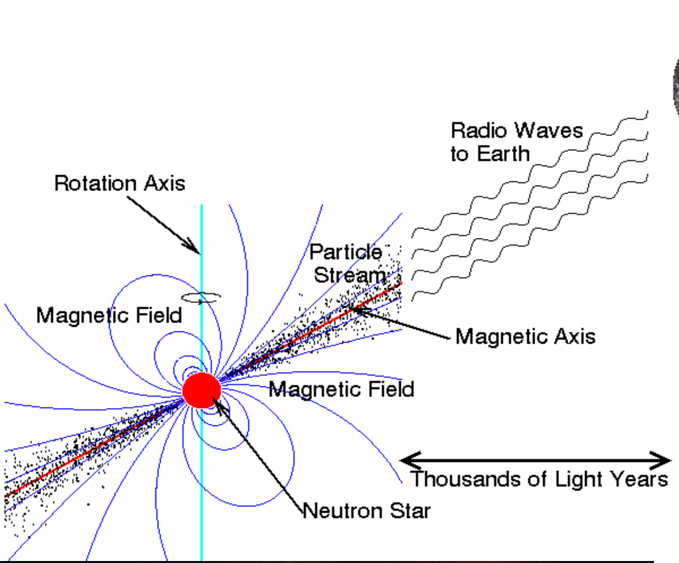
Radio Pulsars

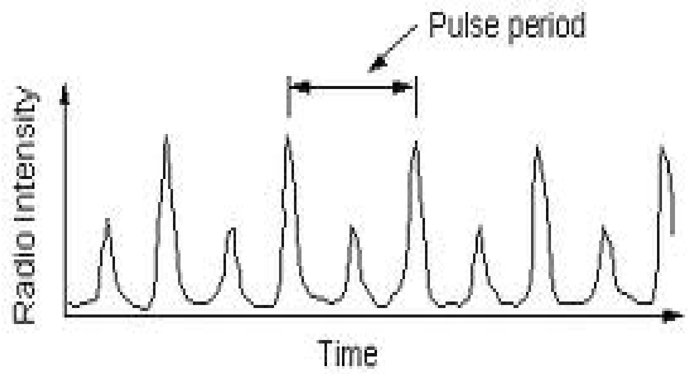
Young NS
High B field
Spin ~ 0.01-5 s

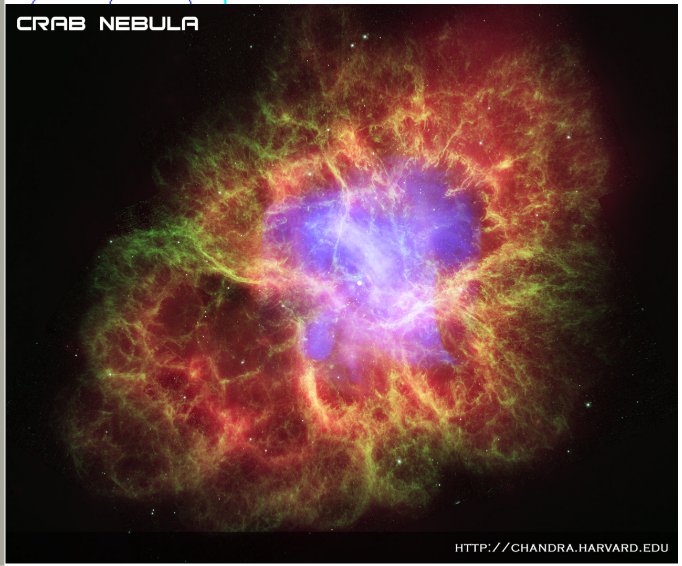
Millisecond Pulsars

Very Old NS
Low B field
Spin ~ 1-10 ms

Pulsar Emission & The Crab







$$E = 2.5\times10^{49}\, erg$$

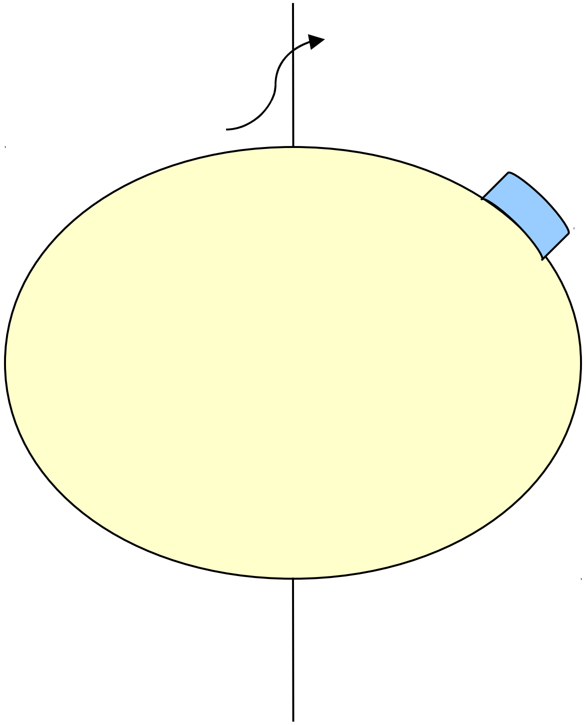
$$\dot{E} = 6.4\times10^{38}\, erg\, s^{-1}$$

$$\dot{E}_{rpulse} \sim 10^{31}\, erg\ s^{-1}$$

Gravitational Radiation Spin Down

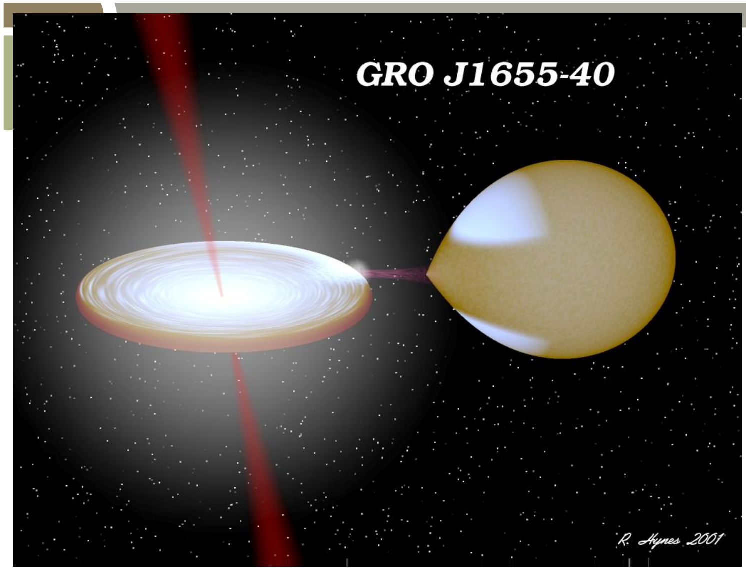
$$\epsilon = \frac{a-b}{(a+b)/2}$$

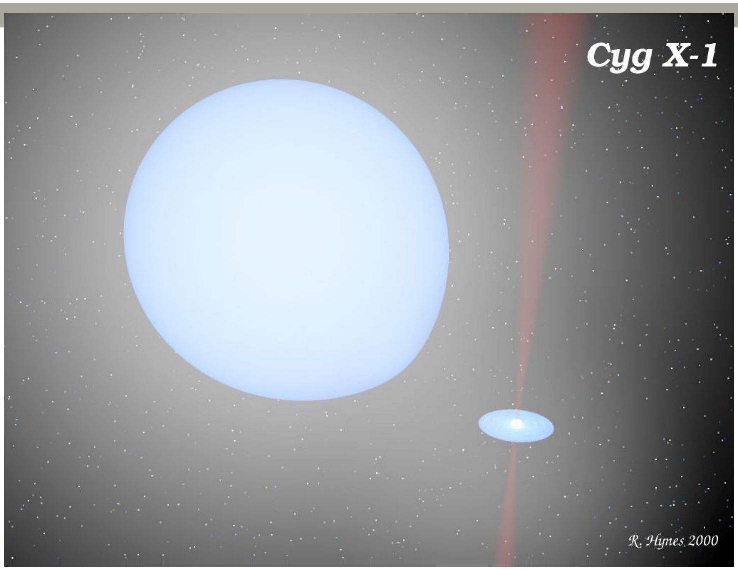
$$\dot{E}_{GW} = -\frac{32}{5}\frac{G}{c^5} I^2 \epsilon^2 \Omega^6$$



The pulsar spin down might be due *also* to other forms of energy loss beside the magneto dipole radiation.
However, if a magnetic field is present, the magneto dipole radiation will *always* be effective.

X-ray binaries





Low mass X-ray binaries
- Roche lobe overflow
- low mass companions
- old NS/BH
- accretion driven by an accretion disc

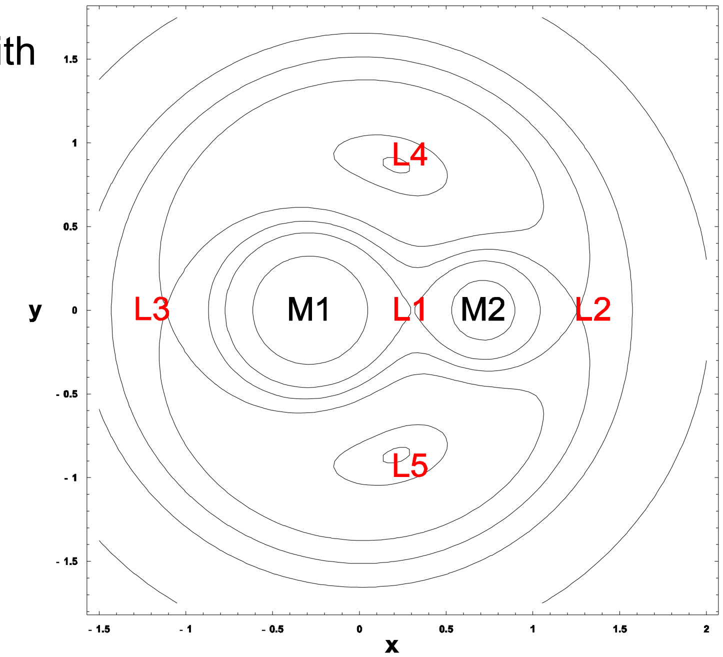
High mass X-ray binaries
- Wind fed accretion
- high mass companions
- young NS/BH
- a disc not always forms

The Roche Potential

In the center of mass reference frame co-rotating with the binary system with angular velocity:

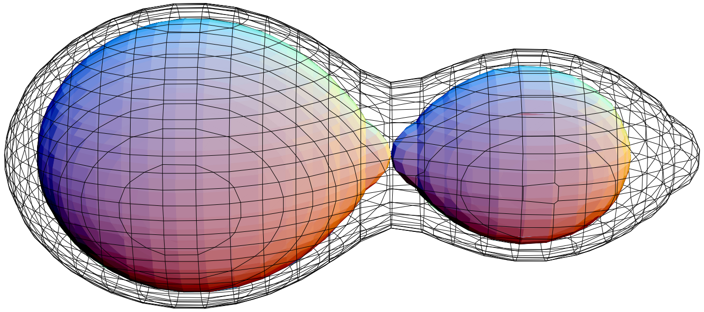
$$\vec{\Omega}_B = \left[\frac{G(M_1 + M_2)}{A^3} \right]^{1/2} \vec{u}$$

(**u** is the unit vector normal to the plane of the orbit)

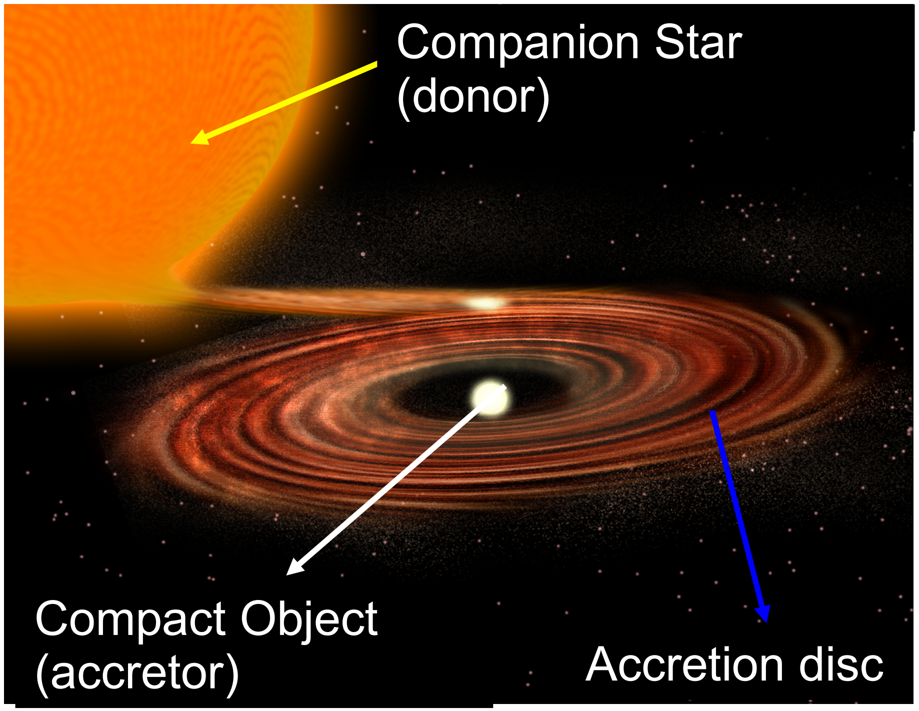


the ***Roche potential*** is given by:

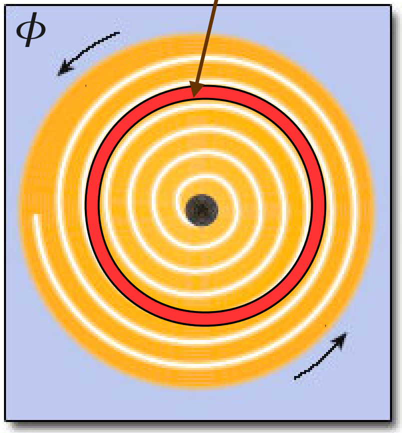
$$\varphi_R = -\frac{GM_1}{|\vec{r} - \vec{r}_1|} - \frac{GM_2}{|\vec{r} - \vec{r}_2|} - \frac{1}{2}\left(\vec{\Omega}_B \times \vec{r}\right)^2$$



Accretion discs



Circularization radius $R_{circ.} = \frac{GM}{v_\phi}$

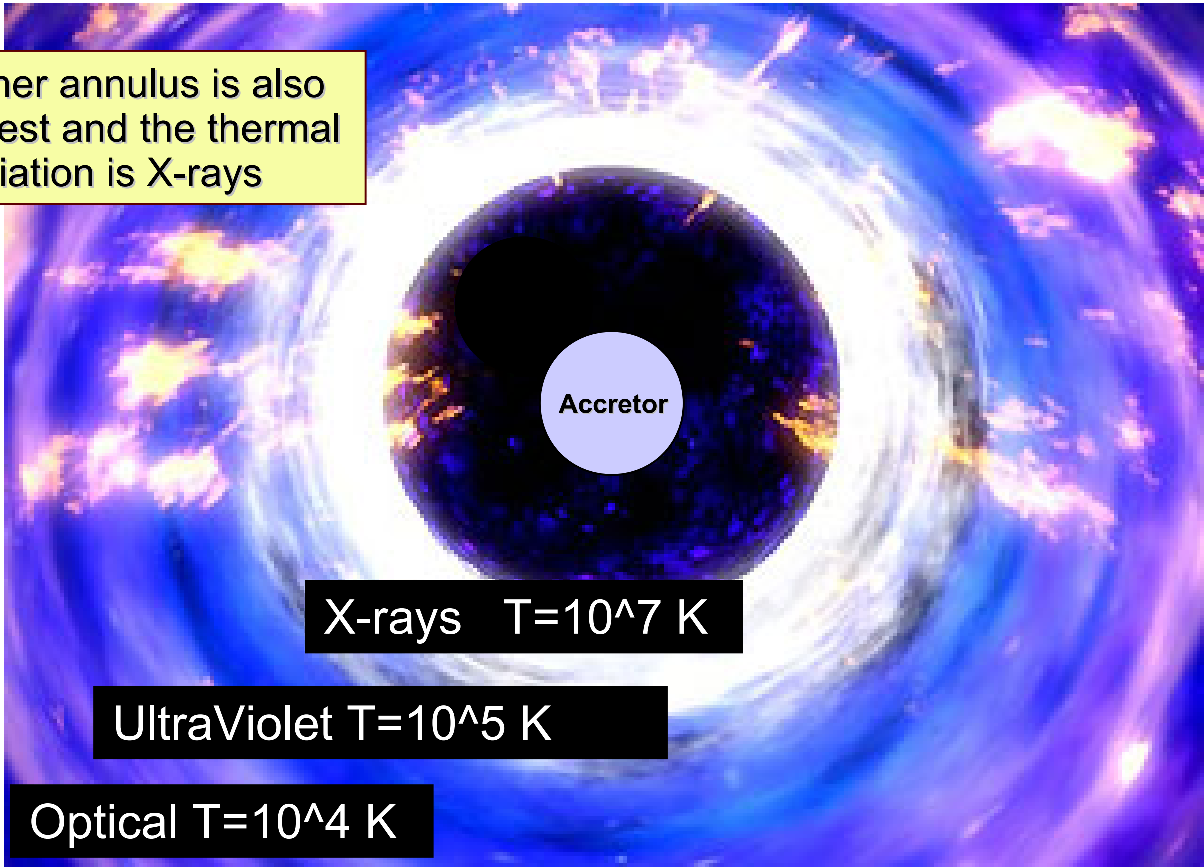


Accretion discs are the "medium" through which the gravitational potential energy which has been transformed into kinetic energy, is finally transformed into thermal energy (mainly radiation)

http://astro.fit.edu/wood/fitdisk.html

Accretion disc structure

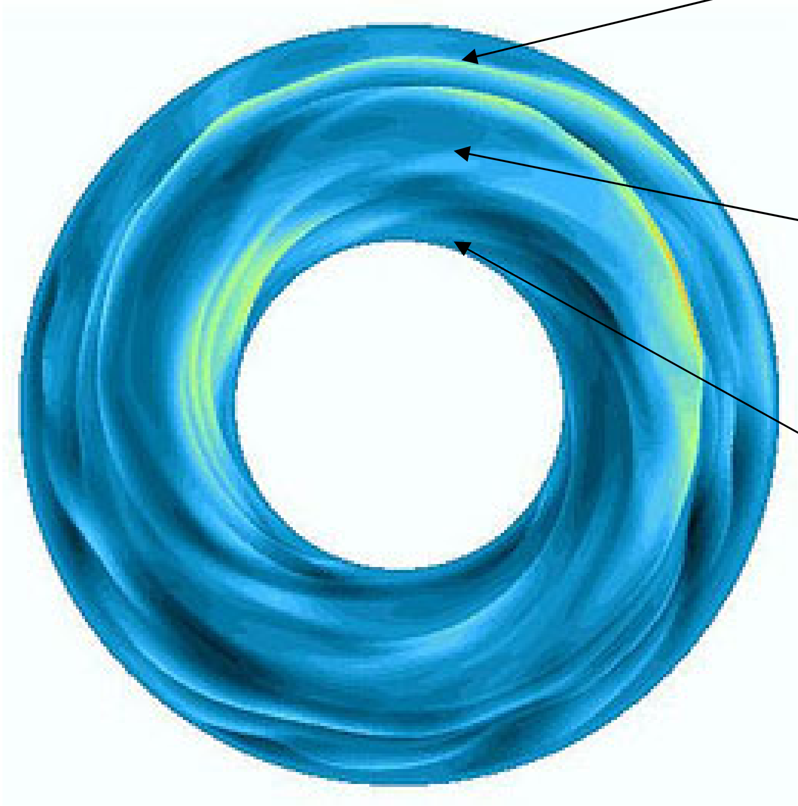
Gas rotates in the disc with “Keplerian frequency”, i.e., the inner annulus rotates faster than the outer one. → $\Omega_K = \sqrt{\frac{GM}{r^3}}$



The inner annulus is also the hottest and the thermal radiation is X-rays

The disc is usually composed mainly by ionized Hydrogen (i.e., a plasma of electrons + protons)

Accretion disc regions

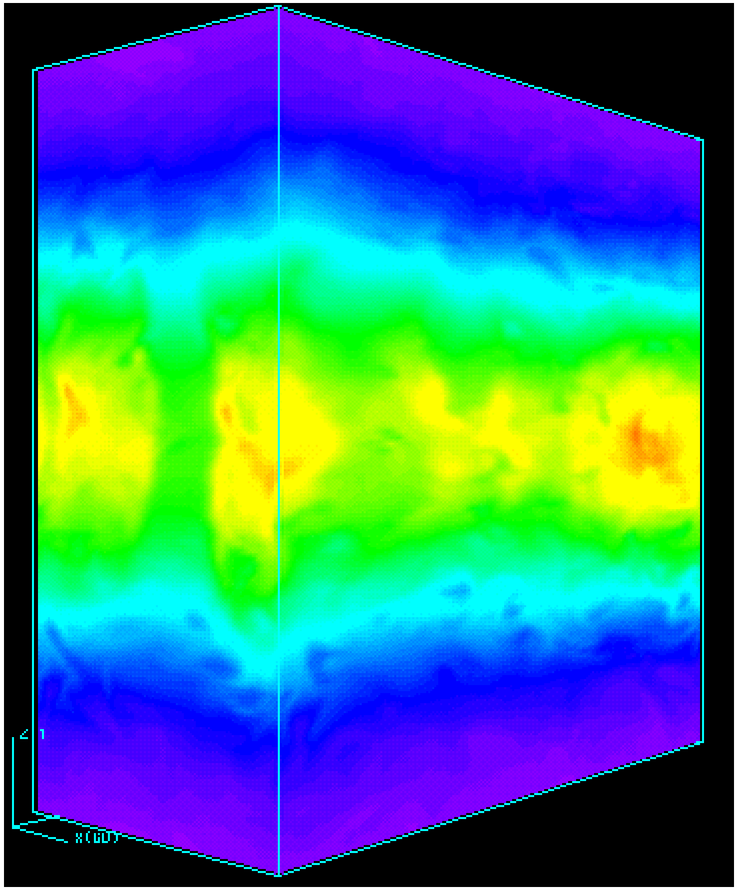


Outer Region: gas pressure dominates. Opacity: free-free absorption (i.e., inverse Bremsstrahlung)

Middle Region: gas pressure dominates
Opacity: electron scattering

Inner Region: radiation pressure dominates.
Opacity: electron scattering

Accretion disc Vertical Structure

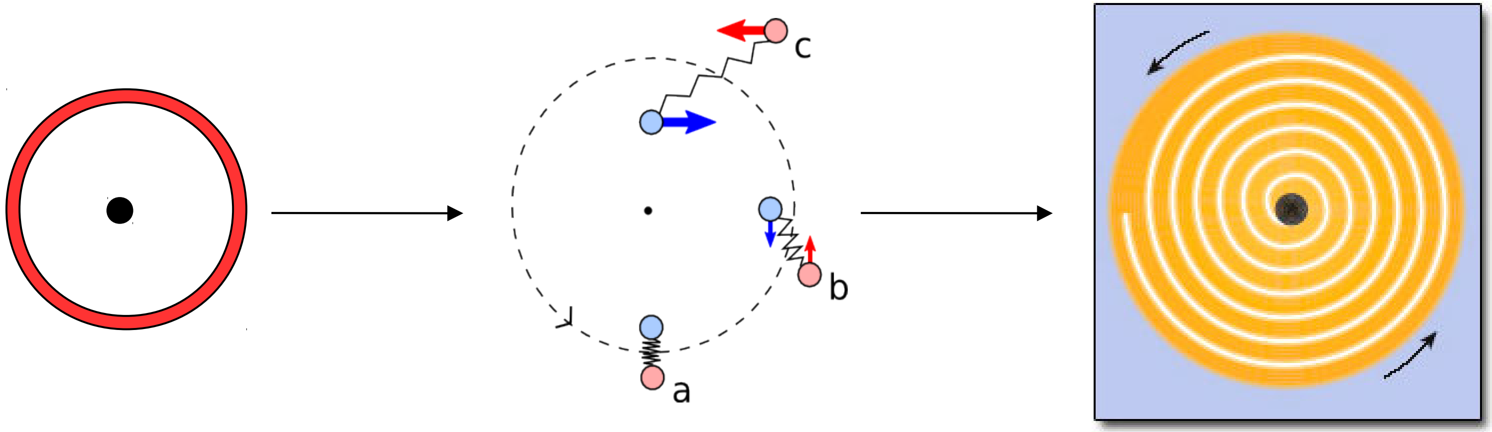


$$\frac{1}{\rho}\frac{dP}{dz} = -\frac{GM}{r^2}\frac{z}{r}$$

$$h = \left(\frac{P}{\rho}\right)^{1/2}\left(\frac{r^3}{GM}\right)^{1/2} \approx \frac{c_s}{\Omega}$$

Vertical disc is in hydrostatic equilibrium

Magneto-Rotational Instability



$$\frac{\partial(R^2\Omega)}{\partial R} > 0$$

Rayleigh Stability $\xrightarrow{\Omega \propto r^{-3/2}}$ Stability: accretion disc will not form

$$\frac{\partial(\Omega^{\text{۲}})}{\partial R} > \text{۰}$$

Rayleigh Stability $\xrightarrow{\Omega \propto r^{-3/2}}$ Instability: accretion disc can form

Lecture Plan

- **L1:** Introduction to the Equation of State of Cold Catalized Matter
- **L2:** Electrostatic corrections to the EoS of CCM and n-p-e gas
- **L3:** Harrison-Wheeler EoS.
- **L4:** White dwarfs, Chandrasekhar Limit.
- **L5:** Introduction to General Relativity, Test particle Motion
- **L6**: Spherically Symmetric Gravitational Field
- **L7**: Neutron Stars
- **L8**: Neutron Stars & Pulsars
- **L9:** Pulsars and Accretion discs
- **L10: Accretion onto magnetized compact objects**
- **L11**: Black Holes
- **L12:** Observational techniques
- **L13:** Cooling of Neutron Stars (?)
- **L14:** Special Session: preparation for written exam

Summary of Lecture 9

- Pulsars convert only a tiny fraction of their magneto dipole radiation into higher e.m. frequency radio beams that we detect on Earth

- If a NS (or a BH) can be found in a binary. If the compact object is accreting gas from the companion, then they form X-ray binaries.

- Accretion proceeds in two steps:

1. From the donor star towards the accretion disc
2. From the accretion disc towards the accretor

- An accretion disc cannot be formed without some form of instability.

- The Magneto-Rotational-Instability seems to be the most promising mechanism to form accretion discs.

What you need to study so far

STUDY:

- Shapiro & Teukolsky (S&T): 2.2, 2.3, 2.4 (up to the end of pp. 32), 2.5, 2.6, 3.3, 3.4, 3.5, 5.1, 5.2, 5.3, 5.5, 5.6, 5.7, 8.1, 8.2, 8.5, 8.6, 9.3, 10.4, 10.5, 10.9, 10.10, 10.5, 13.7
- Lecture Notes 1, 2, 3 (all but Section 3-4 “Minimum NS mass”, “Maximum neutron star mass”), 4, 5.

READING:

- S&T: 2.1, 2.7, 3.2
- Section 3-4 of Lecture Notes 3 (“Minimum NS mass” and “Maximum NS mass”).

<u>IN THIS LECTURE:</u>

Study: S&T: Complete 13.7, 7.1(only pp. 162-163), 15.1, 15.2 + Paper “*Recycled Pulsars*”

Reading: S&T 6.1, 14.5 (with paragraphs from (a) to (i) included, exclude (j))

Literature Discussion (Friday 13 May, Lec. 11)

What a Two Solar Mass Neutron Star Really Means
Topic: Neutron Star EoS (S&T: Chapter 2, 8, 9)
Lattimer, James M.; Prakash, M.

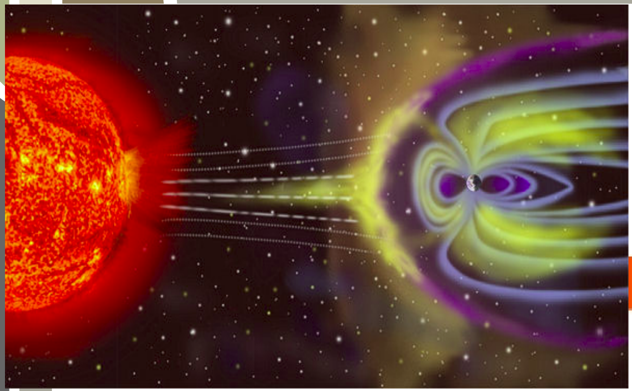
Procyon B: Outside the Iron Box
Topic: White Dwarf composition (S&T: Chapter 2, 3)
Provencal, J. L.; Shipman, H. L.; Koester, Detlev; Wesemael, F.; Bergeron, P.

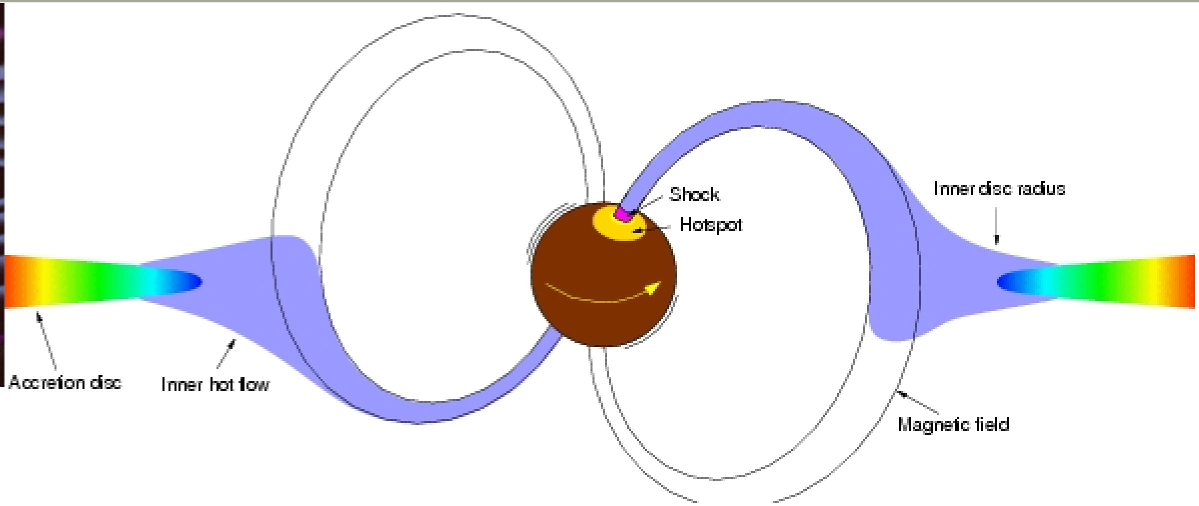
Spin Rates and Magnetic Fields of Millisecond Pulsars
Topic: Accreting Neutron Stars and Pulsars (S&T, Chapter 10, 15)
Lamb, F.; Yu, W.

The disk-magnetospheric interaction in the accreting millisecond pulsar SAX J1808.4-3658
Topic: Accretion onto magnetized objects (S&T: Chapter 14, 15)
Psaltis D. & Chakrabarty D.

The meaning of Relativity (Selected pages Lecture III and IV)
Topic: General Theory of Relativity (S&T: Chapter 5)
Albert Einstein

Accreting Pulsars





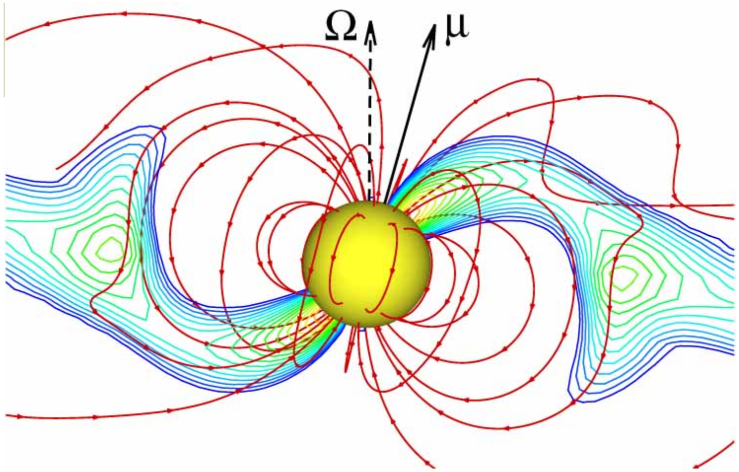
Conservation of angular momentum and viscosity leads to the formation of an accretion disc. The gas flows in the inner part of the primary Roche lobe till the following condition holds:

$$P_{mag} = \frac{B^2}{8\pi} >> (P_{gas}, P_{ram})$$

The gas then flows along the B filed lines and hits the NS surface

$$L_{Edd} \approx 1.3 \times 10^{38} \left(\frac{M}{M_{Sun}} \right) erg/s$$

Accreting Pulsars



$$R_A = \left(\frac{2\mu^2 G^2 M_{NS}^2}{\dot{M}_c} \right) \propto M_{NS}^{1/7} R^{-2/7} L^{-2/7} \mu^{4/7}$$

$$R_{co} = \left(\frac{G M_{NS}}{\omega^2} \right)^{1/3} \approx 2.8 \times 10^3 M_{NS}^{1/3} P_s^{1/2} Km$$

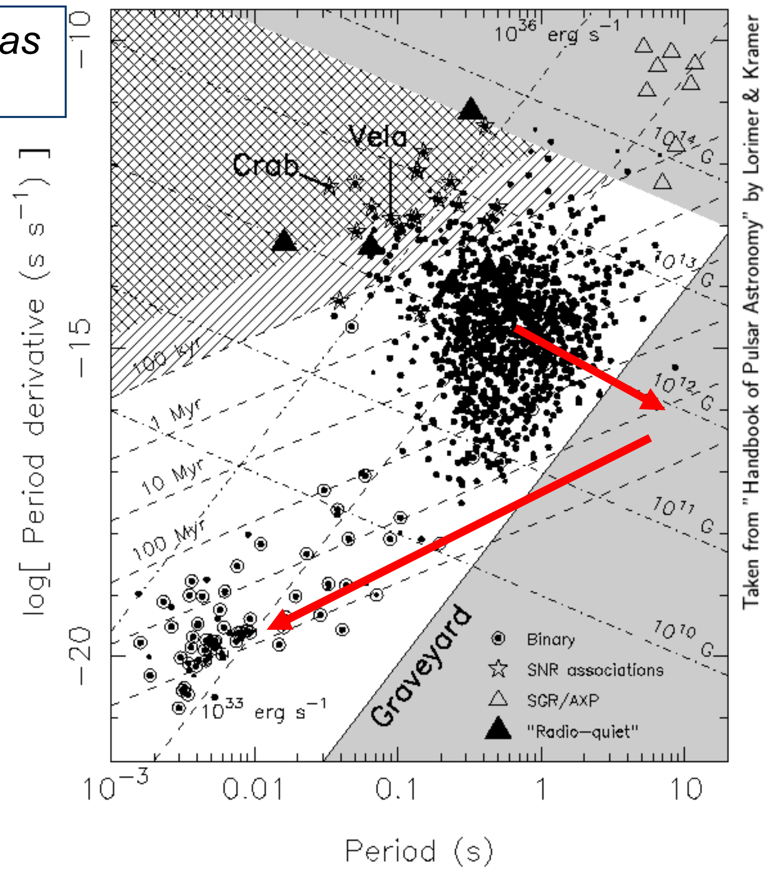
$R_A < R_{co}$ → Accretion is possible. Plasma follows the field line of the NS magnetic field

$R_A > R_{co}$ →

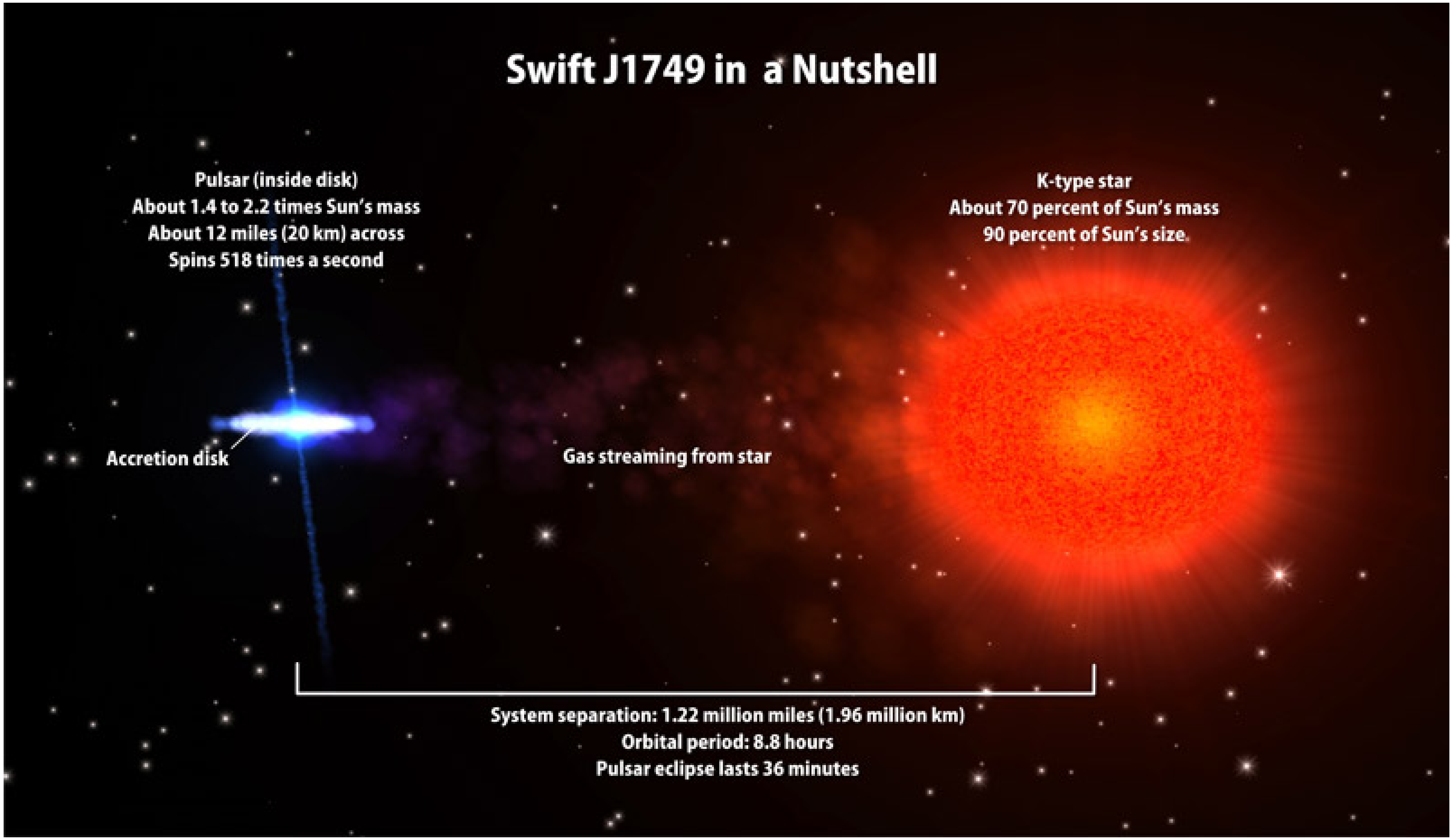
- Strong propeller:Accretion is prevented. Plasma is stopped by the centrifugal barrier of the magnetic field
- Weak propeller:Accretion is reduced by the centrifugal barrier but still can take place

The Recycling Scenario

Angular momentum is transferred from the accreting gas to the neutron star: the neutron star spins up

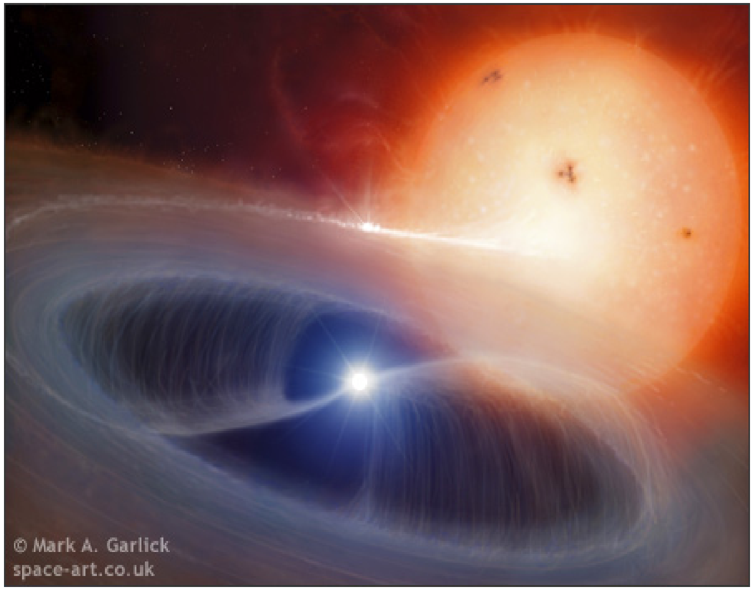


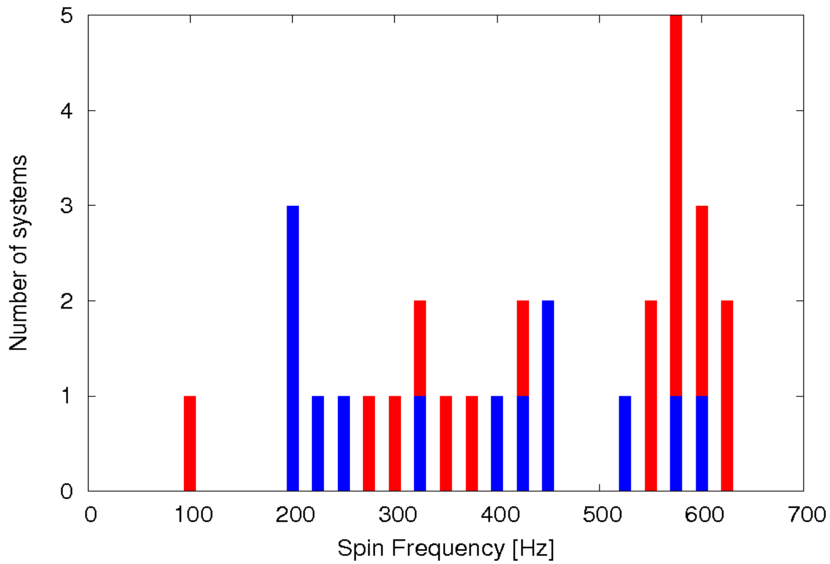
The Eclipsing Accreting Millisecond Pulsar Swift J1749.4-2807



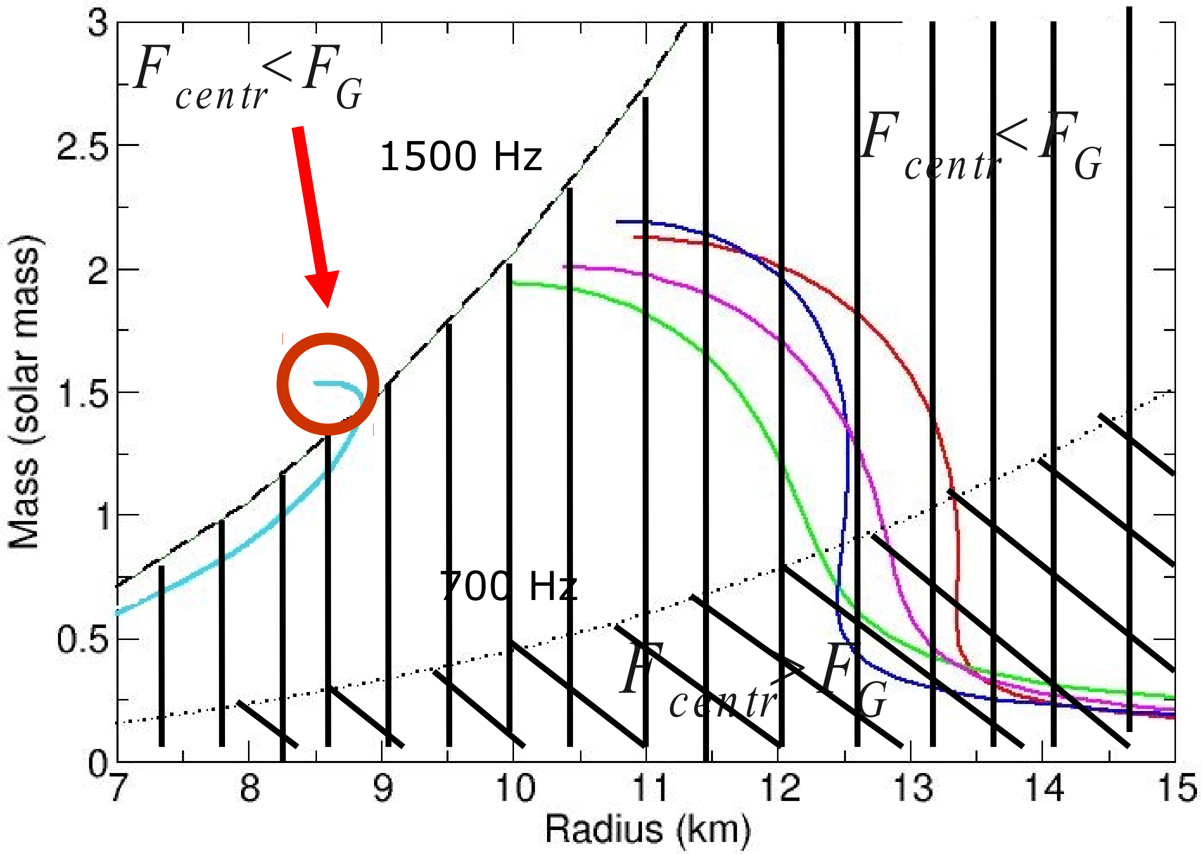
Accreting Pulsars

- ~150 systems
- ~ 30 **known** spin periods





Why do we need more pulsars ?



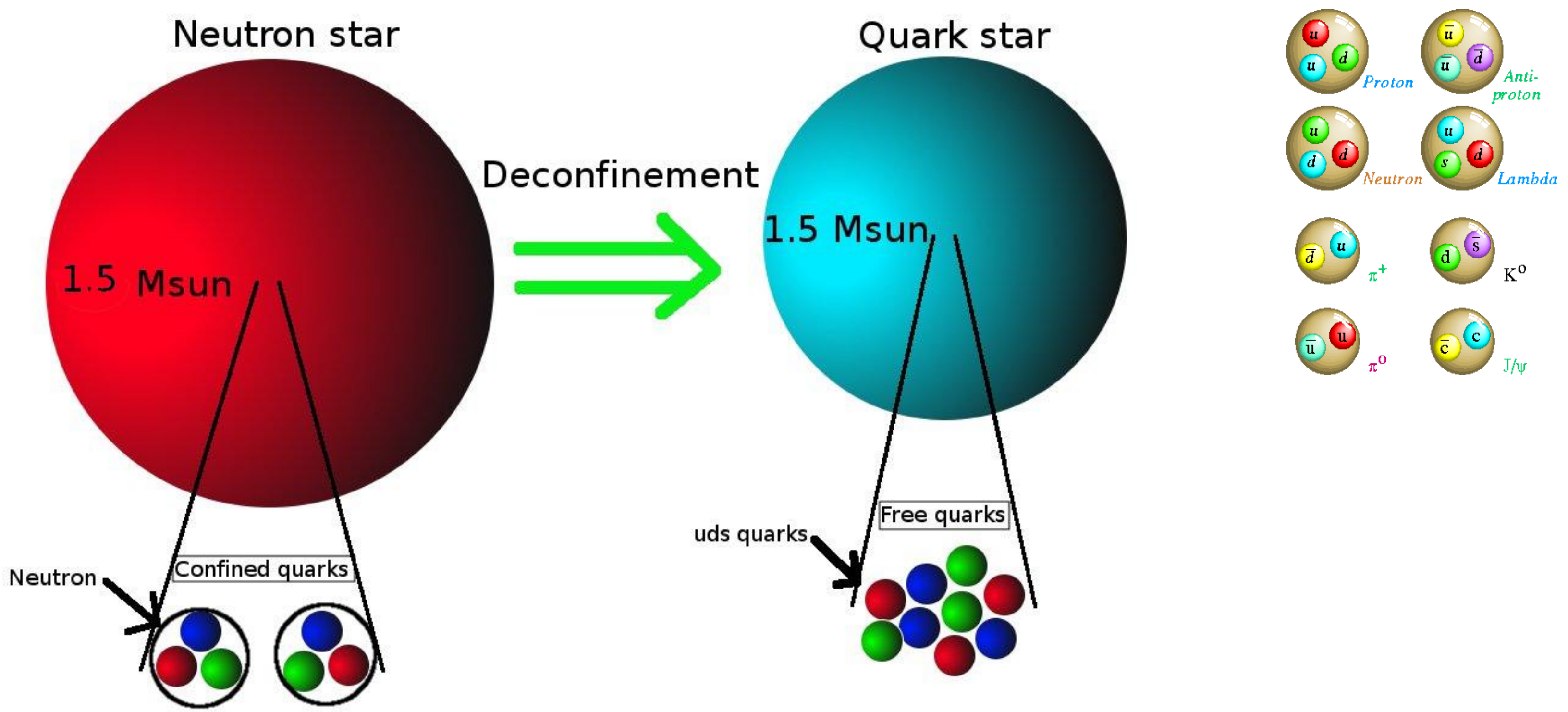
A UNIQUE

MODEL INDEPENDENT

TEST

A BREAKTHROUGH IN NUCLEAR AND PARTICLE PHISICS!

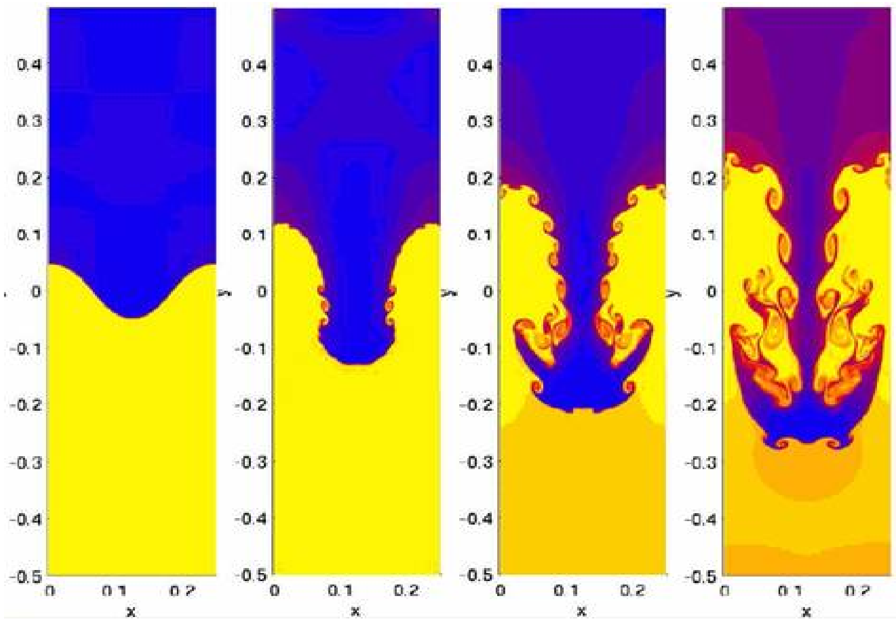
The Mass-Radius relation



Fast Spinning neutron star can give constraints on the EoS of ultradense matter, without a measurement of M and R !

Why the number of accreting pulsars is so small ?

Rayleigh-Taylor instability ?



Lecture Plan

- L1: Introduction to the Equation of State of Cold Catalized Matter
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- L8: Neutron Stars & Pulsars
- L9: Pulsars and Accretion discs
- L10: Accretion onto magnetized compact objects
- **L11: Black Holes + Discussion Paper “What a 2 Solar Mass NS Really means”**
- L12: Observational techniques
- L13: Black Holes ?
- L14: Special Session: preparation for written exam

Summary of Lecture 10

- The plasma flow in the innermost parts of the accretoin disc is controlled by the magnetospheric-disc interaction when the B field is sufficiently strong

- In LMXBs this gives rise to the “Accreting Pulsar” phenomenon

- Accreting Pulsars are the progenitors of radio millisecond pulsars

- Only a small fraction of LMXBs does pulsate. Why ?

- One possibility is the existence of an interchange instability (like Rayleigh-Taylor). However, many problems are still open

What you need to study so far

STUDY:

- Shapiro & Teukolsky (S&T): 2.2, 2.3, 2.4 (up to the end of pp. 32), 2.5, 2.6, 3.3, 3.4, 3.5, 5.1, 5.2, 5.3, 5.5, 5.6, 5.7, 8.1, 8.2, 8.5, 8.6, 9.3, 10.4, 10.5, 10.9, 10.10, 10.5, 13.7, 15.1, 15.2, 7.1 (only pp. 162-163), Paper *"Recycled Pulsars"*
- Lecture Notes 1, 2, 3 (all but Section 3-4 "Minimum NS mass", "Maximum neutron star mass"), 4, 5.

READING:

- S&T: 2.1, 2.7, 3.2, 6.1, 14.5
- Section 3-4 of Lecture Notes 3 ("Minimum NS mass" and "Maximum NS mass").

<u>IN THIS LECTURE:</u>
Study: S&T: 12.3, 12.4.
Reading: S&T 12.1, 12.2, 12.6

Topics in this Lecture

1. What is the definition of “Black Hole” ?

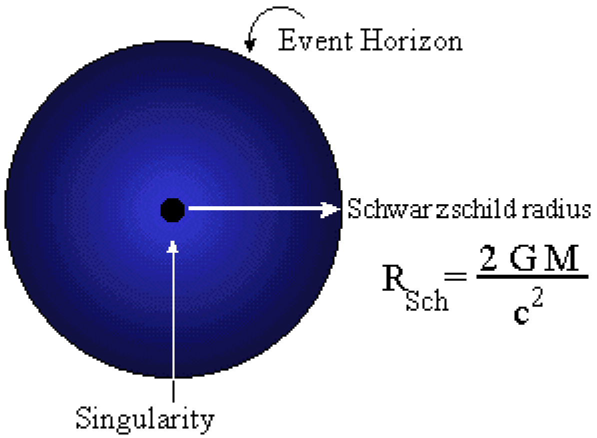
2. What is the simplest BH that exists in Nature ?

3. How does a strong gravity field (around a BH) distort the time and space perception of an observer ?

Black Hole Definition

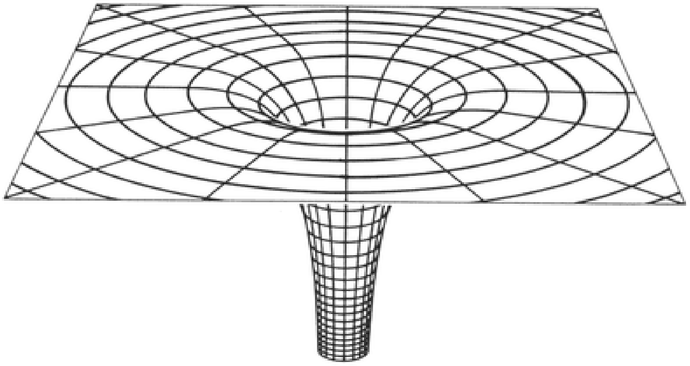
A Black Hole is defined as "*a region of spacetime that cannot communicate with the external universe*"

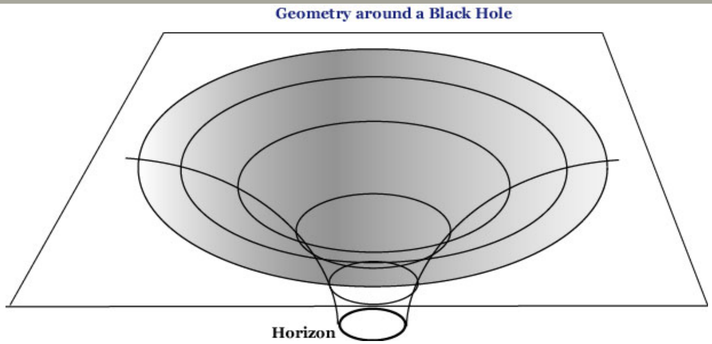
Note: the definition of BH as a body from which light cannot escape is not completely correct.

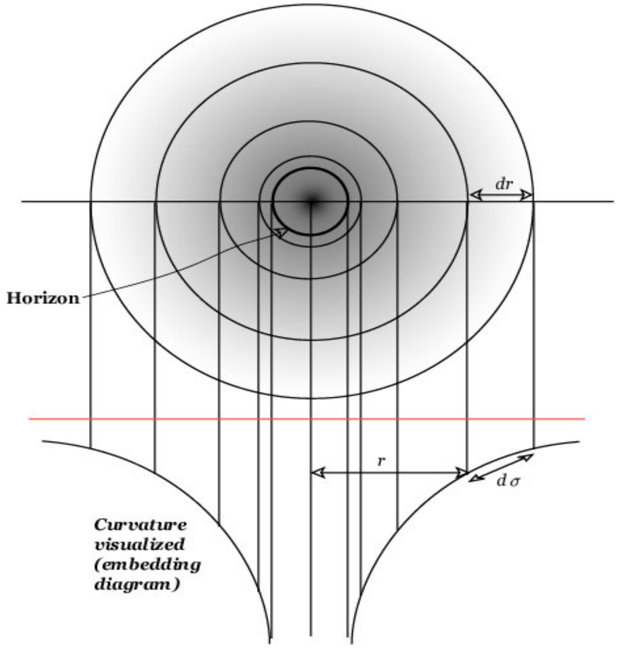


BHs are *not* necessarily very dense objects, if you take the Schwarzschild radius to calculate the volume.

Embedding Diagrams



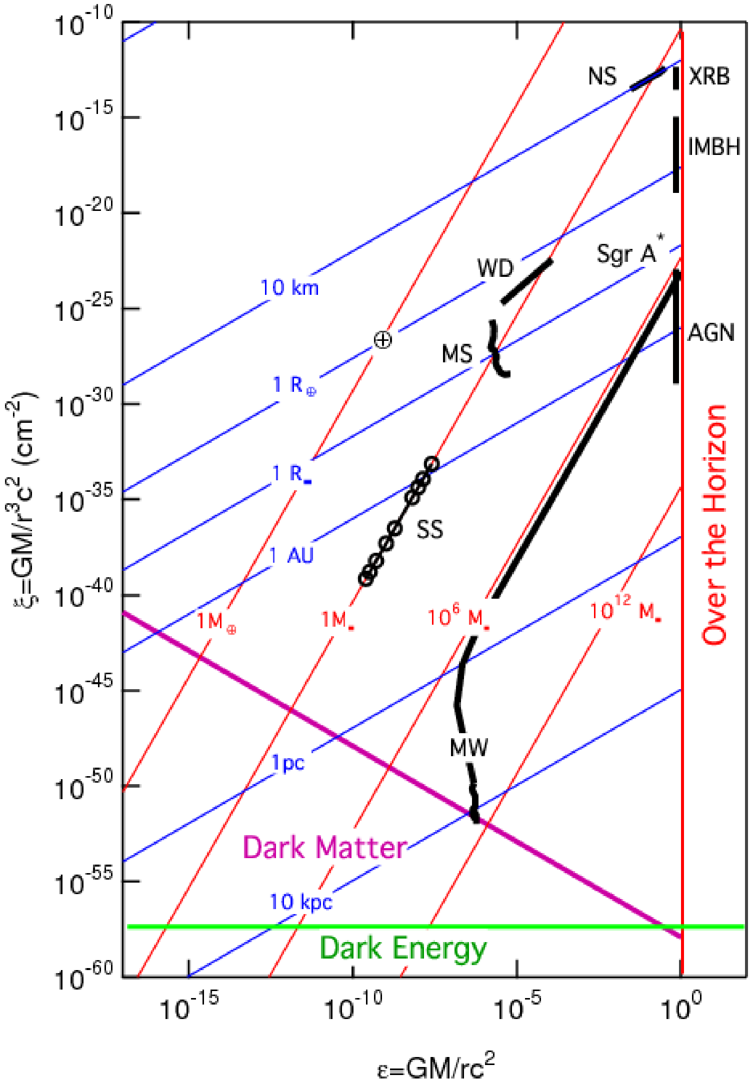




Schwarschild Metric

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

Curvature vs. Strength



Literature Discussion

What a Two Solar Mass Neutron Star Really Means

Lattimer, James M.; Prakash, M.

Rules of the Discussion:

1. The students lead the discussion

2. The discussion is open, i.e., everyone can speak at any moment

3. Everyone if expected to contribute
4. Disagreeing is **alright** !

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What a Two Solar Mass Neutron Star Really Means

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Received Day Month Year
Revised Day Month Year

The determination of neutron star masses is reviewed in light of a new measurement of 1.97 $M_\odot$ for PSR J1614-2230 and an estimate of 2.4 $M_\odot$ for the black widow pulsar. Using a simple analytic model related to the so-called maximally compact equation of state, model-independent upper limits to thermodynamic properties in neutron stars, such as energy density, pressure, baryon number density and chemical potential, are established which depend upon the neutron star maximum mass. Using the largest well-measured neutron star mass, 1.97 $M_\odot$, it is possible to show that the energy density can never exceed about 2 GeV, the pressure about 1.3 GeV, and the baryon chemical potential about 2.1 GeV. Further, if quark matter comprises a significant component of neutron star cores, these limits are reduced to 1.3 GeV, 0.9 GeV, and 1.5 GeV, respectively. We also find the maximum binding energy of any neutron star is about 25% of the rest mass. Neutron matter properties and astrophysical constraints additionally imply an upper limit to the neutron star maximum mass of about 2.4 $M_\odot$. A measured mass of 2.4 $M_\odot$ would be incompatible with hybrid star models containing *significant* proportions of exotica in the form of hyperons, Bose condensates or quark matter.

Keywords: neutron star masses; equation of state.

1. Introduction

One of the most fascinating stories in astrophysics concerns the accumulation of precisely measured neutron star masses. Gerry has had a long-time interest in these measurements, as for many years he has maintained that the neutron star maximum mass is no greater than (1.5-1.6) $M_\odot$. From a theoretical perspective, a maximum mass of 1.5 $M_\odot$ fits his fondness for the effects of kaon condensation proposed by Kaplan and Nelson [1]: the effective kaon mass falls with increasing density and the eventual onset of kaon condensation at a few times the nuclear saturation density softens the equation of state (hereafter, EOS) and leads to a rather small maximum

Literature Discussion

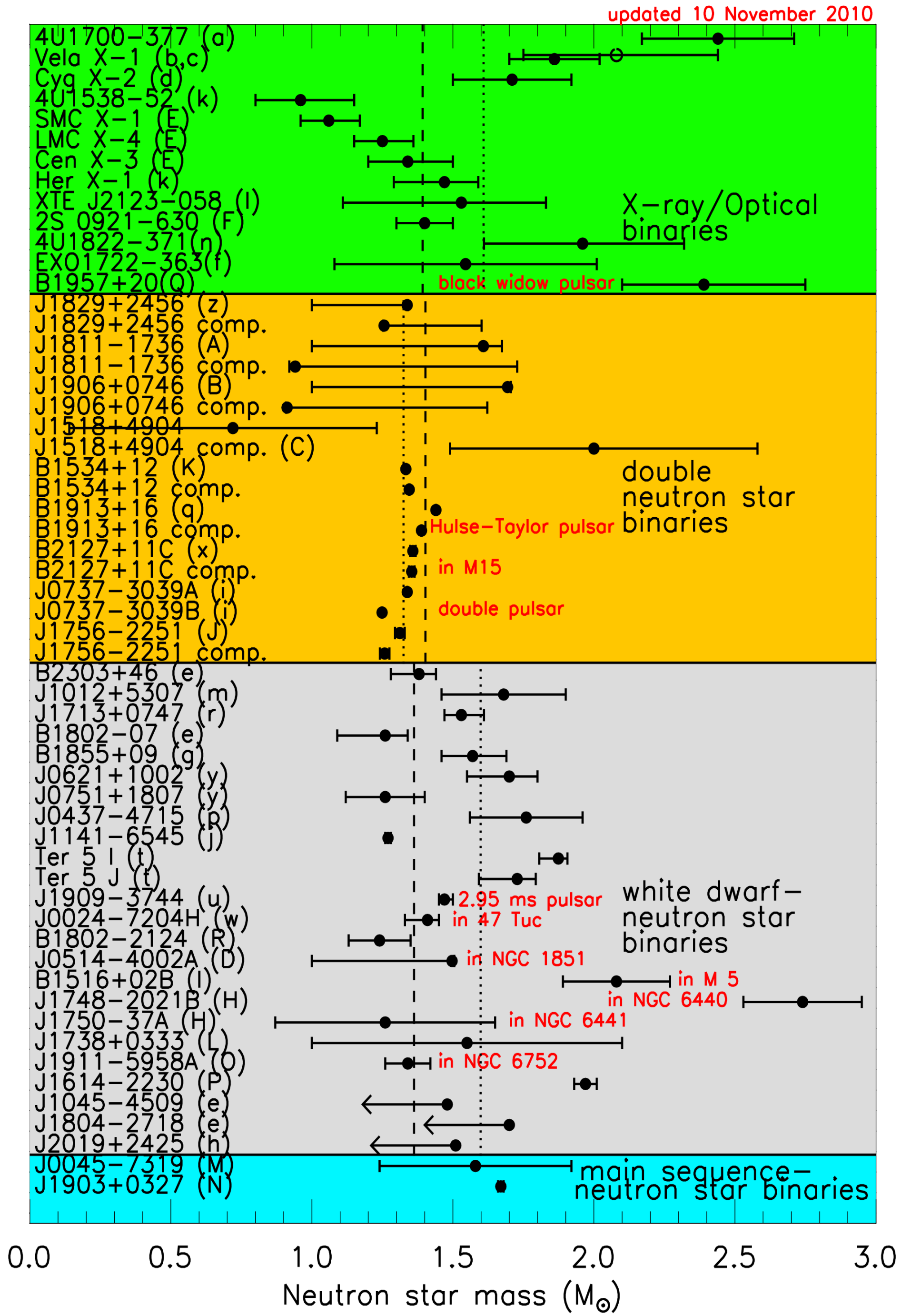
What a Two Solar Mass Neutron Star Really Means
Lattimer, J. M.; Prakash, M.

A few warm up questions:

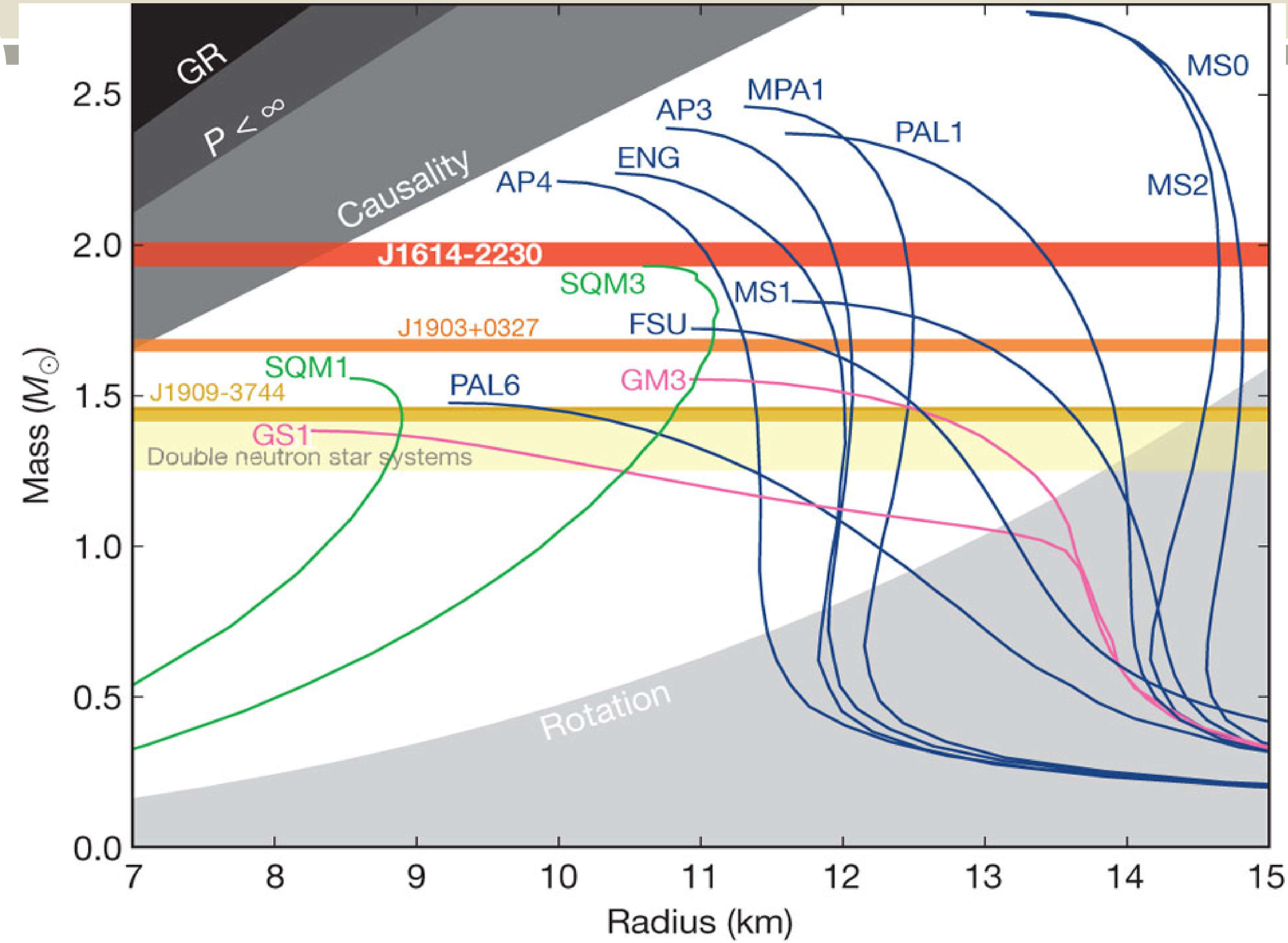
1. What is the main point that the authors tried to make ?
2. Does a 2 Msun neutron star really changes our understanding of NS ?
3. Demorest (who discovered the 1.97 Msun neutron star) says:

"The reported mass of the neutron star is above that predicted by almost all current exotic models, weakening the possibility that neutron stars are made from anything other than neutrons"

Do Lattimer & Prakash agree with this statement ?



updated 10 November 2010
4U1700-377 (a)
Vela X-1 (b,c)
Cyg X-2 (d)
4U1538-52 (k)
SMC X-1 (E)
LMC X-4 (E)
Cen X-3 (E)
Her X-1 (k)
XTE J2123-058 (l)
2S 0921-630 (F)
4U1822-371(n)
EXO1722-363(f)
B1957+20(Q)
black widow pulsar
X-ray/Optical binaries
J1829+2456 (z)
J1829+2456 comp.
J1811-1736 (A)
J1811-1736 comp.
J1906+0746 (B)
J1906+0746 comp.
J1518+4904
J1518+4904 comp. (C)
B1534+12 (K)
B1534+12 comp.
B1913+16 (q)
B1913+16 comp.
Hulse-Taylor pulsar
B2127+11C (x)
B2127+11C comp.
in M15
J0737-3039A (i)
J0737-3039B (i)
double pulsar
J1756-2251 (J)
J1756-2251 comp.
double neutron star binaries
B2303+46 (e)
J1012+5307 (m)
J1713+0747 (r)
B1802-07 (e)
B1855+09 (g)
J0621+1002 (y)
J0751+1807 (y)
J0437-4715 (p)
J1141-6545 (j)
Ter 5 I (t)
Ter 5 J (t)
J1909-3744 (u)
2.95 ms pulsar
J0024-7204H (w)
in 47 Tuc
B1802-2124 (R)
J0514-4002A (D)
in NGC 1851
B1516+02B (I)
in M 5
J1748-2021B (H)
in NGC 6440
J1750-37A (H)
in NGC 6441
J1738+0333 (L)
J1911-5958A (O)
in NGC 6752
J1614-2230 (P)
J1045-4509 (e)
J1804-2718 (e)
J2019+2425 (h)
white dwarf-neutron star binaries
J0045-7319 (M)
J1903+0327 (N)
main sequence-neutron star binaries
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Neutron star mass ($M_\odot$)



GR
P < ∞
Causality
J1614-2230
J1903+0327
J1909-3744
Double neutron star systems
Rotation
MS0
MPA1
AP3
PAL1
ENG
AP4
MS2
SQM3
MS1
FSU
SQM1
PAL6
GM3
GS1
2.5
2.0
1.5
1.0
0.5
0.0
Mass ($M_\odot$)
7
8
9
10
11
12
13
14
15
Radius (km)

Lecture Plan

- L1: Introduction to the Equation of State of Cold Catalized Matter
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- L3: Harrison-Wheeler EoS.
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- L9: Pulsars and Accretion discs
- L10: Accretion onto magnetized compact objects
- L11: Black Holes + Discussion Paper “What a 2 Solar Mass NS Really means”
- **L12: Observational techniques 1**
- L13: Observational techniques 2

L14: Special Session: preparation for written exam

Summary of Lecture 11

- Schwarzschild Black Holes are the simplest black holes in Nature. They can be described by only one parameter: the mass M

- A particle moving around a Schwarzschild BH cannot move in a stable orbit beyond the so called Innermost Stable Cicrular Orbit

- A free falling observer crosses the event horizon in a finite amount of time.

- An observer at infinity will see the free falling observer crossing the event horizon in an infinite amount of time.

What you need to study so far

STUDY:

- Shapiro & Teukolsky (S&T): 2.2, 2.3, 2.4 (up to the end of pp. 32), 2.5, 2.6, 3.3, 3.4, 3.5, 5.1, 5.2, 5.3, 5.5, 5.6, 5.7, 8.1, 8.2, 8.5, 8.6, 9.3, 10.4, 10.5, 10.9, 10.10, 13.7, 15.1, 15.2, 7.1 (only pp. 162-163), 12.3, 12.4
- Paper *“Recycled Pulsars”* + *“What a 2 Solar Mass NS Really Means”*
- Lecture Notes 1, 2, 3 (all but Section 3-4 “Minimum NS mass”, “Maximum neutron star mass”), 4, 5.

READING:

- S&T: 2.1, 2.7, 3.2, 6.1, 14.5, 12.1, 12.2, 12.6
- Section 3-4 of Lecture Notes 3 (“Minimum NS mass” and “Maximum NS mass”).

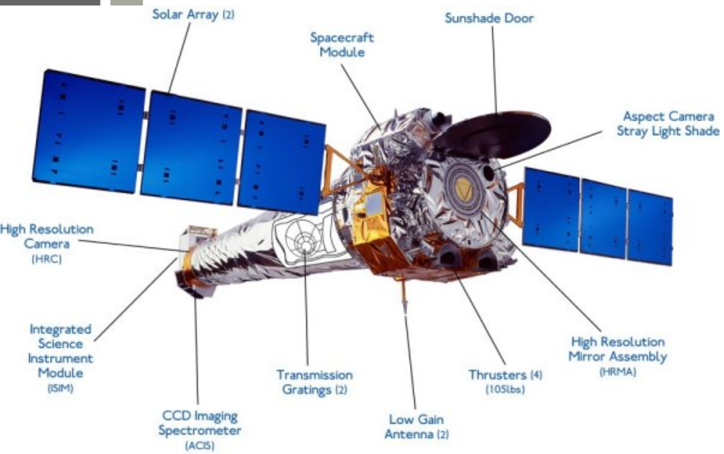
IN THIS LECTURE:

Study: Lecture Notes 6

How to probe Compact Object physics ?

1. X-ray spectra: accretion disc, cooling, cyclotron resonance, etc… (WDs, NS & BHs)
2. Coherent timing: pulse profile shape, torques, timing noise, mass, glitches (WDs, NS & BHs)
3. Thermonuclear bursts (NS)
4. Aperiodic variability: oscillation modes, QPOs (WDs, NS & BHs)

   Use of X-ray telescopes and/or proportional counters aboard space satellites: Chandra, XMM-Newton, RXTE, Suzaku, Swift



How to probe NS physics ?

1. Pulsar timing is one of the most precise and best tested ways to probe neutron star physics.

Use of ground-based radio facilities: Jodrell Band, Parkes, Greenbank, Westerbork, Arecibo, etc...

Pulsar Timing

Second freq. derivative (related with braking index)

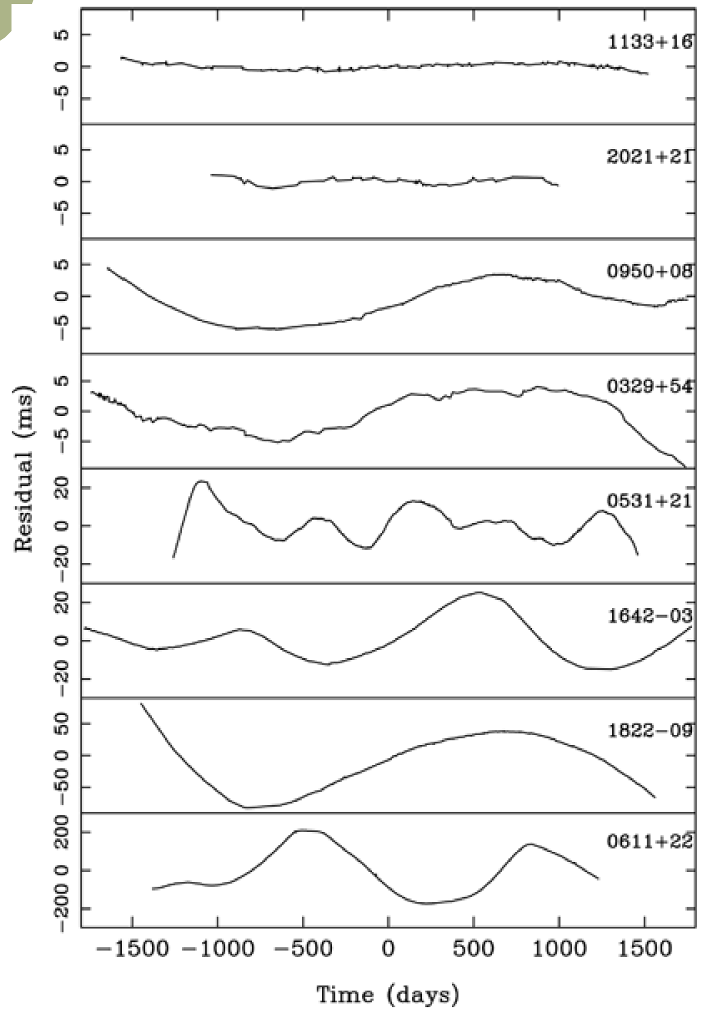
Spin Frequency

$$\phi(t) = \phi(t_0) + \nu_0(t - t_0) + \frac{\dot{\nu}_0\,(t - t_0)^2}{2} + \frac{\ddot{\nu}_0\,(t - t_0)^3}{6}$$

Spin Frequency derivative (related with torques)

Initial phase (fiducial point)

Pulsar Timing Noise



Timing Noise manifests itself as long-term quasi periodic variations in the timing residuals.

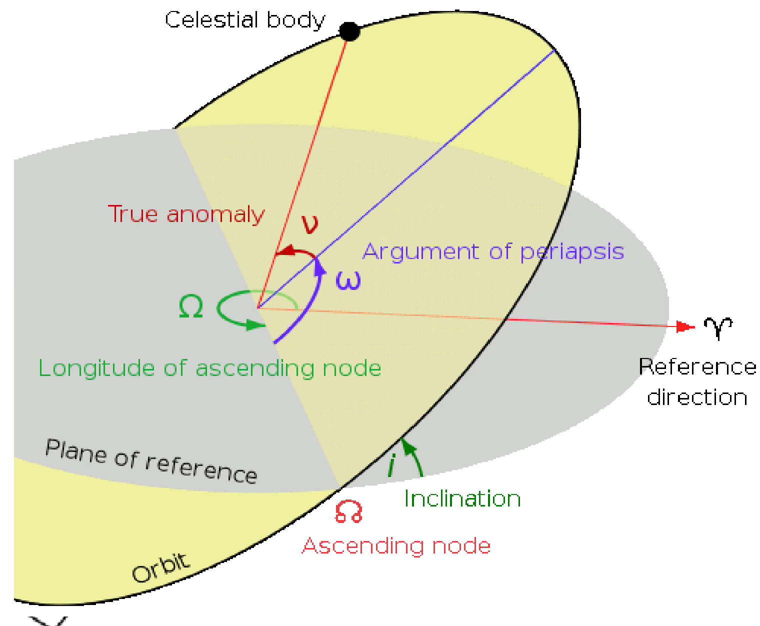
It is observed almost exclusively in young slowly rotating radio pulsars.

Might be connected with superfluid components in the NS interior
Microglitches ?

Orbital Elements

Classical (Keplerian) descriprion of orbital motion is complete with the knowledge of 6 orbital elements.

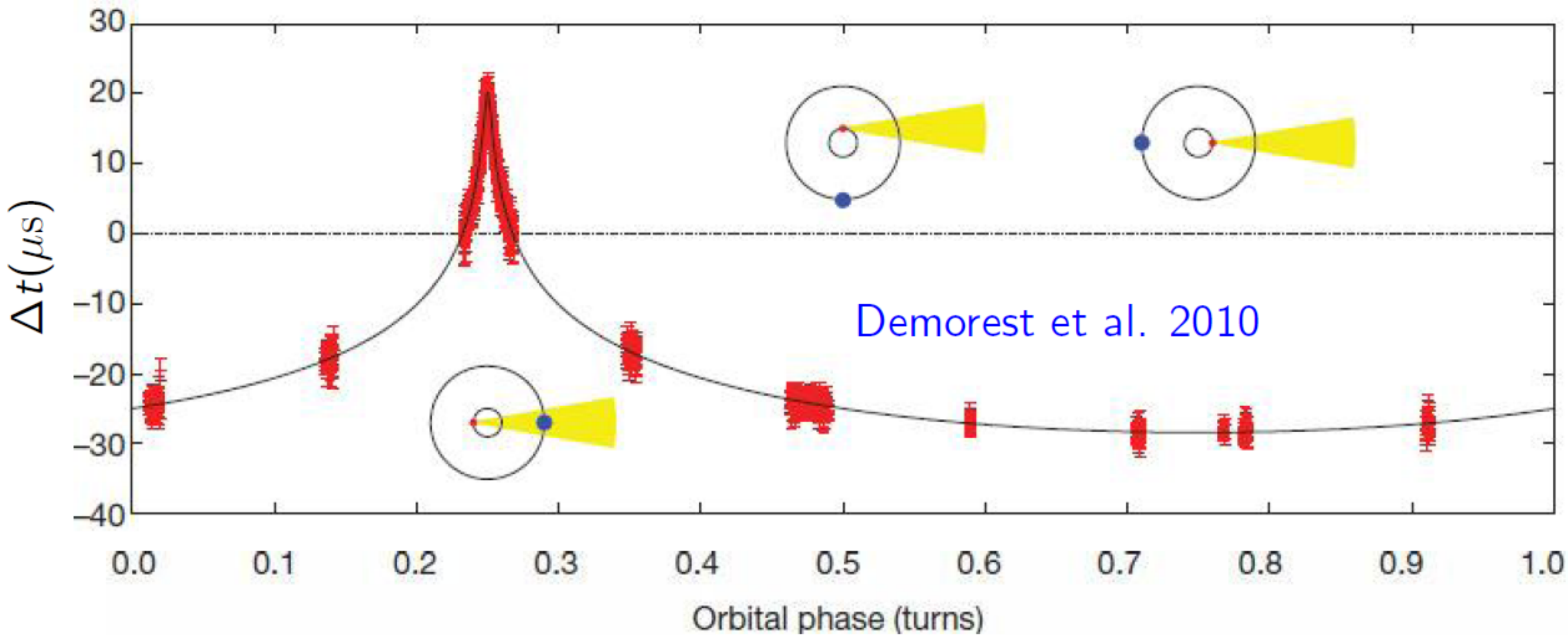
General Relativity requires further parameters, called Post Keplerian (PK) parameters, to take into account effects of strong gravity.



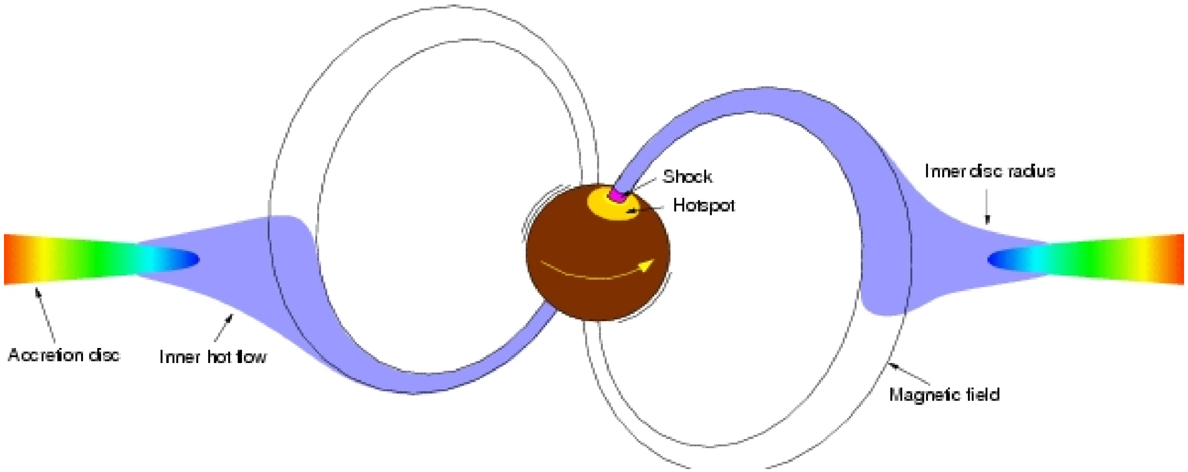
element	name	geometric description
i	inclination	angle between the orbital plane and a reference plane.
Ω	longitude of the ascending node	angle between line of nodes and the zero point of longitude in the reference plane.
a	semimajor axis	half the major axis of an orbit's ellipse.
e	eccentricity	$\sqrt{1-\frac{b^2}{a^2}}$ where a and b are the semimajor and semiminor axes of the orbit's ellipse.
ω	argument of pericenter	the angle from the ascending node to the body in the orbital plane (denoted θ_0 when measured from any point in the orbital plane).
T	time of pericenter passage	a time at which the body passed through pericenter.

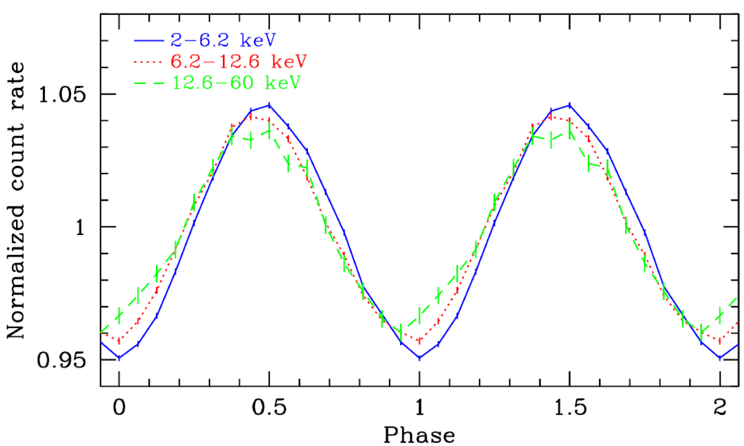
Example: The 1.97 Msun Pulsar

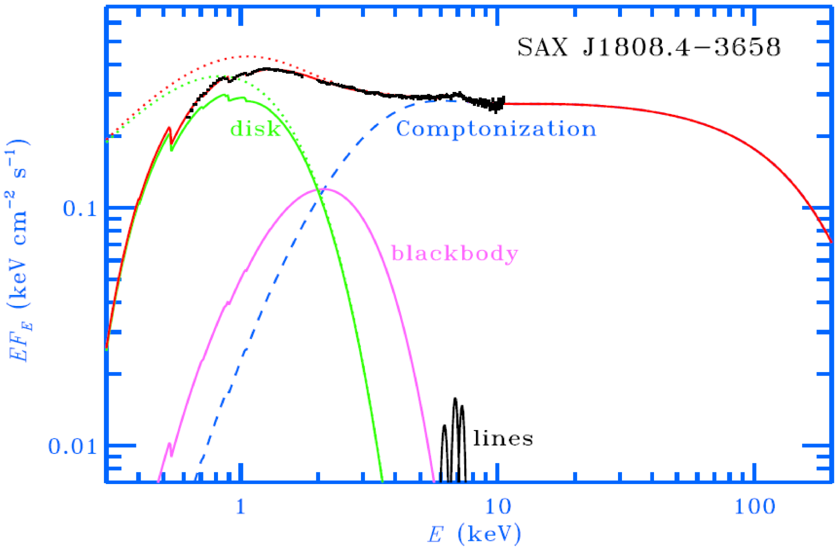
A 3.15 ms pulsar in an 8.69d orbit with an 0.5 $M_\odot$ white dwarf companion.
Shapiro delay yields edge-on inclination: $\sin i = 0.99984$
Pulsar mass is $1.97 \pm 0.04\ M_\odot$
Distance > 1 kpc, $B \simeq \times 10^8$ G



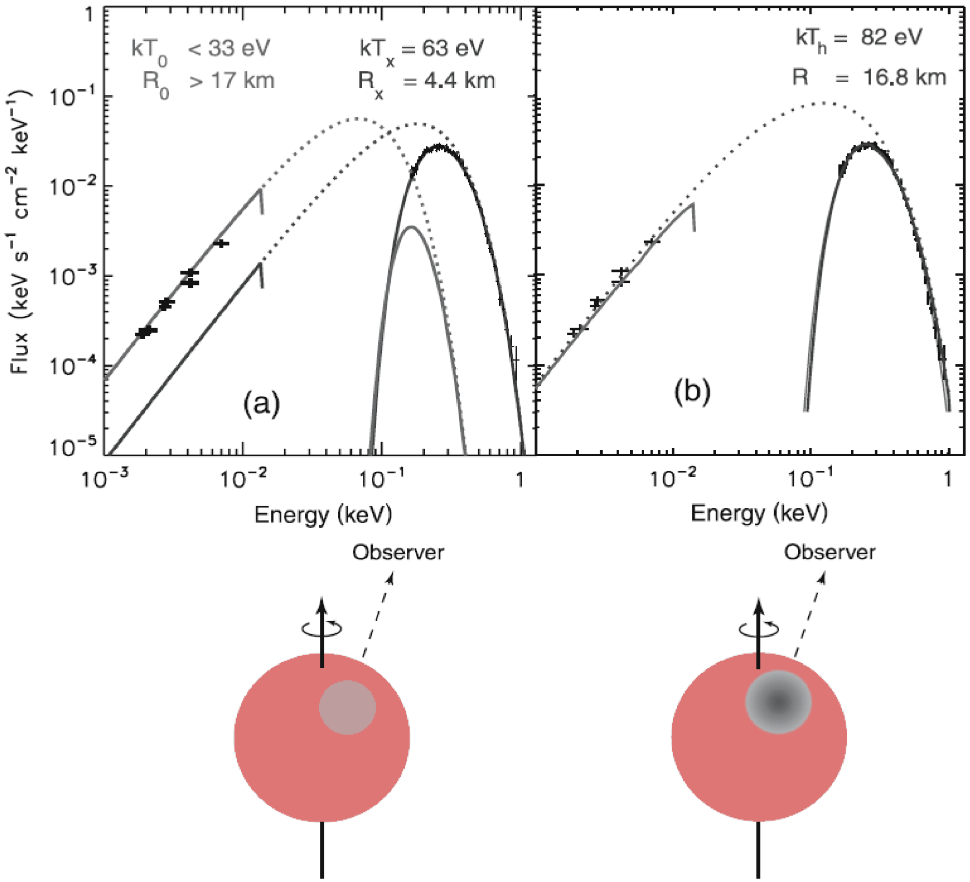
Accreting Magnetized Compact Objects







Dim Isolated Neutron Stars



Very close neutron stars (within ~500 pc from the Earth)

Do not show pulsations, do not show signs of accretion.

They only show thermal emission. Informally known as Magnificent Seven