

## Course Overview I

Lectures: 45 hours.

• April 27 to May 31: sessions of 2h, 11-13 (only May 12), 12:00-14:00 all the other lectures

Teachers:

- Alessandro Patruno (ICE-CSIC & IEEC) 27h patruno @ ice.csic.es
- Abelardo Moralejo (IFAE) 14h moralejo @ ifae.es
- Javier Rico (IFAE) 4h jrico @ ifae.es

#### Exam date: TBD

45% oral presentation on a research paper45% written exam (multiple answers and open questions)10% in class attendance and participation

## Course Overview II

#### High Energy Astrophysics: programme

#### PART 1 – Introduction. Radiative processes - AP

Physical processes associated to the production of photons. Radiative transfer theory. Particle acceleration.

#### PART 2 – Observation methods - AP & AM

X- and gamma-ray instrumentation from space and ground-based. Cosmic-ray detectors. Others

#### PART 3 – The high-energy sky - AP & AM

Accretion-powered sources: compact stars - AP Nova and supernova explosions and their remnants - AP

Pulsars and pulsar wind nebulae – AM

Gamma-ray emission related to nucleosynthesis and matter- antimatter annihilation - AP

Gamma-Ray Bursts – AM, AP

Cosmic rays: origin and propagation; possible acceleration sites - AM

Gamma rays as probes of the intergalactic medium (extragalactic background light, magnetic fields) – AM

### PART 4 – Multi-messenger astronomy, fundamental physics aspects - AM & JR

## Textbooks

§ G.B. Rybicki, A.P. Lightman, "Radiative Processes in Astrophysics" (Wiley, 2004)

§ M. S. Longair "High Energy Astrophysics (Vol.1), Cambridge University Press (2 nd ed. 1992; 3 rd Edition 2011; also available as ebook)

§ J. Frank, A. King, D. Raine, "Accretion power in Astrophysics", Cambridge Univ. Press (3 rd edition, 2002)

§ F. H. Aharonian, "Very high energy cosmic gamma radiation: a crucial window on the extreme Universe", World Scientific (2004)

§ P.A. Charles, F.D. Seward, "Exploring the X-ray Universe", CUP (1995)

§ G.F.Knoll, "Radiation Detection and Measurement", Wiley (3 rd edition, 2000)

§ Several papers provided by teachers

### **Alessandro Patruno**

Lecturer patruno@ice.csic.es

High Energy Astrophysics, Black Holes, Neutron Stars

Slides will be uploaded on the "Campus Virtual". I usually also put the slides on my personal webpage:

https://blacksidus.com/teaching/hea2023

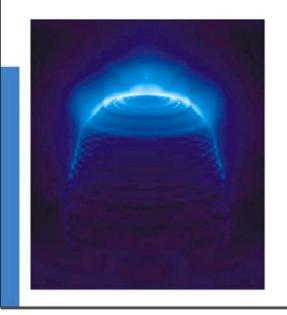
## Part 1: Radiative processes

#### PHYSICS TEXTBOOK

George B. Rybicki Alan P. Lightman

WILEY-VCH

# Radiative Processes in Astrophysics



**Textbook:** "Radiative Processes in Astrophysics" (Rybicki & Lightman)

Other books and interesting material:

- G., Ghisellini "Radiative Processes in High Energy Astrophysics" (2nd edition) (Free book) (https://arxiv.org/abs/1202.5949)

- M. S. Longair "*High Energy Astrophysics* (Vol.1), Cambridge University Press (2<sup>nd</sup> ed. 1992; 3<sup>rd</sup> Edition 2011; also available as ebook)

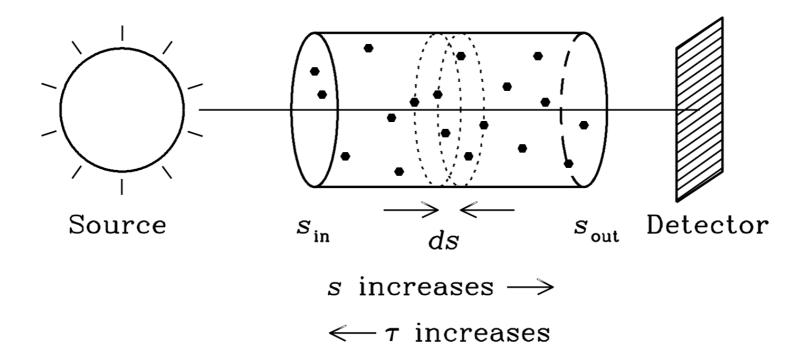
## **Extra Material**

- I will upload notes on lectures, clarifications and exercises/tests on my personal webpage.
- There are also some *tutorials* that you find on the page

www.blacksidus.com/teaching/hea2023

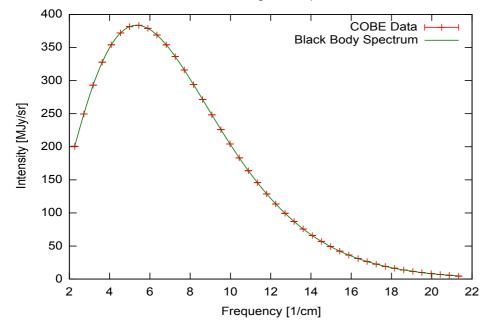
These <u>are not</u> on Campus Virtual because they are interactive webpages.

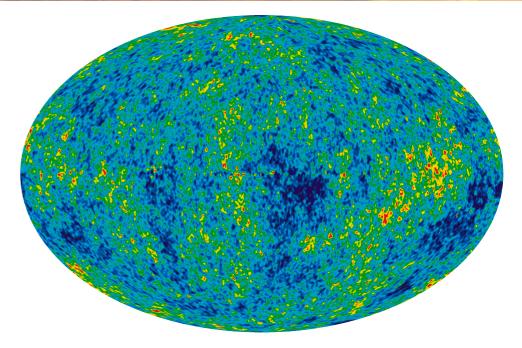
## **Radiative Transfer**

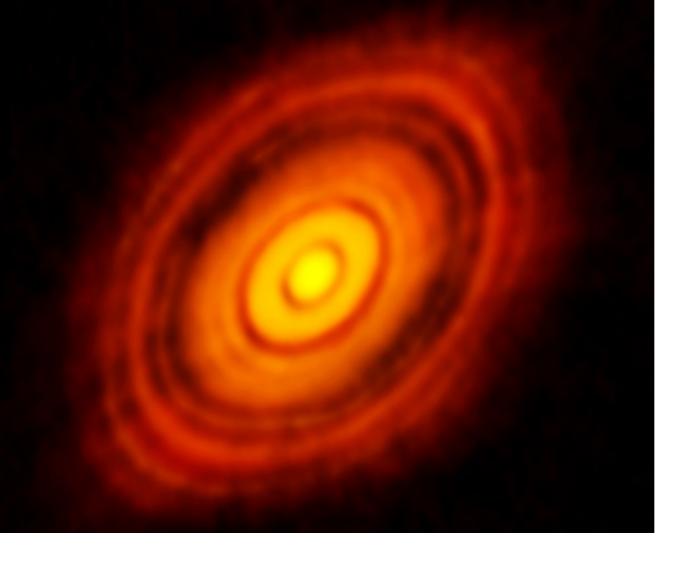


## Thermal and Blackbody Radiation

Cosmic Microwave Background Spectrum from COBE







HL Tauri is a T-Tauri star (pre-main sequence stars) with a huge protoplanetary disk 25 light-hours across.

This image (in the microwave band) shows with unprecedented detail the protoplanetary disk. Gaps (black bands) are seen throughout the disk, which might indicate the presence of planets.

The age of the system is <1Myr. The radiation you see is thermal (specifically blackbody).

Resolution: 35 micro-arcsec (a coin at ~100 km distance)

ALMA is an array of microwave telescopes in Chile. It has an unprecedented sensitivity in this area of the electromagnetic spectrum.

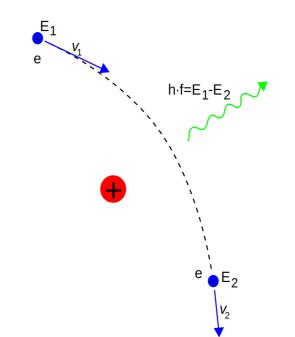
#### **Coma Cluster**

X-Rays

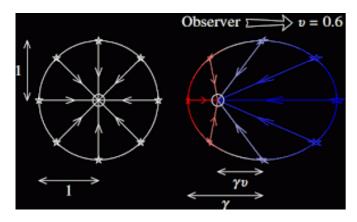
Coma Cluster
Optical

#### Hot intracluster gas

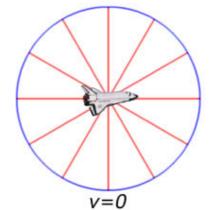
### Bremsstrahlung ("braking radiation")

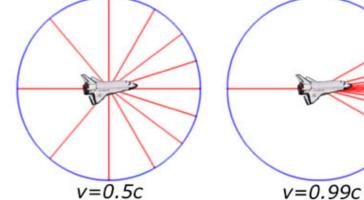


### **Relativistic Phenomena in Radiative Processes**



Relativistic Doppler shift

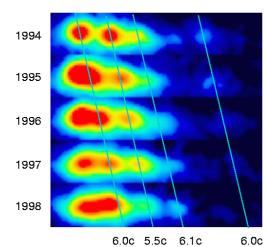




Aberration of light

Superluminal Motion in the M87 Jet





"Superluminal" Motion of Jets

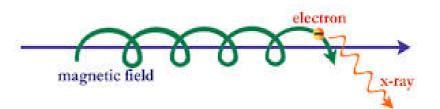


### Crab Nebula

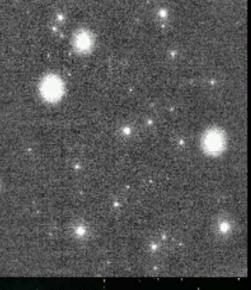
### **Crab Pulsar**

#### Black Hole Jets

### Synchrotron





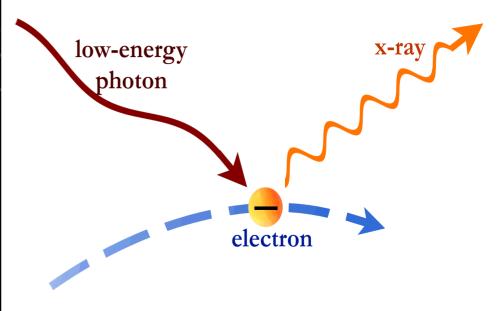


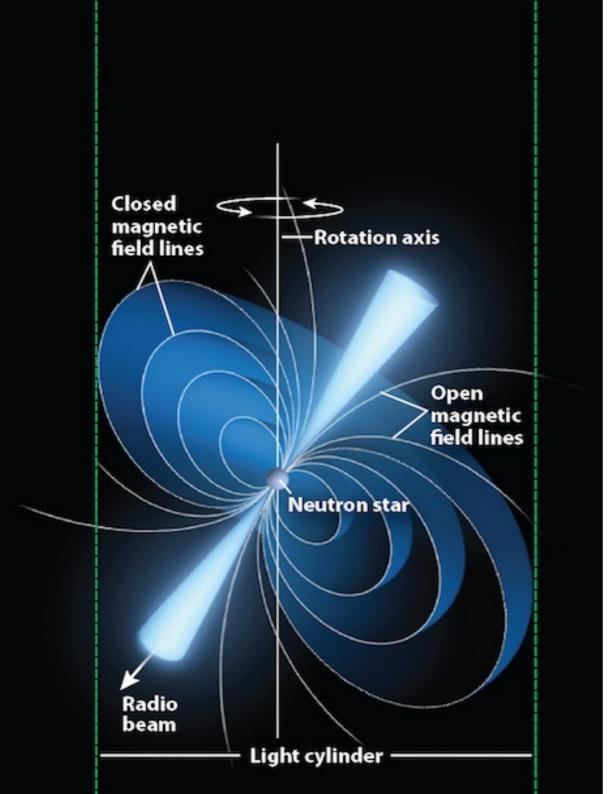




### Gamma Ray Burst

### **Compton Scattering**



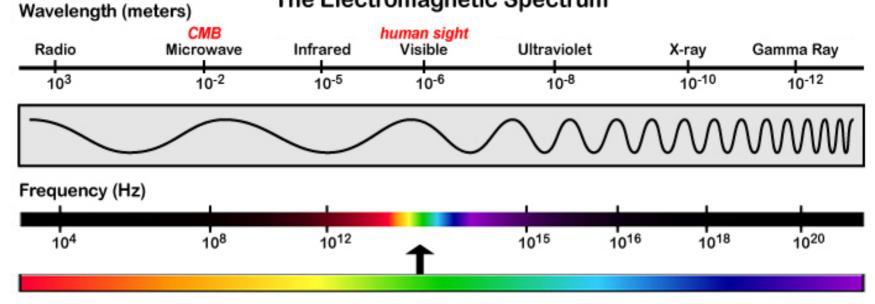


## Curvature Radiation



## **Čerenkov Radiation**

#### The Electromagnetic Spectrum



$$\lambda = \frac{c}{v}$$
$$E = hv = \hbar \omega$$
$$1 \, eV \rightarrow 11,000 \, K \left( E = k_B T \right)$$

 Radio
 > 1 mm

 IR
 700nm - 1mm 

 Optical
 400 - 700nm 

 UV
 100 - 400nm 

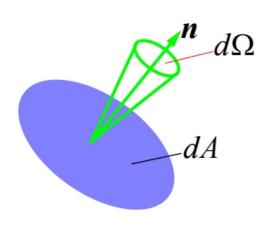
 X-Rays
 0.1 - 100 keV

Gamma > 1 MeV

# Specific Intensity (Brightness)

The flux is a measure of the energy carried by **all rays** passing through a given area.

What if we want to follow each ray instead? Note: in transfer theory (which is the one we're using here) a single ray carries no energy, so we need to consider an infinitesimal amount of energy carried by a set of rays which differ infinitesimally from the given ray.

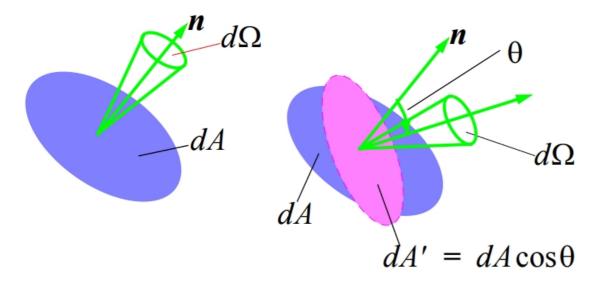


Specific Intensity: energy passing through an area dA normal to the direction of the given ray and consider all rays passing through dA whose direction is within an infinitesimal solid angle of the given ray  $dE = I_v dA d\Omega dt dv$ 

## Momentum Flux

Suppose now that we have a radiation field (rays in all directions) and construct a small element of area dA at some arbitrary orientation **n**.

$$dA' = dA \cos \theta$$
$$dF_{\nu} = I_{\nu} \cos \theta \, d\Omega \rightarrow \frac{dF_{\nu}}{c} = momentum \, flux$$



The net flux in the direction **n**, is obtained by integrating dF over all solid angles

$$F_{\nu} = \int_{\Omega} I_{\nu} \cos \theta \, d \, \Omega$$

## Momentum Flux

Remember now that a photon has momentum E/c

Therefore the magnitude of **momentum** passing through the surface in the same direction is

$$d \mathbf{p}_{\mathbf{v}} = \frac{dE_{\mathbf{v}}}{c} = \frac{I_{\mathbf{v}}}{c} \cos \theta \, d \, \mathbf{v} \, dA \, dt \, d \, \Omega$$

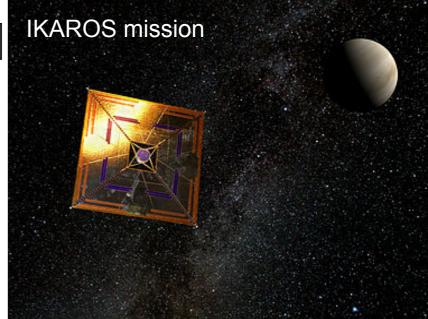
However, **momentum is a vector quantity**. The component of momentum flux normal to the surface **dA** is:

$$dp_{\nu}\cos\theta = \frac{I_{\nu}}{c}\cos^{2}\theta \,d\,\nu\,dA\,dt\,d\,\Omega$$

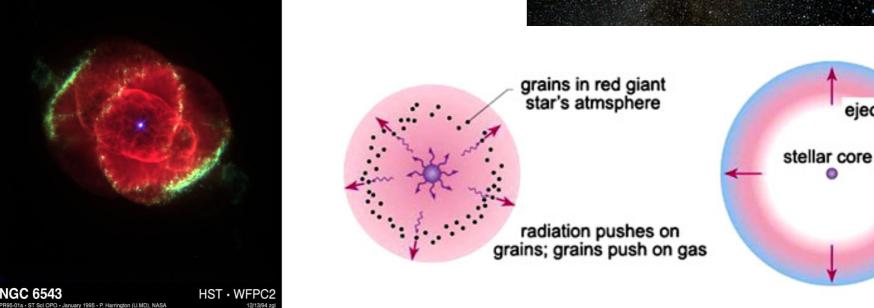
## **Radiation Pressure**

$$p_{\nu} = \frac{1}{c} \int I_{\nu} \cos^2 \theta \, d \, \Omega \, \boldsymbol{\rightarrow} \left[ \, dynes \, cm^{-2} \, Hz^{-1} \right]$$

### **RADIATION PRESSURE (dyne/cm2)** $p = \int p_v dv$



ejected gas



# Radiative Energy Density

The specific energy density  $u_v$  is defined as the energy per unit volume per unit frequency range. To determine this it is convenient to consider first the energy density per unit solid angle  $u_v(\Omega)$  via:

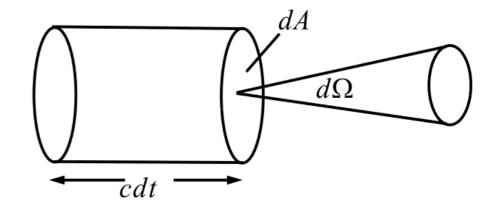
$$dE = u_{\nu}(\Omega) dV d\Omega d\nu = u_{\nu}(\Omega) dA c dt d\Omega d\nu$$

Remember what we said before:

$$dE = I_{\nu} dA d \Omega dt d \nu$$

Equating the two:

$$u_{v}(\Omega) = \frac{I_{v}}{c}$$



## **Radiative Energy Density**

Integrate now over the solid angle:

$$u_{\nu}(\Omega) = \frac{I_{\nu}}{c} \rightarrow u_{\nu} = \int u_{\nu}(\Omega) d\Omega = \frac{1}{c} \int I_{\nu} d\Omega = \frac{4\pi}{c} J_{\nu}$$

where we have defined the mean intensity as:

$$J_{\nu} = \frac{1}{4\pi} \int I_{\nu} d\Omega$$

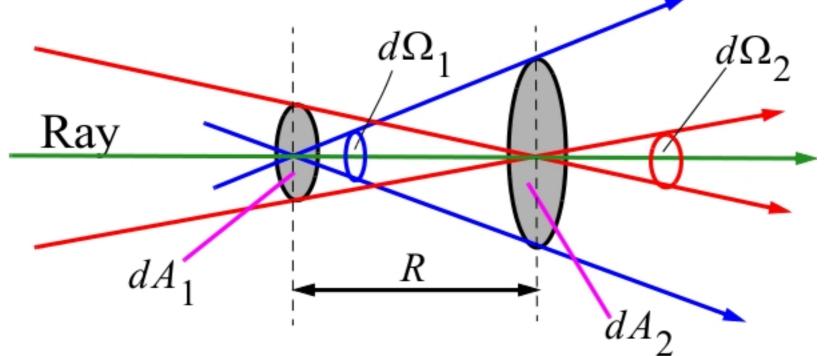
## **Radiative Energy Density**

The total *energy density* **u** is therefore:

$$u = \int u_{\nu} d\nu = \frac{4\pi}{c} \int J_{\nu} d\nu$$

### Constancy of Brightness in Free Space

Consider any ray and any two points along the ray



Consider the energy flux through an elementary surface dA1 within solid angle  $d \Omega_1$ Consider all of the photons which pass through dA1 in this direction which then pass through dA2. We take the solid angle of these photons to be  $d \Omega_2$ 

### **Constancy of Brightness in Free Space**

We use now energy conservation:

$$dE = I_{v_1} dA_1 d\Omega_1 dt dv_1 = I_{v_2} dA_2 d\Omega_2 dt dv_2$$

Remember the definition of solid angle:

$$d\Omega_1 = dA_2/R^2 \qquad d\Omega_2 = dA_1/R^2$$

Since the frequency of the ray is the same (  $dv_2 = dv_1$  ) we have:

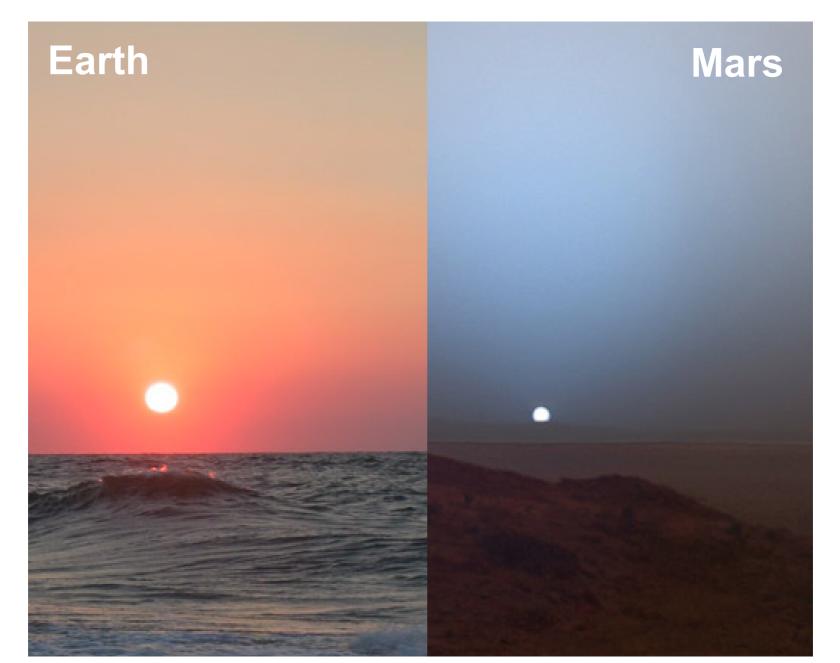
$$I_{v1} = I_{v2} = constant$$

### **Brightness does NOT depend on distance**

## Do we measure Flux or Brightness?



## Do we measure Flux or Brightness?



## Do we measure Flux or Brightness?

Answer: it depends on the object!!

If the telescope cannot resolve the object then we measure flux. If the telescope can resolve the object, then we measure intensity.

Why? Consider the case when the source is unresolved. Now imagine pushing the source to farther distance. As the distance increases, the number of photons falls as r<sup>2</sup>. The flux is measured.

If instead the the source is resolved, then as the source is pushed farther away, more area of the source would be included with the solid angle, which compensates for the the increased distance and the collected number of photons remain the same.

# What is radiative transfer?

- Radiative transfer is the physical phenomenon of energy transfer in the form of electromagnetic radiation
- The propagation of radiation through a medium is affected by absorption, emission and scattering processes.
- The equation of radiative transfer describes these interactions mathematically.
- Applications apart from astrophysics include optics, atmospheric science, and remote sensing.
- Analytical solutions exist in a few (simple) cases, but more realistic cases need a numerical treatment.

## Why do we study transfer theory?

The light we detect arrives at us in two steps:

- first, it is created by some radiative process (e.g., blackbody, synchrotron, etc etc...)
- then it propagates through space where it might be (partially) scattered and absorbed

Scattering, absorption and emission are thus three fundamental steps to generate the light we see.

Transfer theory tells us how the specific intensity of an object is affected by absorption, scattering and emission.



### **Constancy of Brightness in Free Space**

We use now energy conservation:

$$dE = I_{v_1} dA_1 d\Omega_1 dt dv_1 = I_{v_2} dA_2 d\Omega_2 dt dv_2$$

Remember the definition of solid angle:

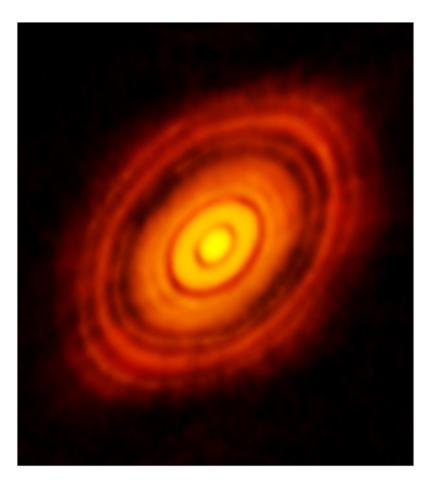
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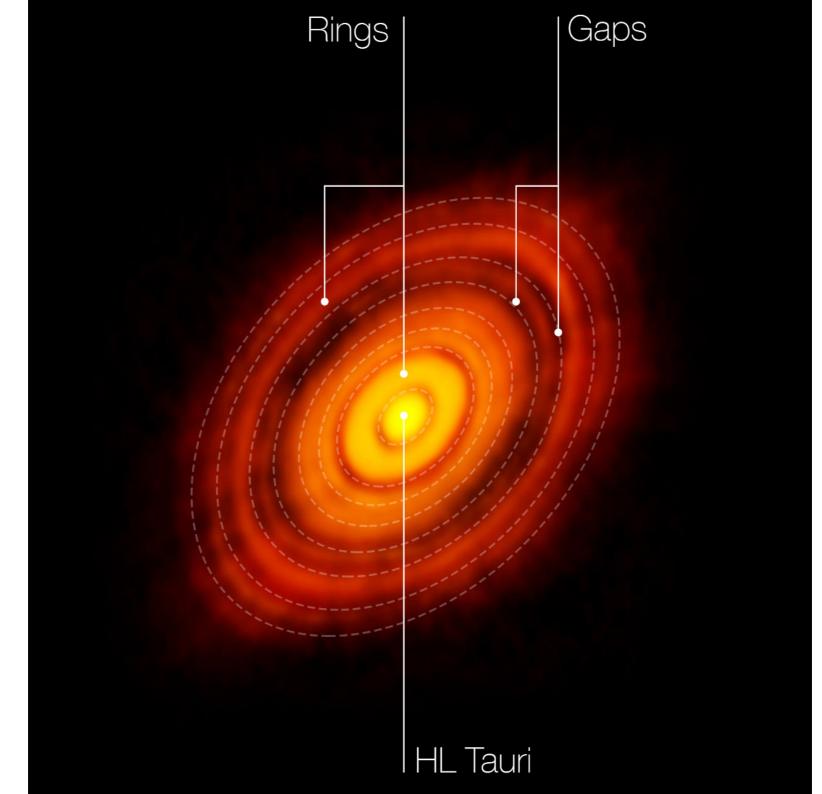
$$I_{v1} = I_{v2} = constant$$

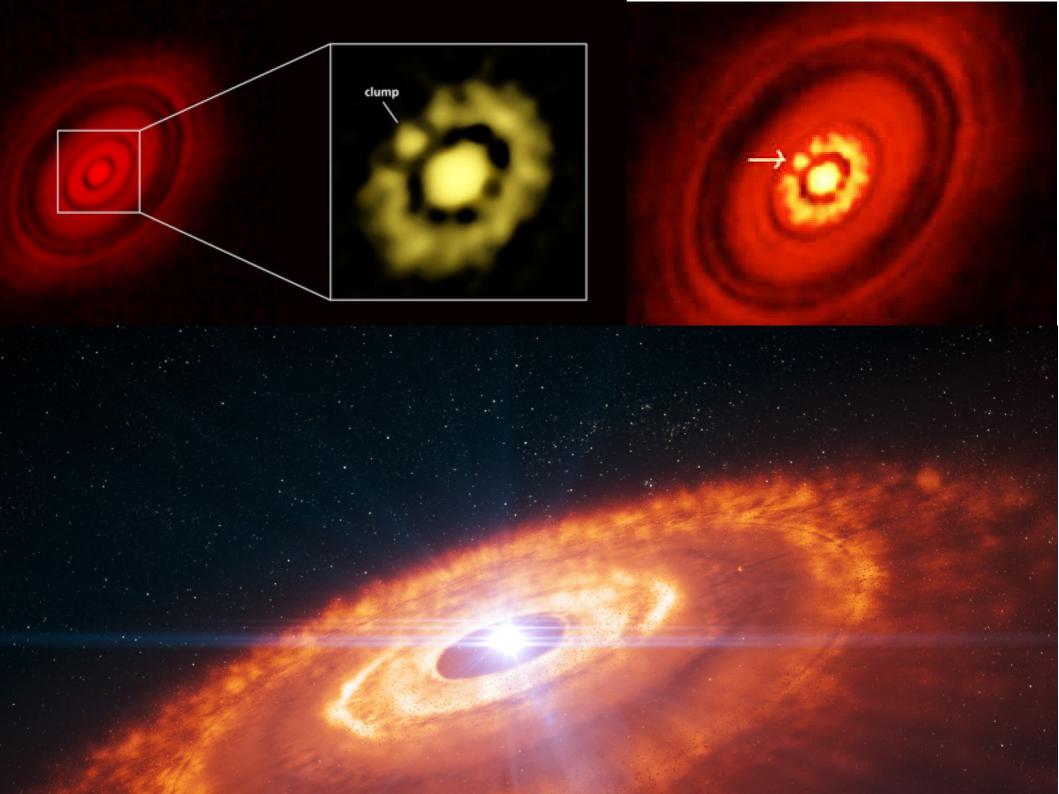
### **Brightness does NOT depend on distance**

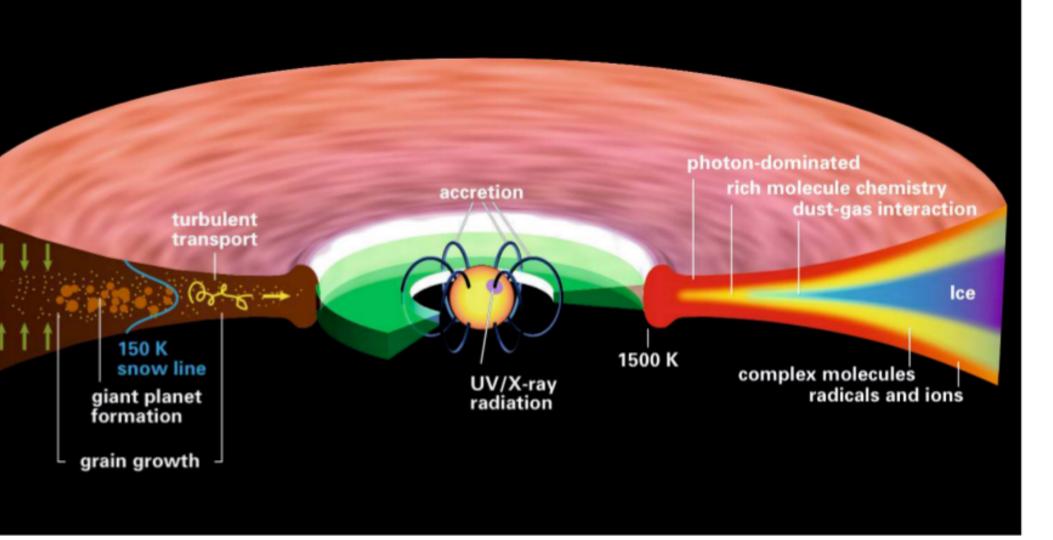
## HL Tau



- ALMA radio image of protoplanetary disk around young star
- Rings show up because dust is cleared out by protoplanets
- Resolution 35 microarcsec (penny at 110 km distance)

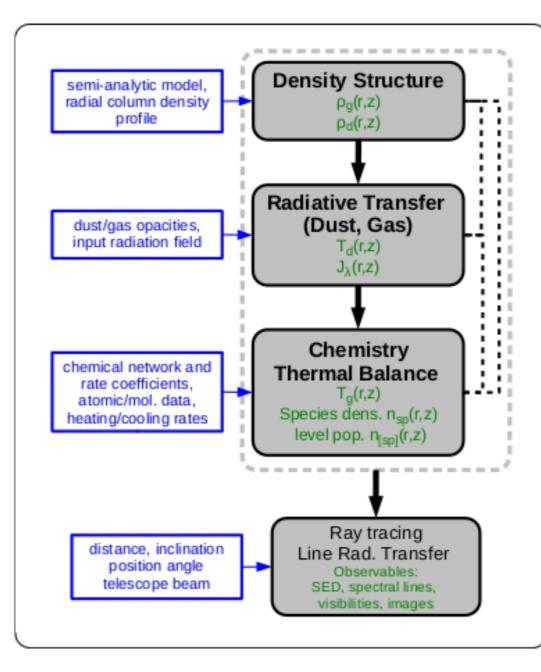






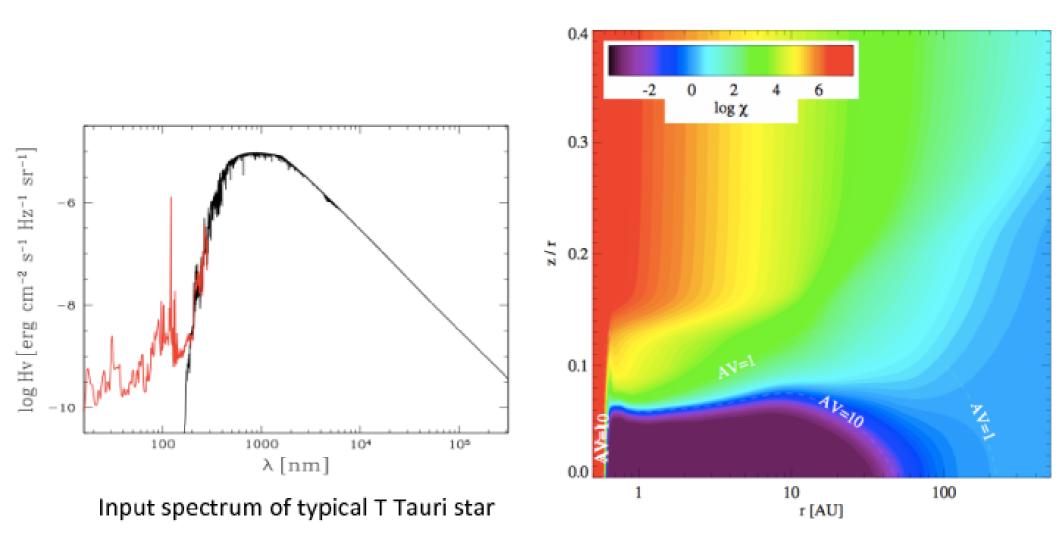
Credit: Henning and Semenov (2013)

#### ProDiMo - Code overview



Vertical Structure
$\frac{1}{\rho} \frac{\mathrm{d}p}{\mathrm{d}z} = -\frac{z \mathrm{GM}_*}{\left(r^2 + z^2\right)^{3/2}}$
Radiative Transfer
$\frac{\mathrm{d}I_{\lambda}}{\mathrm{d}\tau_{\lambda}} = I_{\lambda} - S_{\lambda}$
Chemistry & Thermal Balance
$\frac{\mathrm{d} n_i}{\mathrm{d} t}  =  \sum_{jkl} R_{jk \to kl} (T_{\mathrm{g}) n_j n_k}$
$-n_i \sum_{jkl} R_{il \to jk} n_l \dots$
$\frac{\mathrm{d}e}{\mathrm{d}t} = \sum_{k} \Gamma(T_{\mathrm{g}}, n_{\mathrm{sp}}) - \sum_{k} \Lambda(T_{\mathrm{g}}, n_{\mathrm{sp}})$

## Radiative transfer in the disk



Radiation field throughout the disk

## Why do we need radiative transfer?

- Crucial to determine radiation contribution throughout the object of interest
- It is key in determining the heating and cooling processes, and as a result the density, thermal and chemical structure
- Finally, dust and line radiative transfer will provide dust and emission characteristics to be observed with telescopes.

## Radiative transfer

- If a ray passes through a medium, energy can be added or subtracted by emission and absorption
- Therefore: Specific intensity will usually not remain constant when passing through the interstellar medium.

## Emission (1)

- The spontaneous emission coefficient *j* is defined as the energy emitted per unit time per unit solid angle per unit volume: *dE* = *j d*V *d*Ω *dt*
- Or when the emission is monochromatic:  $dE = j_v dV d\Omega dt dv$
- If the emission is isotropic, we can write:  $j_v = 1/(4\pi) P_v$ , with the power per unit volume per unit frequency.

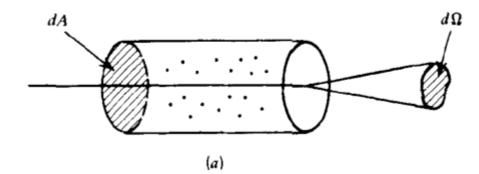
#### Emission (2)

- In going a distance ds, a beam of cross section dA travels through a volume dV = dA ds
- The intensity added to the beam is then:  $dI_v = j_v ds$
- Or compare the specific intensity and the emission coefficient:

*j*<sub>v</sub> [erg cm<sup>-3</sup> s<sup>-1</sup> ster<sup>-1</sup> Hz<sup>-1</sup>] to *I*<sub>v</sub> [erg cm<sup>-2</sup> s<sup>-1</sup> ster<sup>-1</sup> Hz<sup>-1</sup>]

#### Absorption

- The absorption coefficient α [cm<sup>-1</sup>] is defined by the following equation: dI<sub>v</sub> = -α<sub>v</sub> I<sub>v</sub> ds, representing the loss of intensity in a beam as it travels a distance ds.
- This  $\alpha$  can be defined by:  $\alpha_v = n\sigma_v$  or  $\alpha_v = \rho\kappa_v$



#### **Radiation transport**

- The decrement of I<sub>ν</sub> when passing through a path of length *ds*:
   dI<sub>ν</sub> = -α<sub>ν</sub> I<sub>ν</sub> ds
- Inside a source, a contribution to  $I_v$  can be made from emitters. The increment is:  $dI_v = j_v ds$
- The basic equation of transport is:

$$\frac{dI_{\nu}}{ds} = -\alpha_{\nu}I_{\nu} + j_{\nu}$$

#### Simple solutions (1)

• Emission only:

$$\frac{dI_{\nu}}{ds} = j_{\nu} \rightarrow I_{\nu} = I_{\nu,0} + \int_0^S j_{\nu} ds$$

with S the total emission path.

 The increase in the specific intensity is equal to the emission coefficient integrated along the line of sight

#### Simple solutions (2)

• Absorption only:

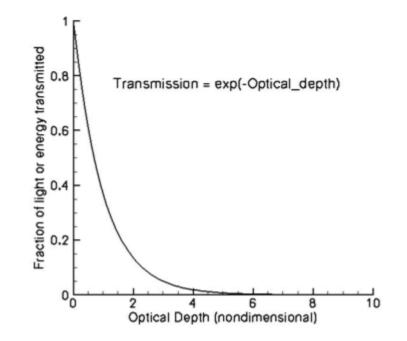
$$\frac{dI_{\nu}}{ds} = -\alpha_{\nu}I_{\nu} \to \frac{dI_{\nu}}{I_{\nu}} = -\alpha_{\nu}ds$$
$$I_{\nu}(s) = I_{\nu}(s_0) e^{-\int_{s_0}^s \alpha_{\nu}(s')ds'}$$

 The brightness decreases along the ray by the exponential of the absorption coefficient integrated along the line of sight

#### Optical depth (1)

• We now introduce the quantity optical depth:

$$d\tau_{\nu} = \alpha_{\nu} ds = n\sigma_{\nu} ds$$



#### Optical depth (2)

- The intensity decreases as follows:
   I<sub>ν</sub> = I<sub>ν,0</sub> exp(-τ<sub>ν</sub>)
- What follows from this:
  - − τ=1 → 1/e (37%)
  - $-\tau$ >>1  $\rightarrow$  optically thick
  - $\tau << 1 \rightarrow$  optically thin

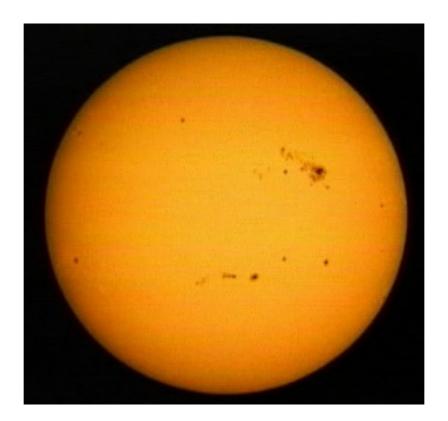
Optical depth is connected with more familiar concepts like reflectivity, for example a very good mirror usually reflects 90-95% of light. This means that some fraction (5-10%) of radiation is "absorbed" (it is actually transmitted).

You can see this effect very clearly when you have multiple reflections in a mirror. The brightness of the reflected object goes down very quickly.

You can check this at home, but be careful: the effect you see with your eye is not the same that you record on camera. Why?



#### The "surface" of the Sun



The "surface" of the Sun is usually defined as the location in the photosphere where the optical depth is equal to 2/3.

This is just a definition, and it is used because about 50% of photons we see are coming from this region.

# Transport equation and source function

The rewritten equation of transport becomes

$$\frac{dI_{\nu}}{\alpha_{\nu}ds} = -I_{\nu} + \frac{j_{\nu}}{\alpha_{\nu}} \to \frac{dI_{\nu}}{d\tau_{\nu}} = -I_{\nu} + \frac{j_{\nu}}{\alpha_{\nu}}$$

• We define  $S_{\nu}$  the source function as follows

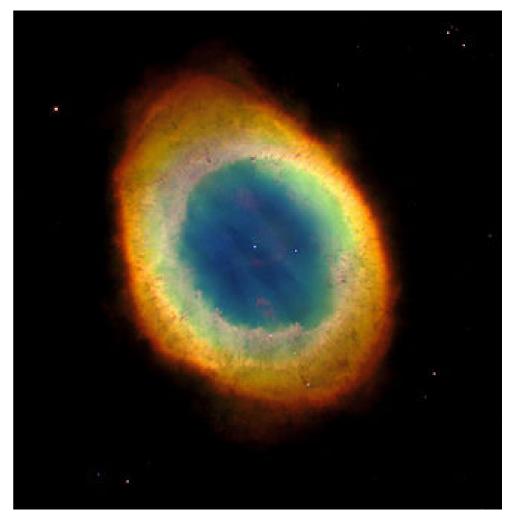
$$S_{\nu} = \frac{j_{\nu}}{\alpha_{\nu}}$$

(The source function can be thought in a way as the local input of radiation. For example when optical depth >>1 then source function ~ specific brightness which means that little is contributed to the to the specific brightness from matter far from the location of interest).

#### Light comes from a white dwarf illuminating the surrounding material



Helix Planetary Nebula (remnant of a low mass star) Distance: 220 pc Size: about 0.8 pc across



Ring Planetary Nebula (remnant of a low mass star) Distance: 700 pc Size: about 0.4 pc across

#### Equation of transfer

• This yields the format solution of the EOT:

$$I_{\nu}(\tau_{\nu}) = I_{\nu,0} e^{-\tau_{\nu}} + \int_{0}^{\tau_{\nu}} e^{-(\tau_{\nu} - \tau_{\nu}')} S_{\nu}(\tau_{\nu}') d\tau_{\nu}'$$

• When  $S_v$  constant:  $I_v(\tau_v) = I_{v,0} \exp(-\tau_v) + S_v(1-\exp(-\tau_v))$ 

• 
$$\tau >>1: I_{\nu} \rightarrow S_{\nu}$$

•  $\tau <<1: I_{\nu} \rightarrow I_{\nu,0} + S_{\nu}\tau_{\nu}$ 

#### A special case

• When  $S_{\nu}$  is constant throughout the source, this can be rewritten as:

$$I_{\nu}(\tau_{\nu}) = I_{\nu,0} e^{-\tau_{\nu}} + S_{\nu} \left(1 - e^{-\tau_{\nu}}\right)$$

Question: What is the intensity of this source for small and large optical depth when it has size R?

#### Answer

• If 
$$I_{\nu,0} = 0$$
 then  $I_{\nu}(\tau_{\nu}) = \frac{\jmath_{\nu}}{\alpha_{\nu}} \left(1 - e^{-\tau_{\nu}}\right)$ 

• A little trick. First, we multiply by the source size s=R:

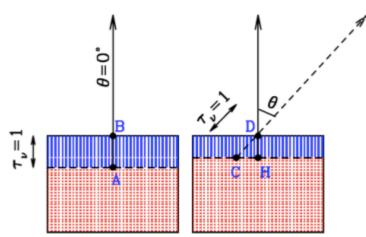
$$I_{\nu}(\tau_{\nu}) = \frac{j_{\nu}R}{\alpha_{\nu}R} \left(1 - e^{-\tau_{\nu}}\right) = j_{\nu}R\left(\frac{1 - e^{-\tau_{\nu}}}{\tau_{\nu}}\right)$$

• Optically thin ( $\tau \ll 1$ ):  $1 - \exp(-\tau) = 1 - 1 + \tau = \tau$  $\rightarrow I_{\nu}(\tau_{\nu}) = j_{\nu}R$ 

• Optically thick (
$$au >> 1$$
):  $I_{\nu}(\tau_{\nu}) = \frac{J_{\nu}r_{\nu}}{\tau_{\nu}}$ 

#### The $cos(\theta)$ law

- We often hear the expression that radiation from an optically thick source comes from its surface
- We do mean that the emission we see is emitted from a layer with τ = 1.
- The emitting volume the observer sees depends on inclination.



#### Mean free path (1)

- The mean free path is the average distance traveled by a photon before being absorbed.
- The probability of a photon to travel at least an optical depth τ<sub>ν</sub> is exp(-τ<sub>ν</sub>). The mean optical depth is thus unity:

$$\langle \tau_{\nu} \rangle \equiv \int_{0}^{\infty} \tau_{\nu} e^{-\tau_{\nu}} d\tau_{\nu} = 1$$

 In a homogeneous medium, the average distance traveled is defined as I<sub>v</sub>:

$$\langle \tau_{\nu} \rangle = \alpha_{\nu} l_{\nu} = 1$$
 or  $l_{\nu} = \frac{1}{\alpha_{\nu}} = \frac{1}{n\sigma_{\nu}}$ 

#### Mean free path (2)

 A source with radius R and total optical depth τ > 1 has a mean free path:

$$\ell_{\nu} = \frac{1}{\sigma_{\nu}n} = \frac{R}{\sigma_{\nu}nR} = \frac{R}{\tau_{\nu}}$$

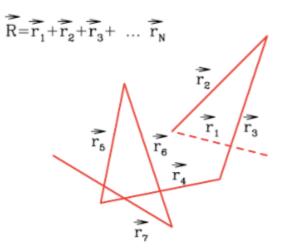
#### Scattering effects: random walks

- Assume a photon that interacts through scattering inside a source R and optical depth τ > 1
- How many times does it scatter before escaping?
- How much time does it take?

#### Random walks

- The total net displacement after N scatterings is: R = r<sub>1</sub> + r<sub>2</sub> + r<sub>3</sub> + ... + r<sub>N</sub>
- If we want the distance |R| traveled by a typical photon we need to calculate the square displacement:

$$\begin{aligned} \langle \vec{R}^2 \rangle &= \langle \vec{r}_1^2 \rangle + \langle \vec{r}_2^2 \rangle + \langle \vec{r}_3^2 \rangle \\ &+ 2 \langle \vec{r}_1 \cdot \vec{r}_2 \rangle + 2 \langle \vec{r}_1 \cdot \vec{r}_3 \rangle + \dots \end{aligned}$$



#### Random walks

 The cross products vanish for isotropic scattering

$$\langle \vec{R}^2 \rangle \, = \, N \langle \vec{r}_i^2 \rangle \, = \, N \ell^2 \, \rightarrow \, \sqrt{\langle \vec{R}^2 \rangle} = \sqrt{N} \ell$$

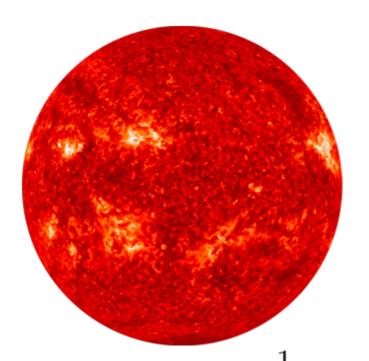
• Question 1:

$$\sqrt{N} = \frac{R}{\ell} = R\sigma n = \tau \to N = \tau^2$$

• Question 2:

$$t_{\text{tot}} = Nt_1 = \tau^2 \frac{\ell}{c} = \tau^2 \frac{R}{R\sigma nc} = \frac{R}{c} \frac{\tau^2}{\tau} = \tau \frac{R}{c}$$

#### How long before light escapes the Sun?



Sun's mean density: 1.4 g/cm3

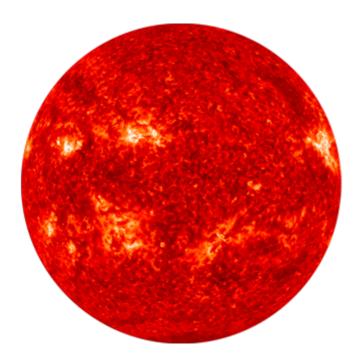
Assume Thomson scattering:  $\sigma_{\rm T} = 6.7 \times 10^{-25} \text{ cm}^2$  $R \odot = 7 \times 10^{10} \text{ cm}$  $m_{\rm H} = 1.66 \times 10^{-24} \text{ g}$ 

Assume Sun is made by pure hydrogen Then:  $n=\rho/m_{\rm H}$  =  $8.4\times10^{23}~{\rm cm}^{-3}$ 

Mean free path:  $\ell = \frac{1}{n\sigma_{\rm T}} \sim 1 \, \text{cm.}$  The time required to escape the Sun will be the time to travel one mean free path times N scatter events. Since each mean free path is done at speed *c*, we have:

$$t = N \frac{\ell}{c}.$$

#### How long before light escapes the Sun?



The Sun is clearly optically thick, so  $N\approx \tau^2$  Therefore:  $\tau=R_\odot/\ell\,$  and by squaring it we get:

$$t = \left(\frac{R_{\odot}}{\ell}\right)^2 \frac{\ell}{c}$$

By plugging the numbers in, we obtain a number of the order of a few thousand years.

#### Why The Sky is Blue and the Sun is Red at Sunset



We know that the Rayleigh scattering has a strong dependence on frequency.

Indeed the cross-section of the scattered light scales in the following way:

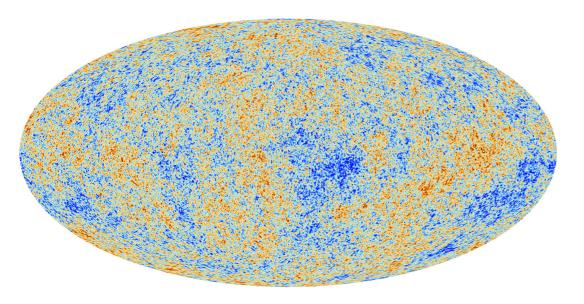
 $\sigma_{v} \propto v^{4}$ 

The scattering therefore is very large for blue light which has a much larger random walk in the sky. Indeed the mean free path for blue light:

$$l_{v} = \frac{1}{\sigma_{v} n}$$

is very small and the number of scatterings N is very large and so is the optical depth. Red light instead has a much larger mean free path, N is small and the region of the sky affected by scattering is smaller.

#### **Example: relation between flux and brightness**



The Cosmic Microwave Background is an isotropic radiation field that permeates the whole universe.

The specific intensity (or specific brightness) at 100 GHz is 10^-15 erg/cm2/s/sr/Hz

What is the specific flux observed by the Planck Satellite at that frequency?

#### Hint:

Flux has units of erg/cm2/s Specific Flux has units of erg/cm2/s/Hz