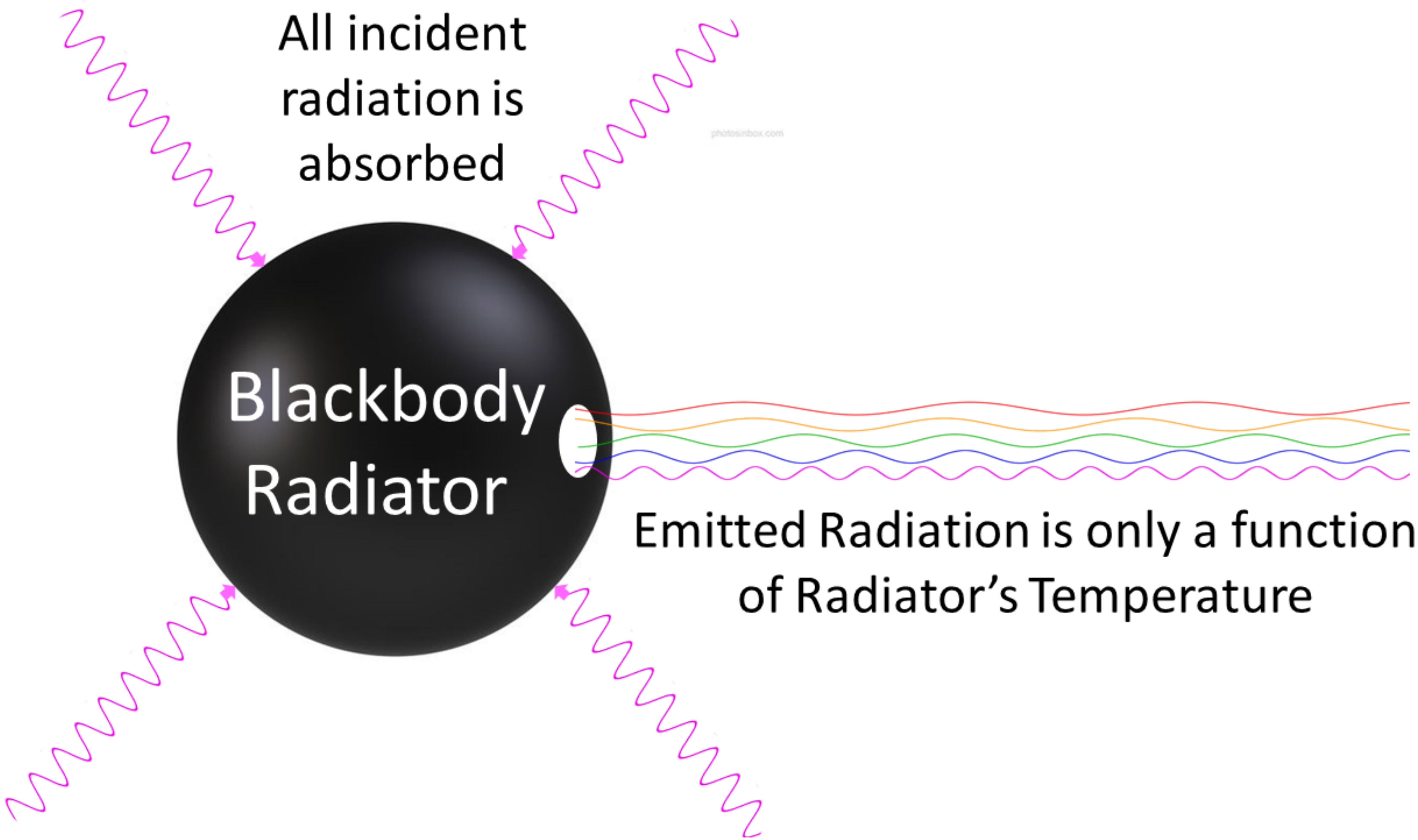
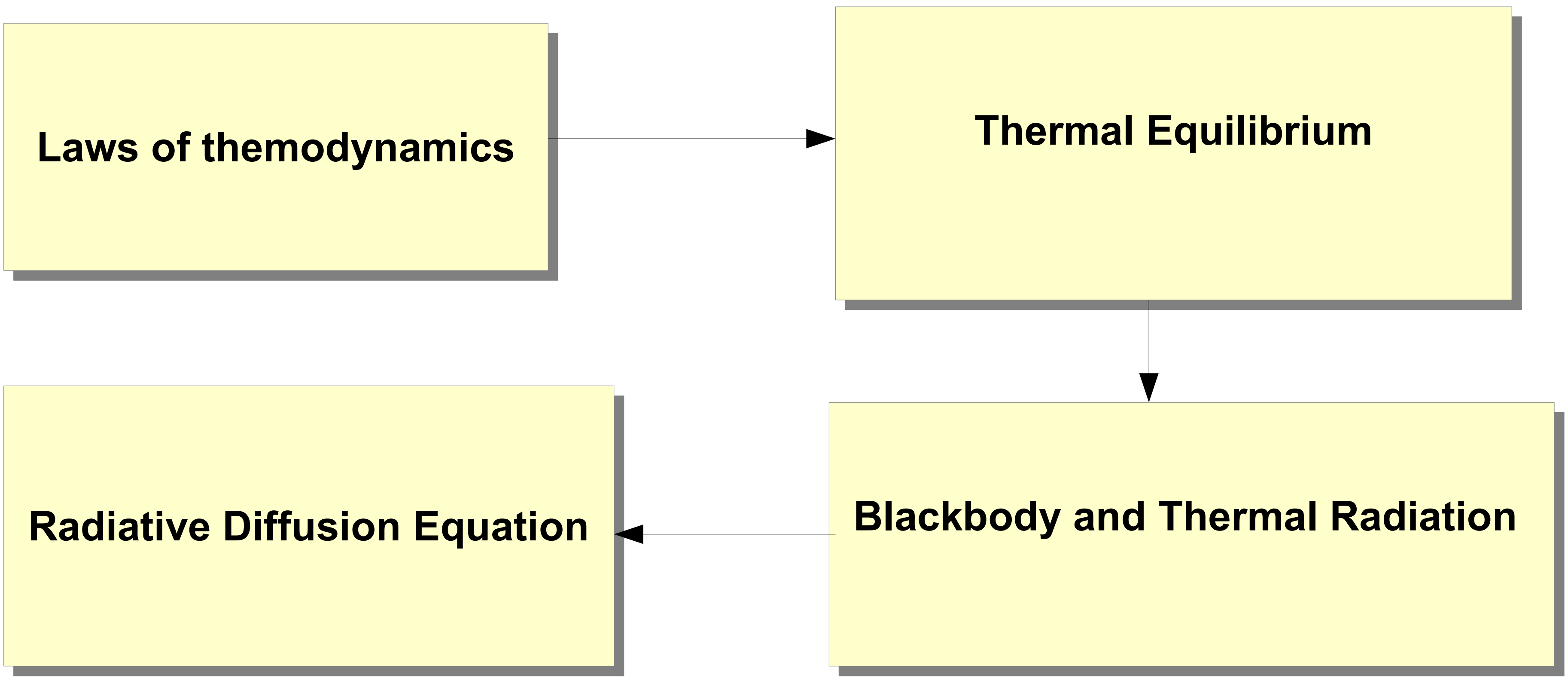
Blackbody Radiation



Outline

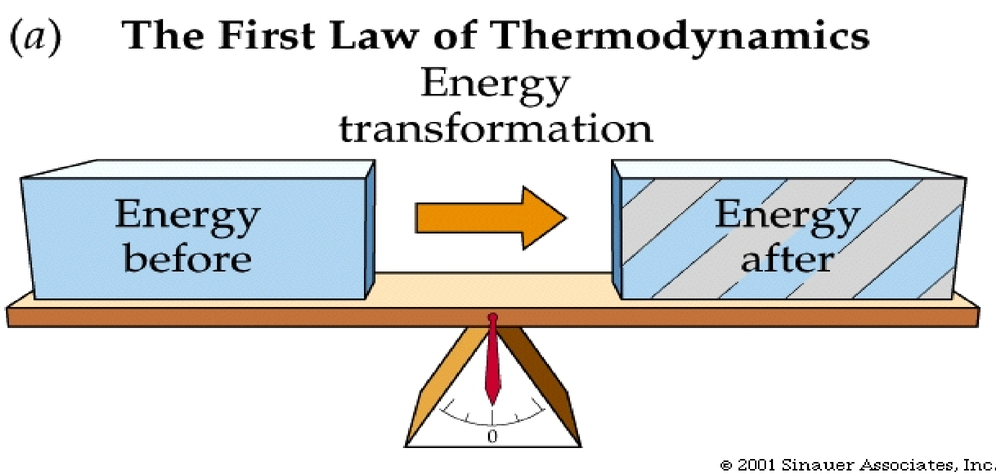
Today we will learn what is **thermal radiation**



Useful to describe radiation emitted by stars, accretion disks, nebulae, stellar atmospheres, etc...

Laws of Thermodynamics

First Law: Energy can be changed from one form to another, but it cannot be created or destroyed. (Conservation of Energy)



$$\Delta U = Q - W$$

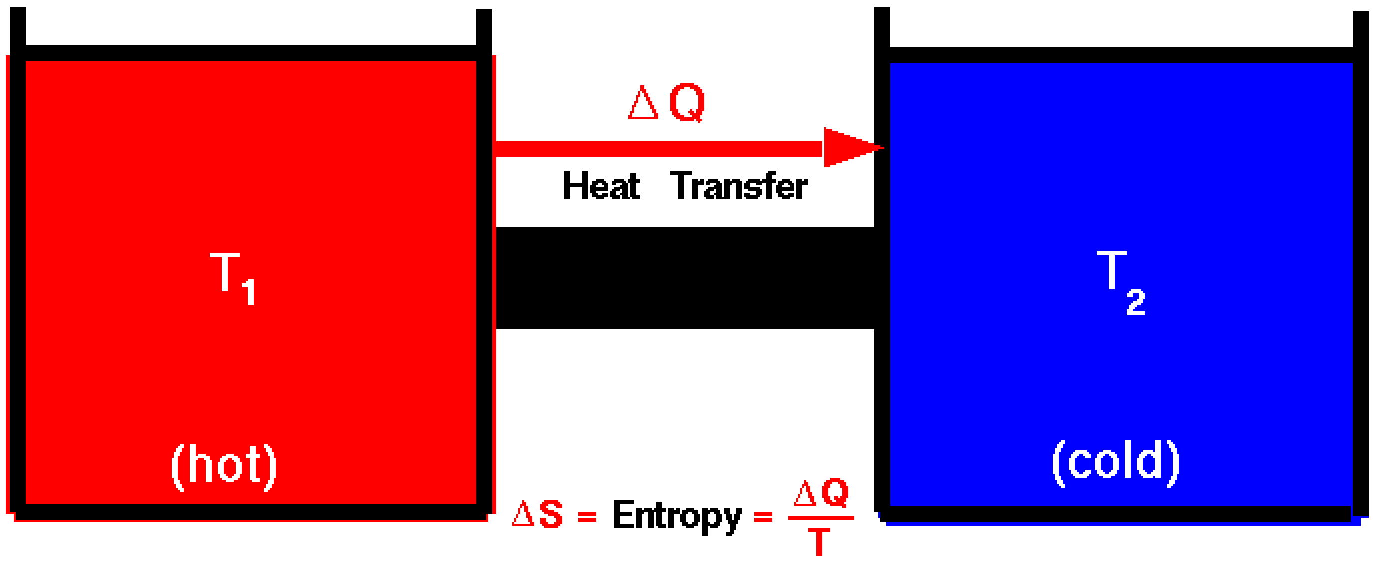
Change in internal energy

Heat added to the system

Work done by the system

Laws of Thermodynamics

Second Law: Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time (Clausius formulation)



Thermal Equilibrium

Thermal Radiation is radiation emitted by **matter** in thermal equilibrium

Thermal Equilibrium: Two physical systems are in thermal equilibrium if no heat flows between them when they are connected by a path permeable to heat.

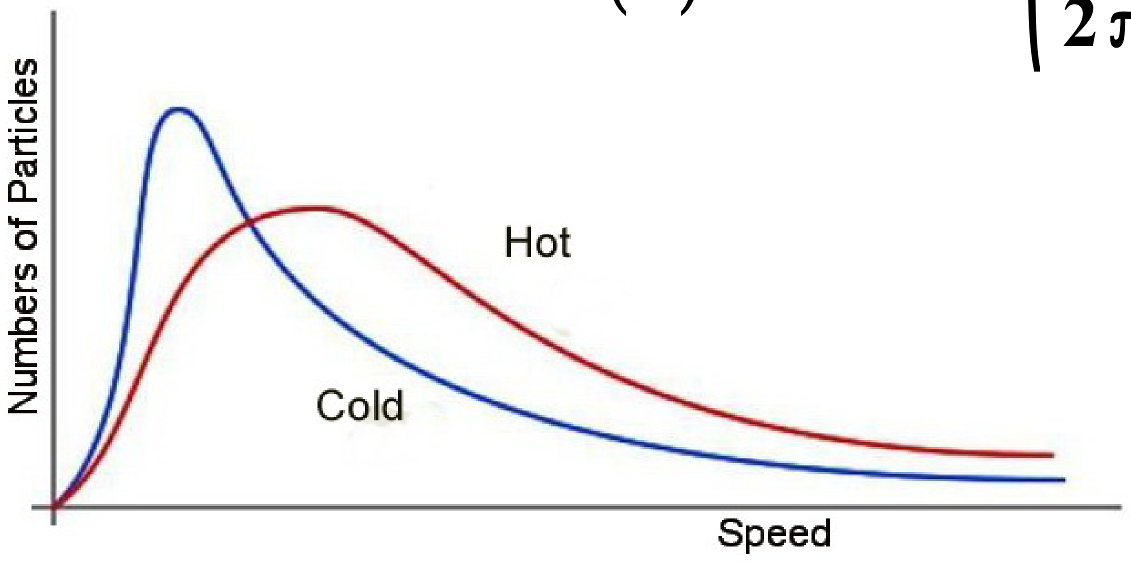
Important: there is a reason why I highlighted the word **matter**. Indeed radiation does not necessarily need to be in thermal equilibrium to have thermal radiation.

Matter in Thermal Equilibrium

Suppose to have a plasma in thermal equilibrium (*thermal plasma*). What does this mean in terms of micro-physical properties of the matter?

Probability distribution function of (non-relativistic) velocities is the Maxwell-Boltzmann distribution:

$$F(v)\,dv = 4\pi v^2 \left(\frac{m}{2\pi k T}\right)^{3/2} e^{-mv^2/2kT}\,dv$$



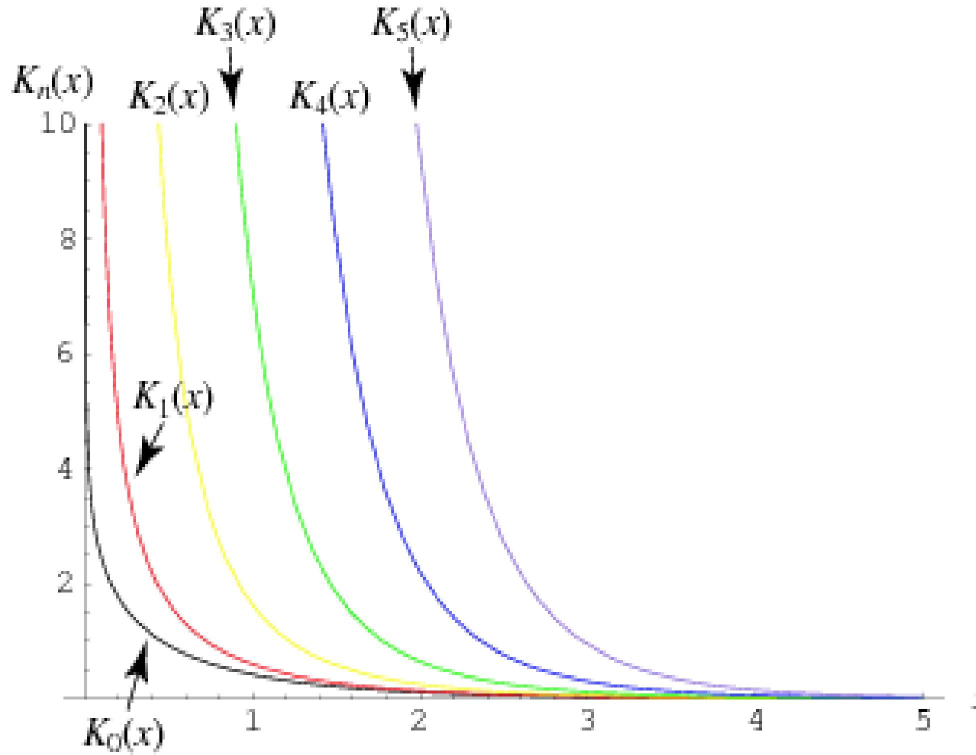
Matter in Thermal Equilibrium

The Maxwell-Boltzmann distribution written in this way is valid only for non-relativistic particles. We will find many astrophysical systems where particles have relativistic speeds and they emit thermal radiation.

$$p = \gamma \beta m c$$

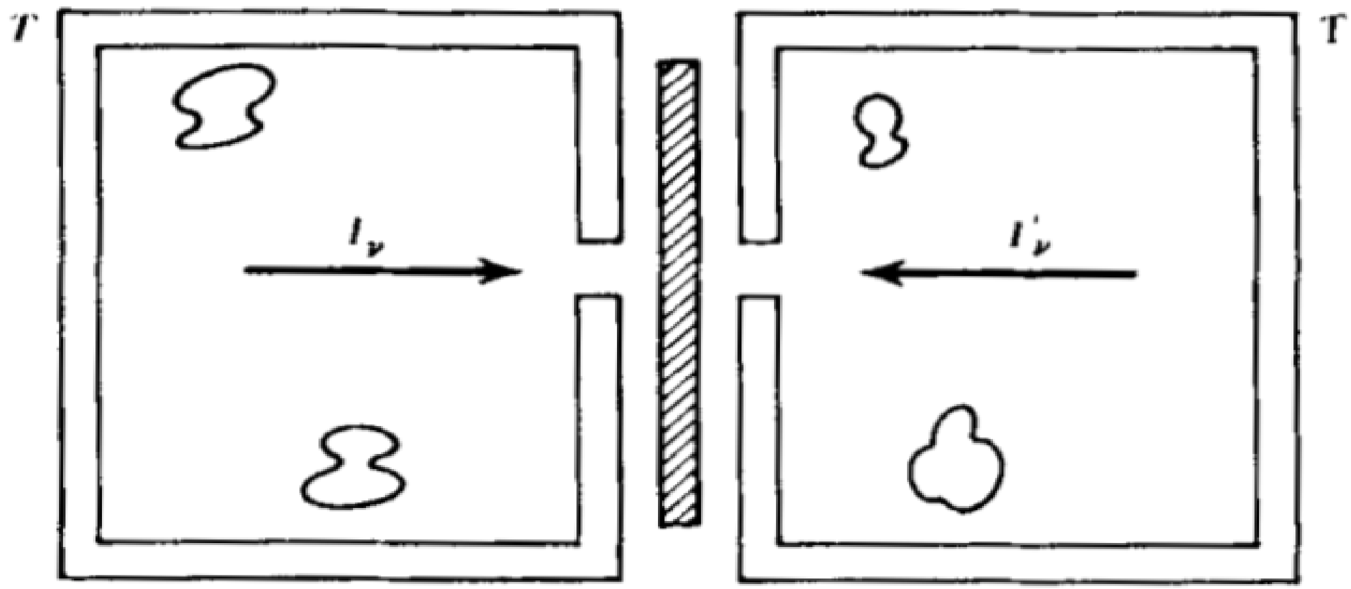
$$F(p)\,dp = \frac{p^2 e^{-\gamma\Theta}}{\Theta m^3 c^3 K_2(1/\Theta)}\,dp$$

Modified Bessell
Function of the 2nd kind



This form is valid in BOTH the relativistic and non-rel. limit

Blackbody Radiation



1. Take an enclosure (of arbitrary shape) at temperature T and do not let radiation flow in or out until equilibrium is achieved. Matter and radiation are in this case in thermal equilibrium.
2. Now open a **small** hole in the first enclosure so that you do not disturb equilibrium.
3. Join a second enclosure with the same properties as the first (temperature T, arbitrary shape) and place a “filter” between the holes so that only a specific frequency can pass through it.

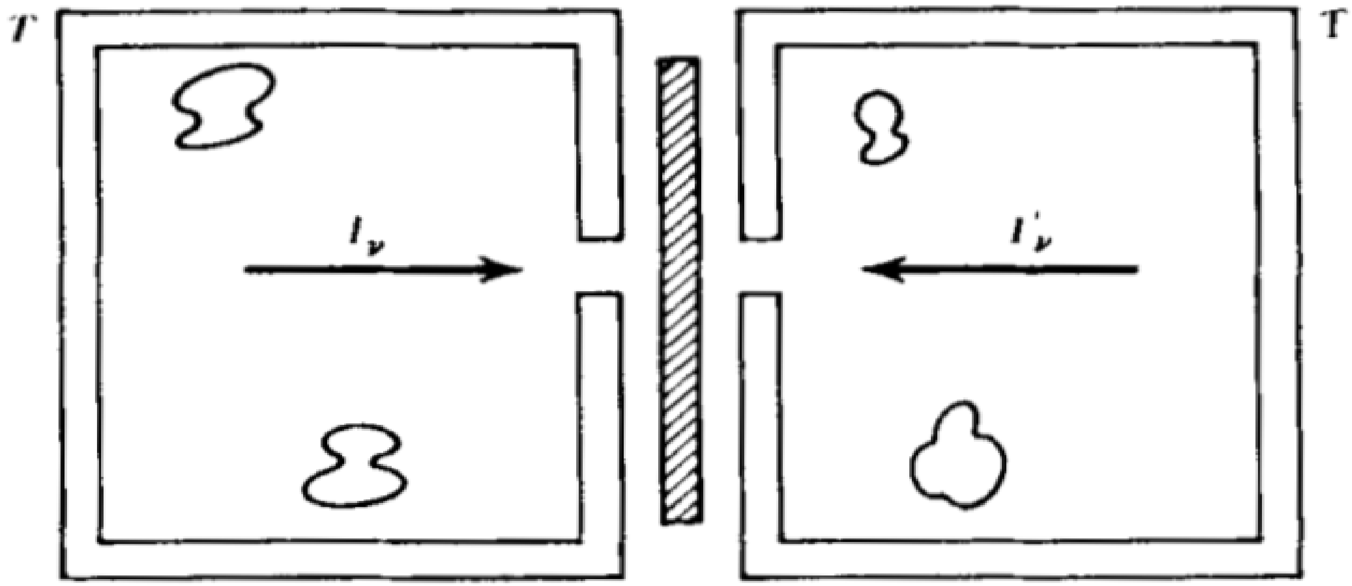
What will happen?

Blackbody Radiation

We have said nothing about the emissivity and absorption properties of the cavity walls. Why?

Should we expect the radiation field inside the cavity to depend on the properties of the wall or not?

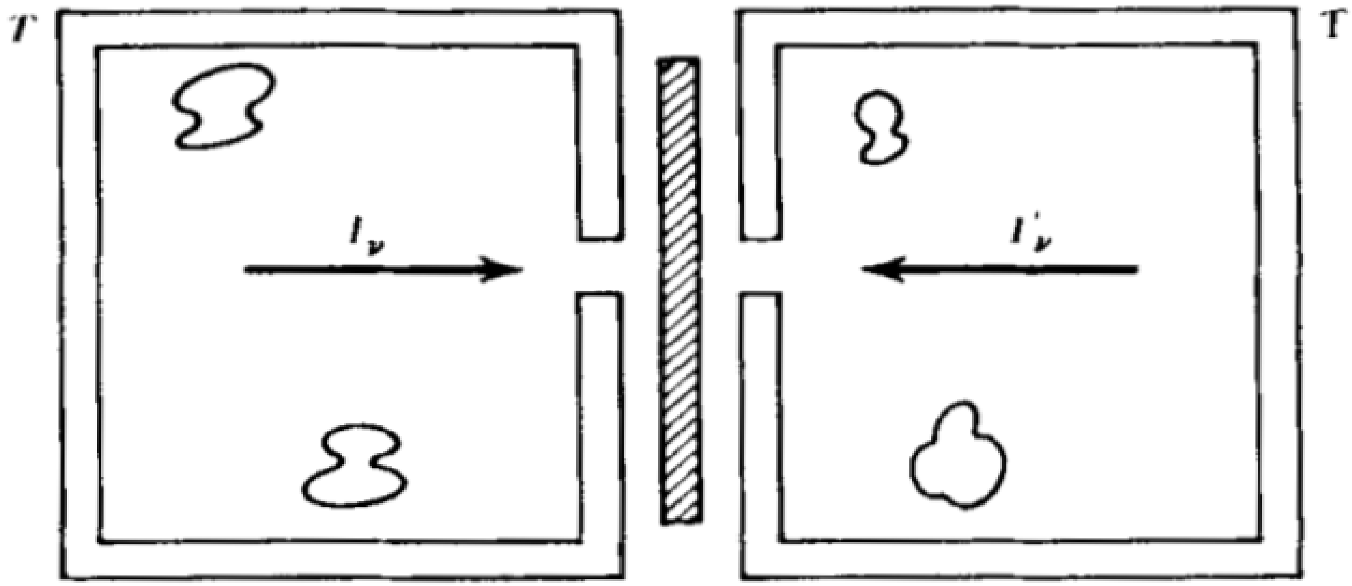
Blackbody Radiation



If radiation flows between the two systems you've just violated the second law of thermodynamics since the two enclosures have the same temperature!
So no ***net*** radiation will flow between the enclosures

This is true regardless of the properties of the cavity!

Blackbody Radiation



The conclusion is therefore simple: the specific brightness of your enclosure must be a universal function of temperature and frequency alone.

$$I_\nu = B_\nu(T)$$

This universal function is called the Planck function

Properties of the Planck Spectrum

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

We will not derive this function here, but read the Section "The Planck Spectrum" on the textbook R&L.

Properties of the Planck Spectrum

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT)-1}$$

The “2” is there because light has two polarizations (left and right circular polarization)

Properties of the Planck Spectrum

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

“h” of course is telling you
that photons are quantized

Properties of the Planck Spectrum

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

“c” is there because of the propagation velocity of photons

Properties of the Planck Spectrum

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

This term comes from the fact that photons are **bosons (integer spin particles)** so they obey the Bose-Einstein statistics.

hv is the energy of a photon
The chemical potential of photons is zero (otherwise you'd have seen there (hv – mu)/kT)

Properties of the Planck Spectrum

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

The “-1” is there because photons are bosons, so there can be multiple photons with the same quantum number.

Properties of the Planck Spectrum

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT)-1}$$

The frequency appears there in this way because it can be easily demonstrated that the density of states is proportional to $2\nu^2/c^3$

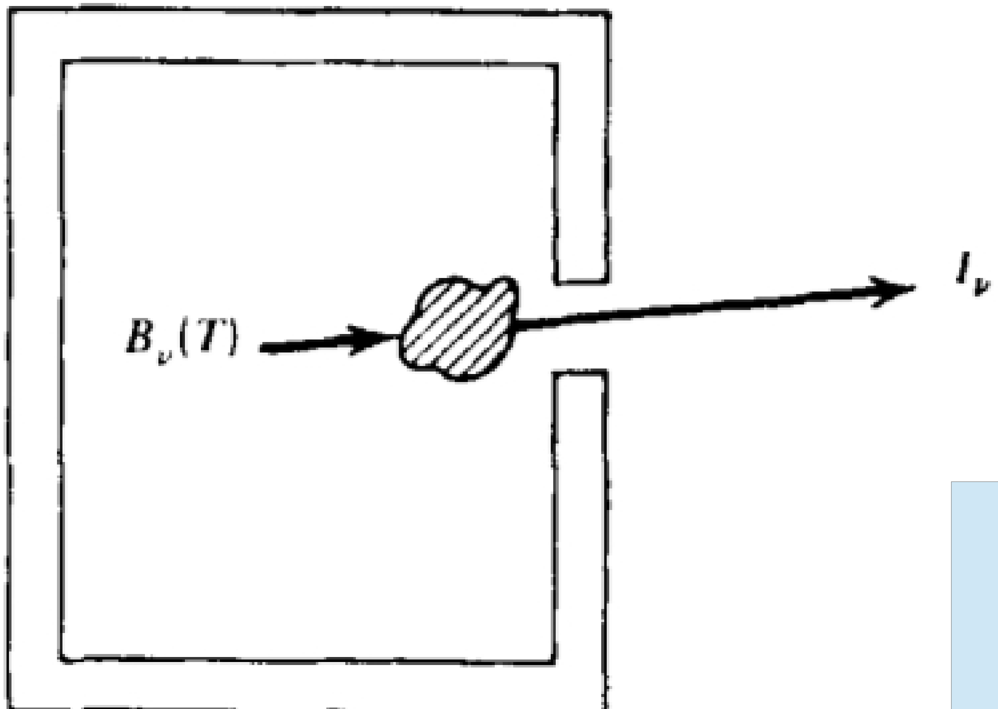
You then need to multiply this by the average energy per state

$$E_{avg} = \frac{h\nu}{\exp(h\nu/kT)-1}$$

Kirchoff's Law

If you place a body with temperature T inside the cavity, what will happen?

The cavity + body is *still a blackbody enclosure* at temperature T (remember we just said that it does not matter how the enclosure is made)



Therefore the source function of the body is:

$$S_\nu = B_\nu(T) \rightarrow j_\nu = \alpha_\nu B_\nu(T)$$

Thermal Radiation	$S_\nu = B_\nu(T)$
Blackbody Radiation	$I_\nu = B_\nu(T)$

Planck Spectrum

As we have stated before an example of thermal radiation is blackbody radiation.
But the opposite is not generally true: thermal radiation is not necessarily blackbody radiation

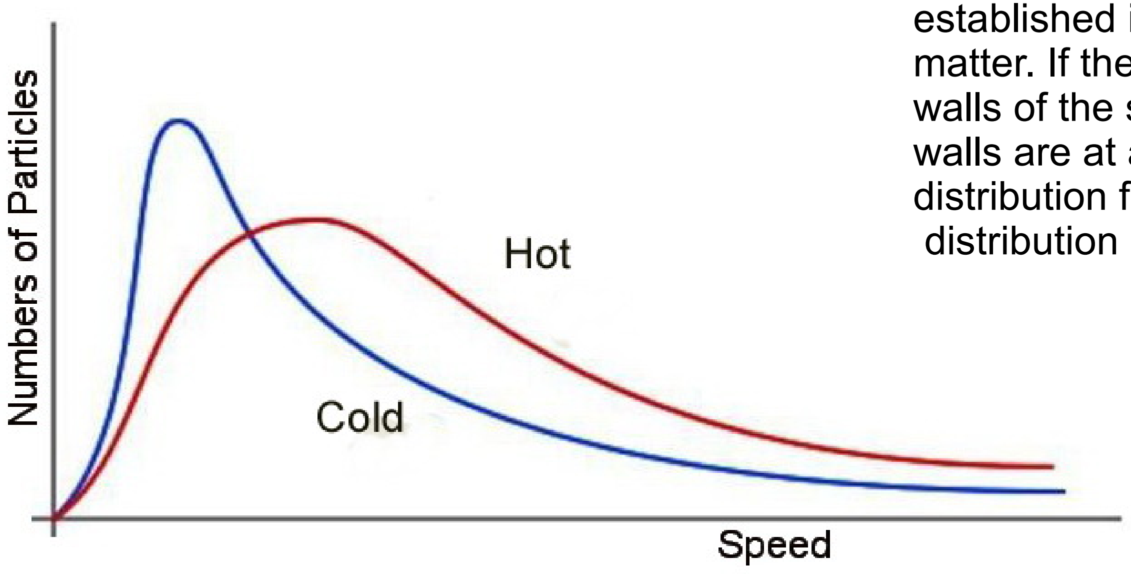
Blackbody → Thermal

Thermal $\not\rightarrow$ **Blackbody**

Blackbody radiation is generated by an optically thick medium emitting thermal radiation

Blackbody Radiation as a Photon Gas

In a classical ideal gas with massive particles, the energy of the particles is distributed according to a Maxwell–Boltzmann distribution. This distribution is established as the particles collide with each other, exchanging energy (and momentum) in the process. In a photon gas, there will also be an equilibrium distribution, but photons do not collide with each other (except under very extreme conditions, see two-photon physics), so the equilibrium distribution must be established **by other means**.

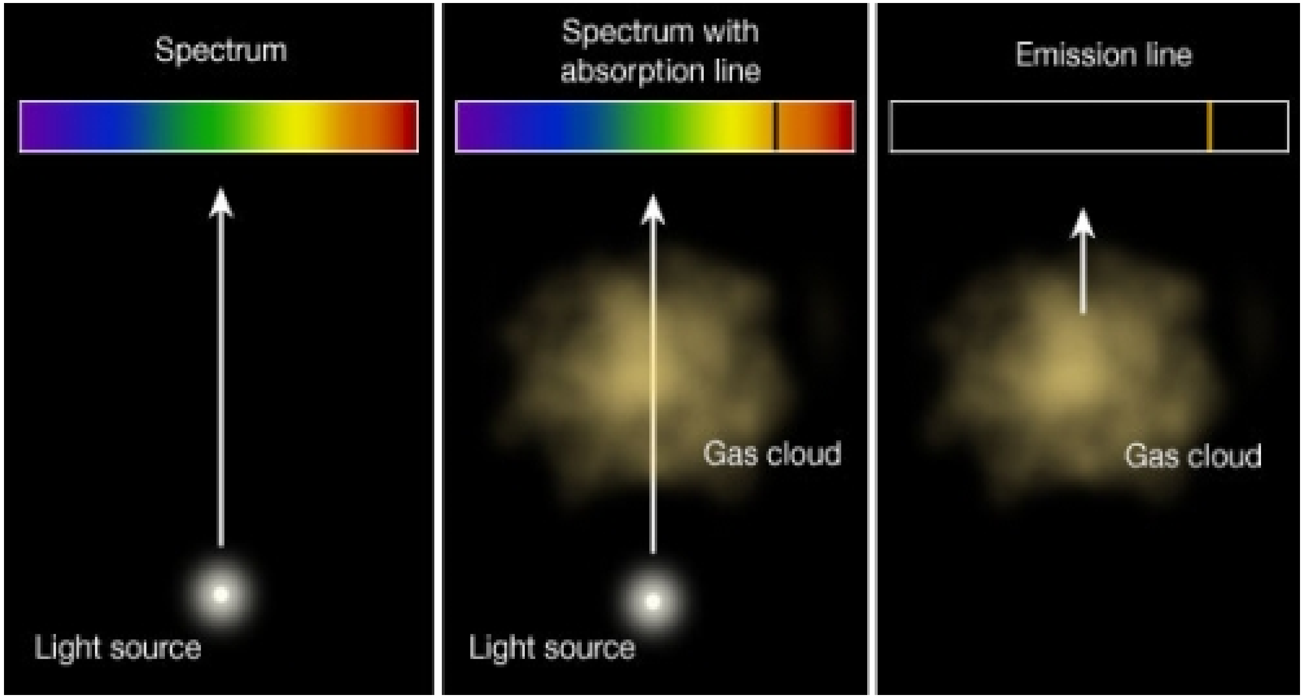


The most common way that an equilibrium distribution is established is by the interaction of the photons with matter. If the photons are absorbed and emitted by the walls of the system containing the photon gas, and the walls are at a particular temperature, then the equilibrium distribution for the photons will be a black-body distribution at that temperature.

Understanding Kirchhoff's Law

Now we are in position to understand Kirchhoff's laws in the light of the equation of radiative
transfer.

1. A hot dense gas produces light with a continuous spectrum.
2. A hot dense gas surrounded by a cool tenuous gas produces light with a continuous spectrum which has gaps at discrete wavelengths.
3. A hot tenuous gas produces light with emission lines at discrete wavelengths



Understanding Kirchhoff's Law

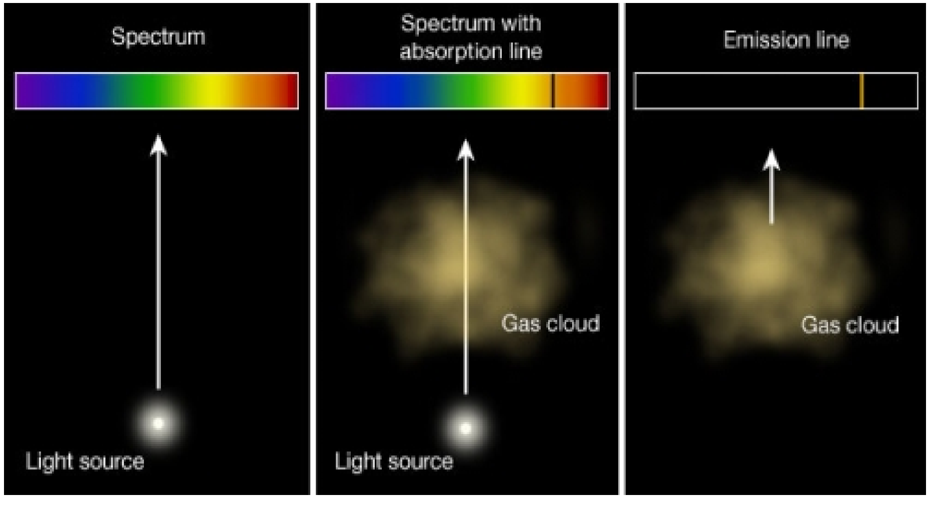
1. A hot dense gas produces light with a continuous spectrum.

Consider (for simplicity) that the initial surface brightness is zero $I_\nu(0)=0$

Then: $I_\nu=S_\nu\left(1-e^{-\tau_\nu}\right)$

Since the gas is dense, the optical depth is >>1. Therefore: $I_\nu=S_\nu=B_\nu$

And this implies that what you will see is a blackbody spectrum (Planck spectrum)



Understanding Kirchhoff's Law

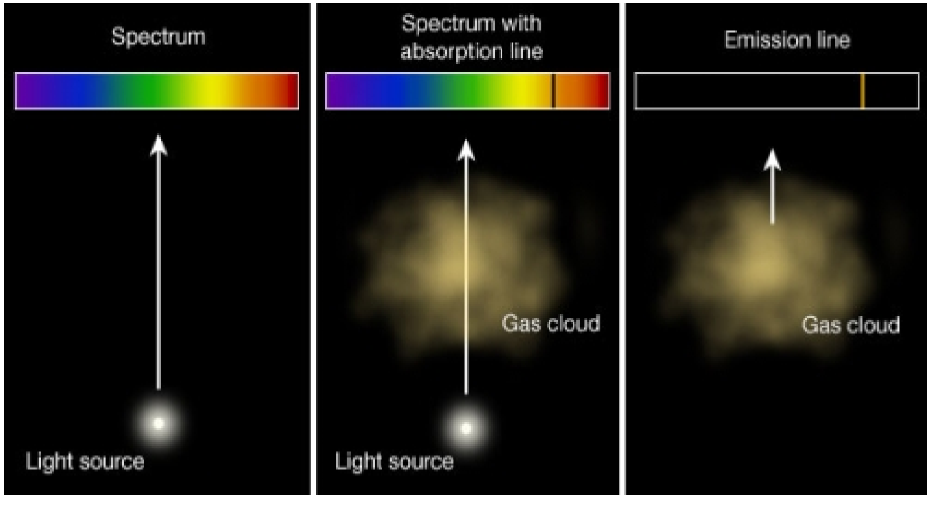
2. A hot dense gas surrounded by a cool tenuous gas produces light with a continuous spectrum which has gaps at discrete wavelengths.

In this case there is a background (initial) specific emissivity, i.e., the one of the hot ga which we know is a blackbody. Therefore: $I_\nu(0)=B_\nu$

The cold gas instead is not emitting (or, more correctly, has negligible emission when compared to the hot gas and thus: $S_\nu \approx 0$

Therefore: $I_\nu = B_\nu e^{-\tau_\nu}$

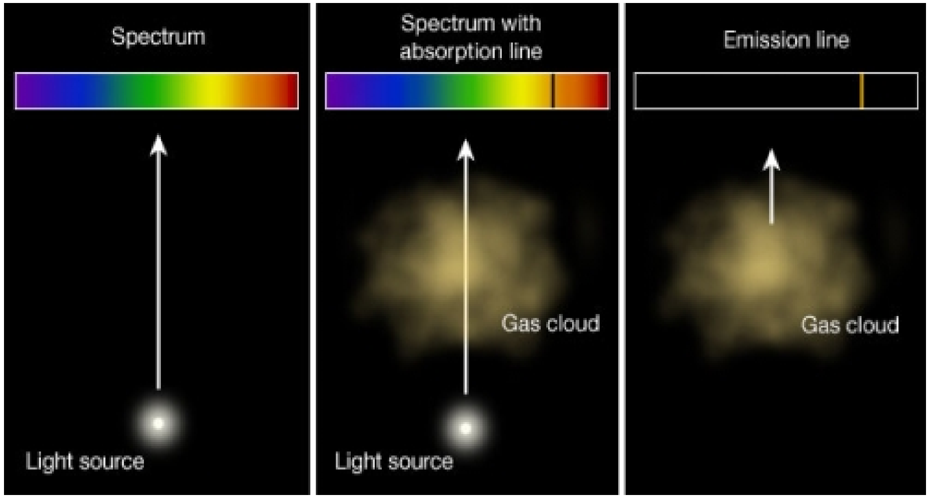
The intensity is a Planck continuum, lowered where the optical depth is high. And where does the optical depth become high? Only at those frequencies that correspond to the absorption energies of the cold atoms. Thus, we see absorption lines.



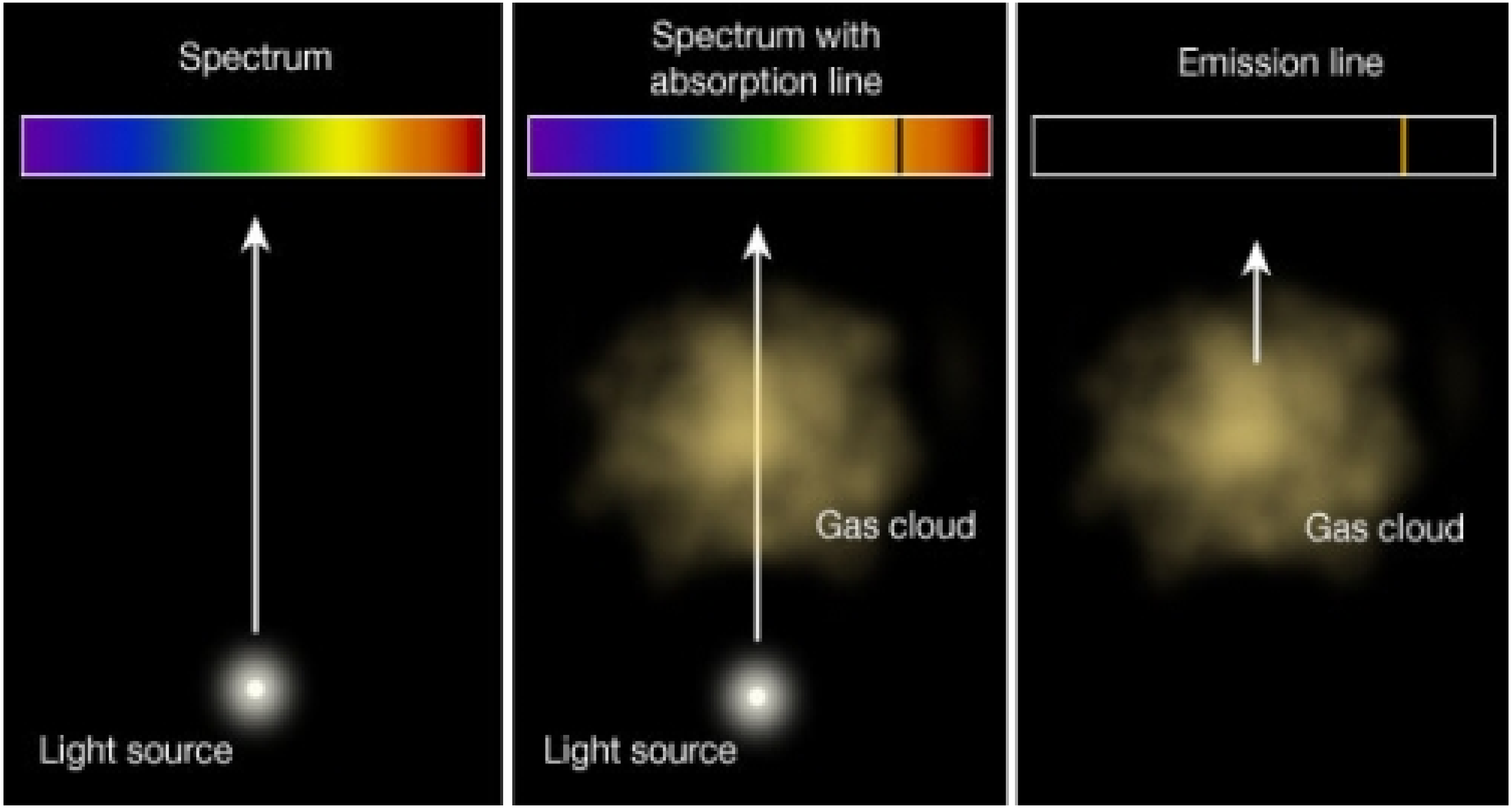
Understanding Kirchhoff's Law

3. A hot tenuous gas produces light with emission lines at discrete wavelengths

In the optically thin case $I_\nu = S_\nu(1 - e^{-\tau_\nu}) \approx S_\nu(1 - 1 + \tau_\nu) = S_\nu \tau_\nu$
The intensity will be high where the optical depth is high. Since there is no background intensity, these are seen as emission lines.



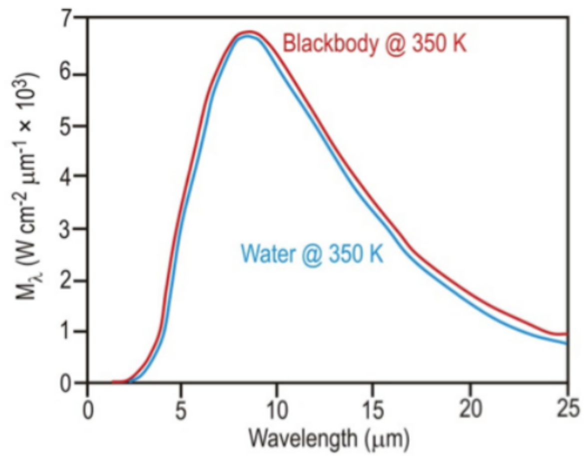
Principle of Detailed Balance

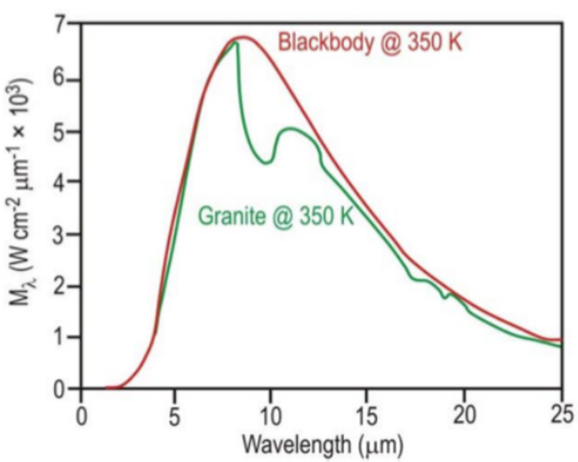


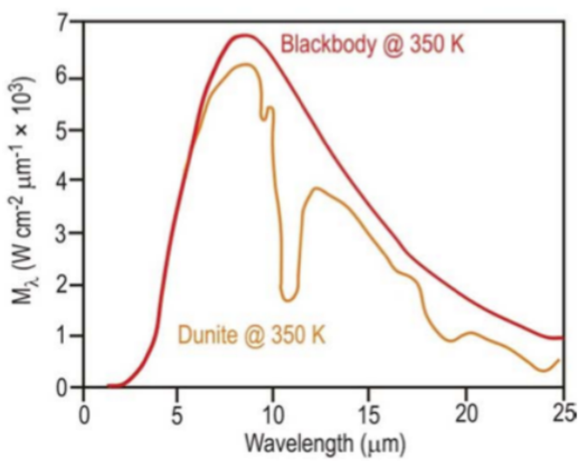
At equilibrium, each elementary process should be equilibrated by its reverse process.

This principle has been derived here by using Kirchhoff's law, which requires that matter is in thermal equilibrium. However, as we will see it is valid also for nonthermal emission

Clarification on Kirchhoff's Law







$$j_\nu = \alpha_\nu B_\nu(T)$$

A good absorber is a good emitter
A good emitter is a good absorber

It was proved for objects in fully thermodynamic equilibrium but it is applicable to any object in thermal equilibrium.
It is NOT applicable for non-thermal emitters (e.g., synchrotron, shocks, nuclear explosions)

Thermodynamics of Blackbody Radiation

We can derive an important property of blackbody radiation by treating it as a photon gas and then applying the laws of thermodynamics.

$$dQ = dU + p\,dV$$ 1st law of thermodynamics

$$dS = \frac{dQ}{T}$$ 2nd law of thermodynamics

We have seen in the first lecture how to calculate *u,* the energy density of radiation:

$$u = \frac{4\pi}{c}\int J_\nu \, d\nu$$ And here $J_\nu = B_\nu(T)$

We thus know what the total energy of radiation is:

$$U = uV$$

We have also seen how to calculate the radiation pressure *p,* which is 1/3 the energy density

$$p = \frac{u}{3}$$

Thermodynamics of Blackbody Radiation

$$dS = \frac{V}{T}\frac{du}{dT}dT + \frac{u}{T}dV + \frac{1}{3}\frac{u}{T}dV = \frac{V}{T}\frac{du}{dT}dT + \frac{4u}{3T}dV$$

From this expression you see that *dS* is an exact diffrential. Let's remember the definition of exact differential. A differential *df* is exact (or perfect) if:

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy,$$

Therefore: $\left(\frac{\partial S}{\partial T}\right)_V = \frac{V}{T}\frac{du}{dT}$ $\quad$ $\left(\frac{\partial S}{\partial V}\right)_T = \frac{4u}{3T}$

Let's now differentiate the first partial derivative again with respect to V:

$$\frac{\partial^2 S}{\partial T \partial V} = \frac{1}{T}\frac{du}{dT} = -\frac{4u}{3T^2} + \frac{4}{3T}\frac{du}{dT} \longrightarrow \frac{du}{u} = 4\frac{dT}{T}$$

Thermodynamics of Blackbody Radiation

Solution: $\log u = 4 \log T + \log a$ $\longrightarrow$ $u(T) = aT^4$

This is the *Stefan-Boltzmann* law.
For isotropic radiation:

$$u = \frac{4\pi}{c} \int B_\nu(T)\, d\nu = \frac{4\pi}{c} B(T) = \frac{ac}{4\pi} T^4$$

But there is another result which is very profound and comes from this discussion: **blackbody radiation is the type of radiation with *maximum* possible entropy.**

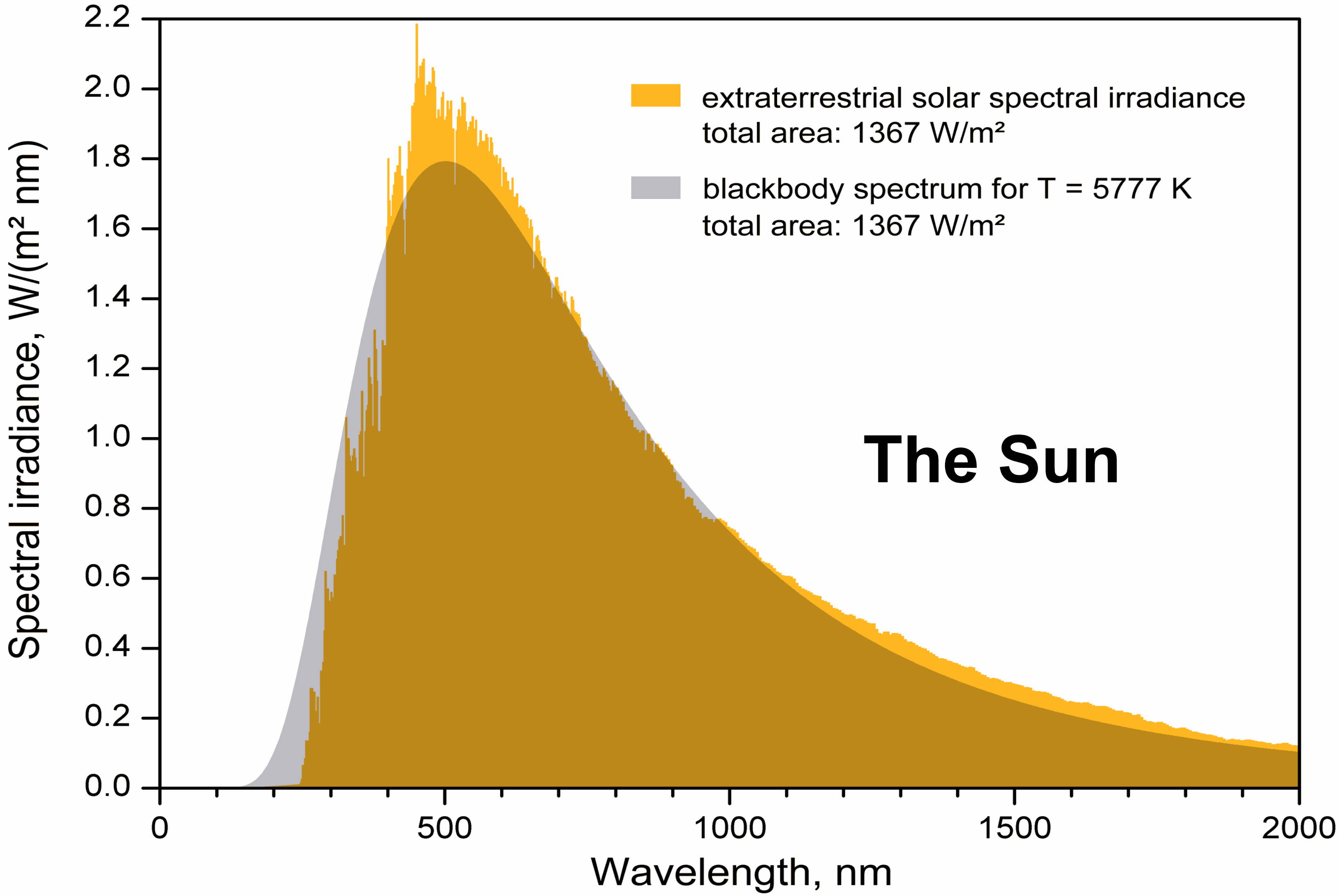
What is the entropy of blackbody radiation? Since u(T)=aT^4 we can write:

$$\left(\frac{\partial S}{\partial T}\right)_V = 4aVT^2, \qquad \left(\frac{\partial S}{\partial V}\right)_T = \frac{4}{3} aT^3$$

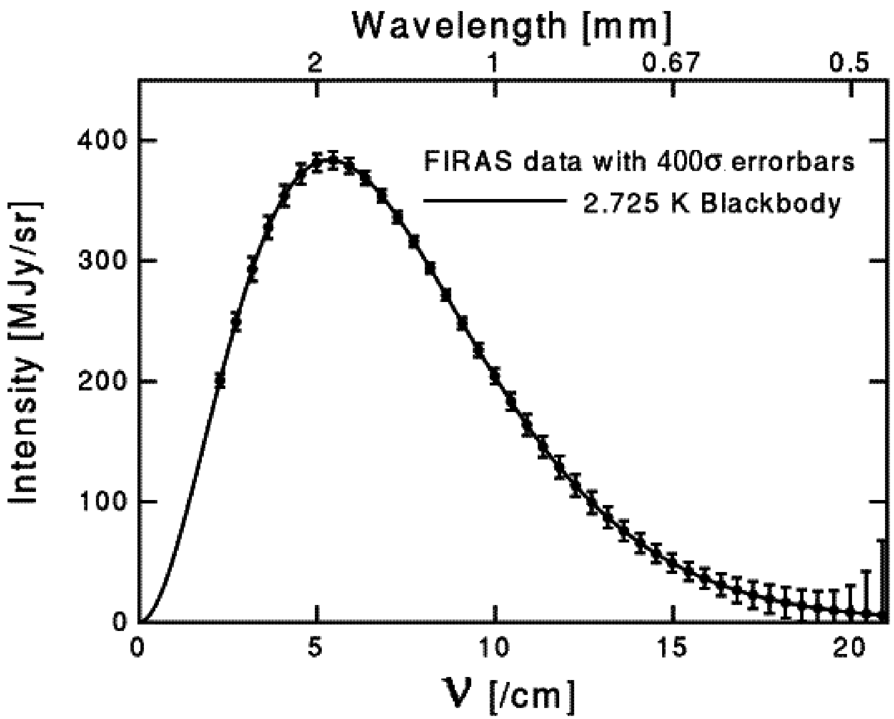
We can thus write: $S = (4/3)\; aVT^3 + \text{constant}$

(here the constant is actually zero, since S→ 0 when T→ 0).

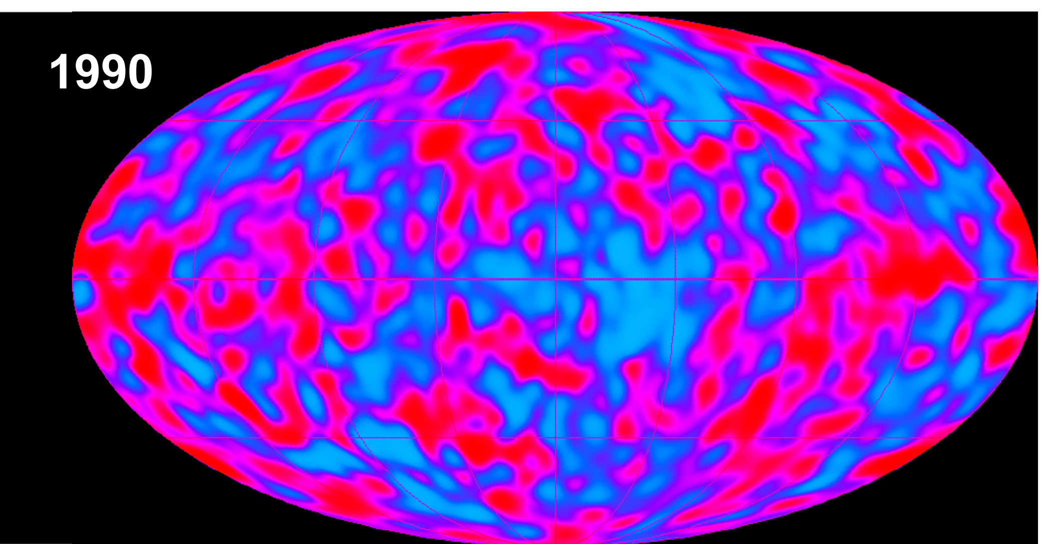
Stellar Spectra: blackbody emitters

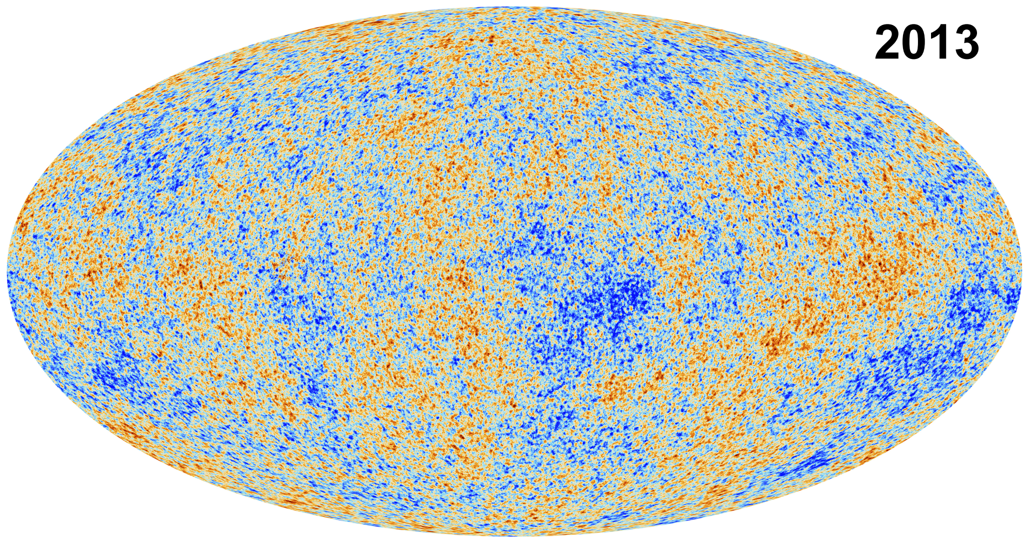


Cosmic Microwave Background



Deviation from a 2.725 K Blackbody: 1/100.000





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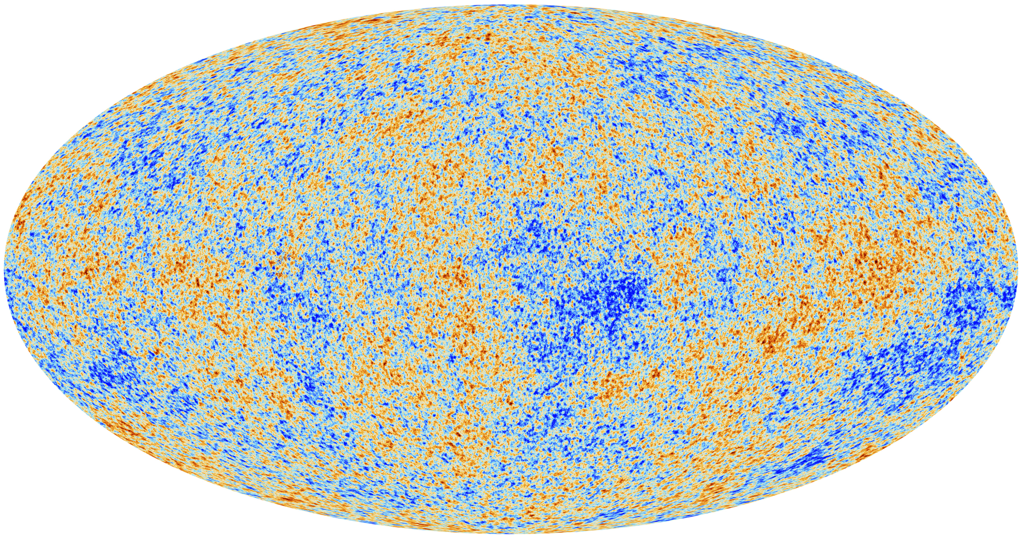
Cosmic Microwave Background



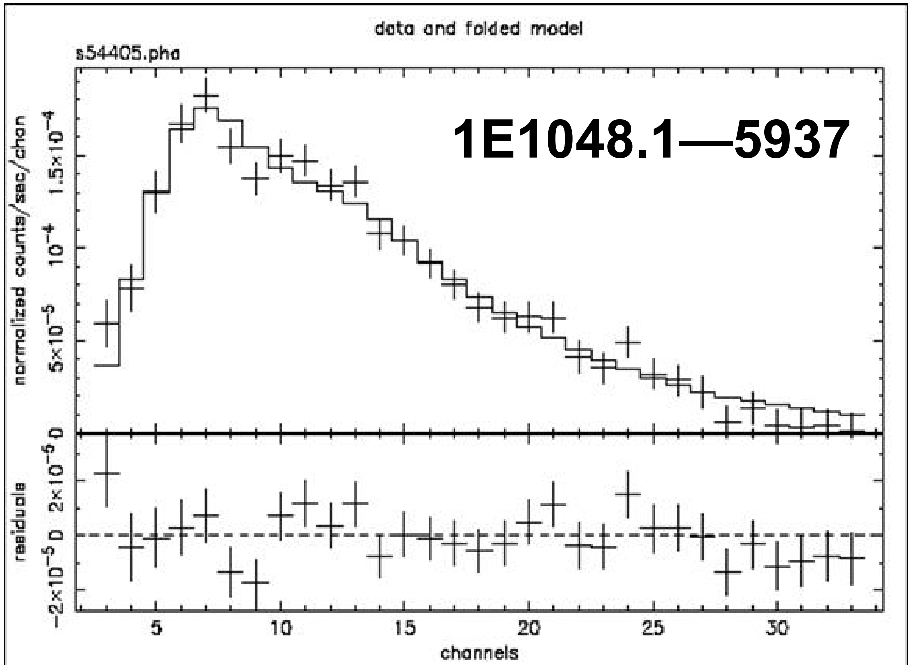
CMB shows that radiation in the early universe was close to a perfect BB.

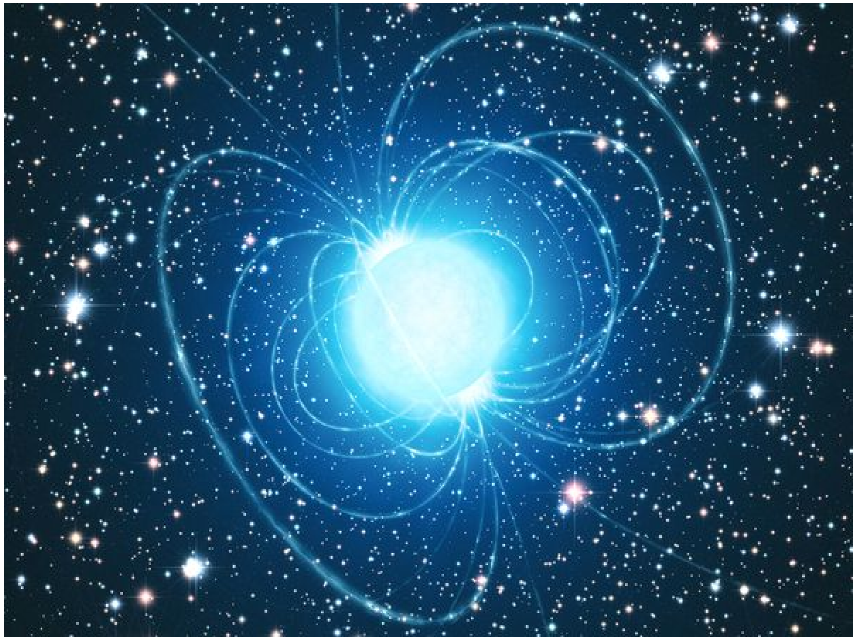
Therefore, since a BB has maximum entropy, radiation was in thermal equilibrium.

Matter was also in thermal equilibrium since it has thermalized radiation in such a perfect way.

Question: why do we say then that the Universe was born with a very **low** entropy? CMB seems to suggest the exact opposite. How is this possible?

Neutron Stars



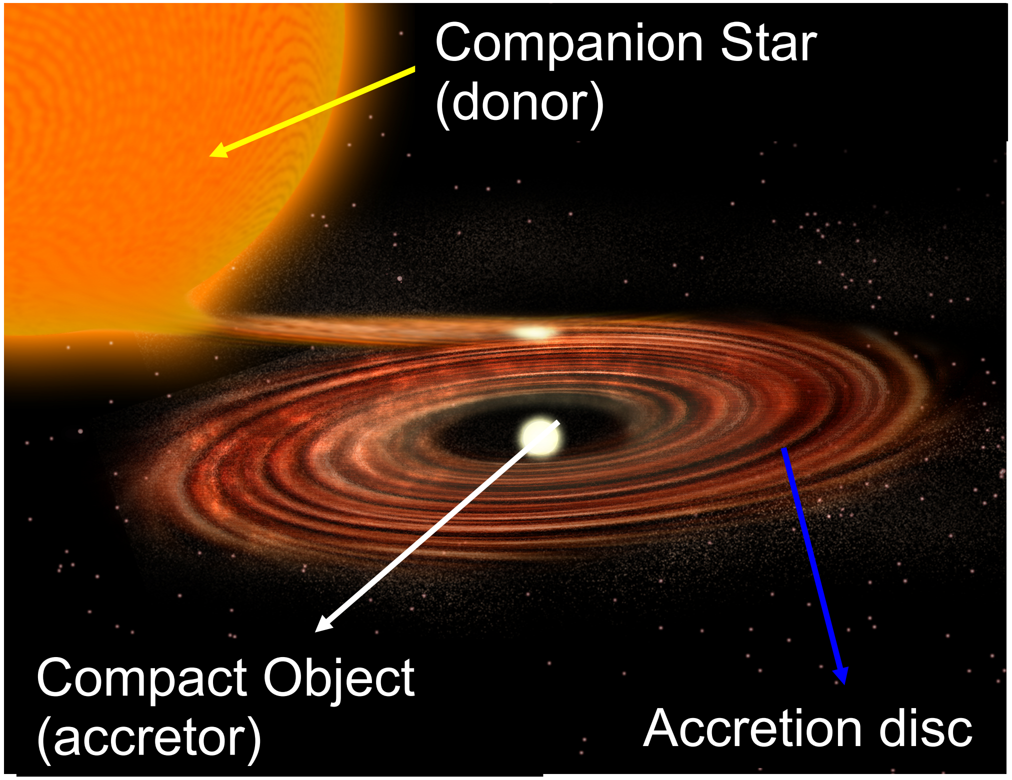


6-s period Anomalous X-ray Pulsar

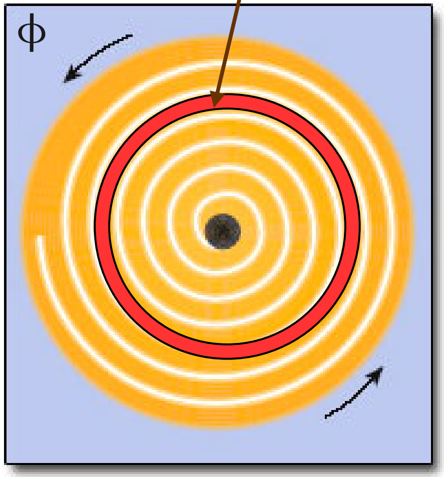
Check also the **Magnificent Seven:**

http://en.wikipedia.org/wiki/The_Magnificent_Seven_(neutron_stars)

Accretion discs



Circularization radius $R_{circ.} = \frac{GM}{v_\phi}$



Accretion discs are the “medium” through which the gravitational potential energy which has been transformed into kinetic energy, is finally transformed into thermal energy (mainly radiation)

http://astro.fit.edu/wood/fitdisk.html

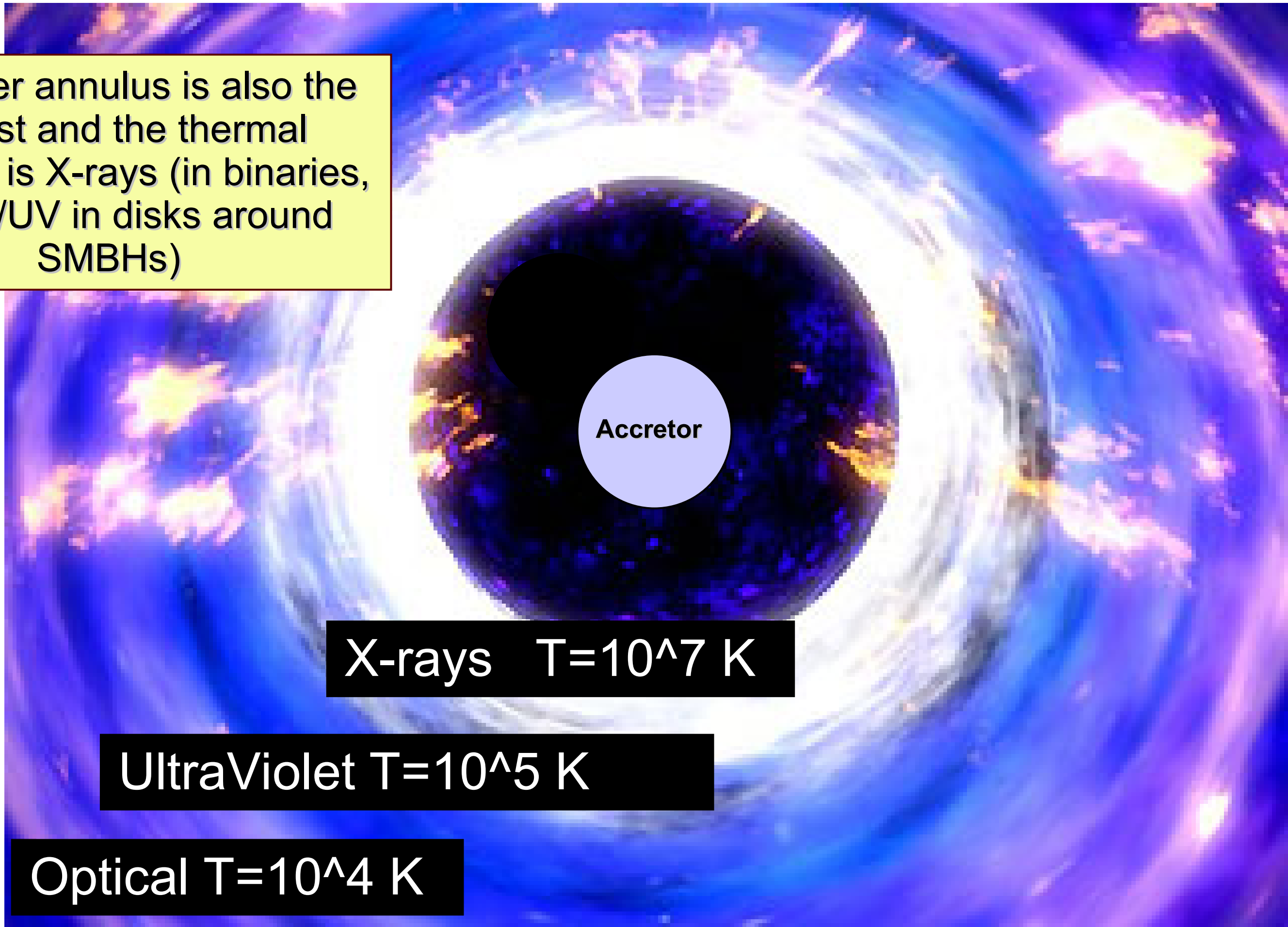
Accretion disc structure

Gas rotates in the disc with "Keplerian frequency", i.e., the inner annulus rotates faster than the outer one.

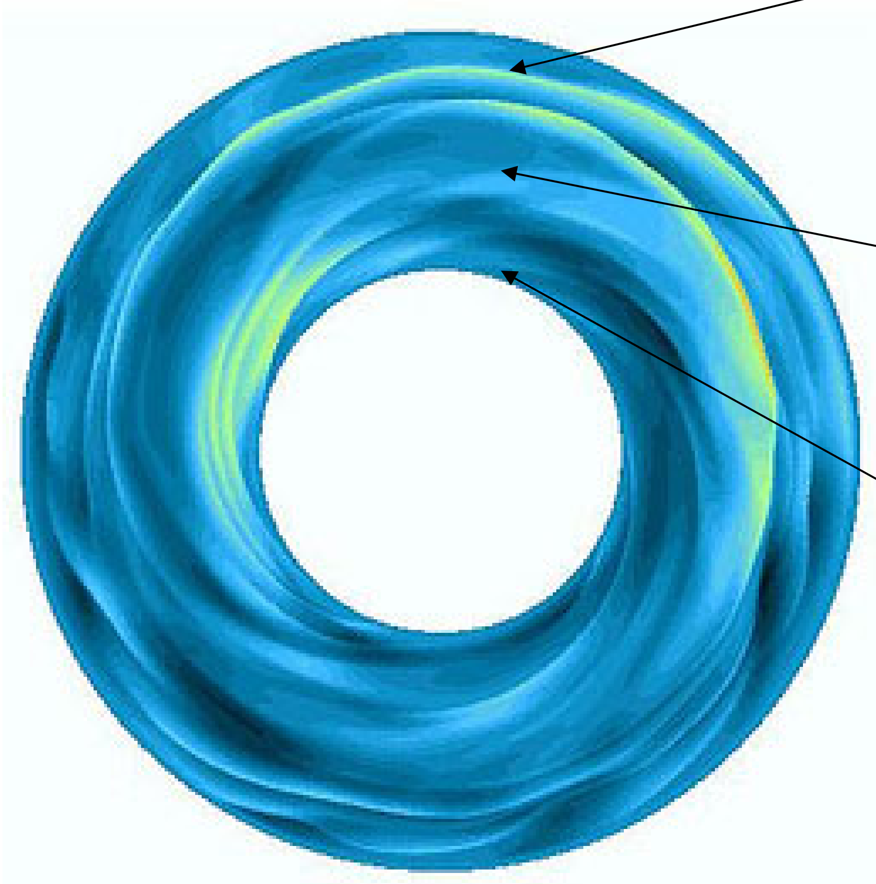
$$\longrightarrow \quad \Omega_K = \sqrt{\frac{GM}{r^3}}$$

The inner annulus is also the hottest and the thermal radiation is X-rays (in binaries, optical/UV in disks around SMBHs)

The disc is usually composed mainly by ionized Hydrogen (i.e., a plasma of electrons + protons)



Accretion disc regions

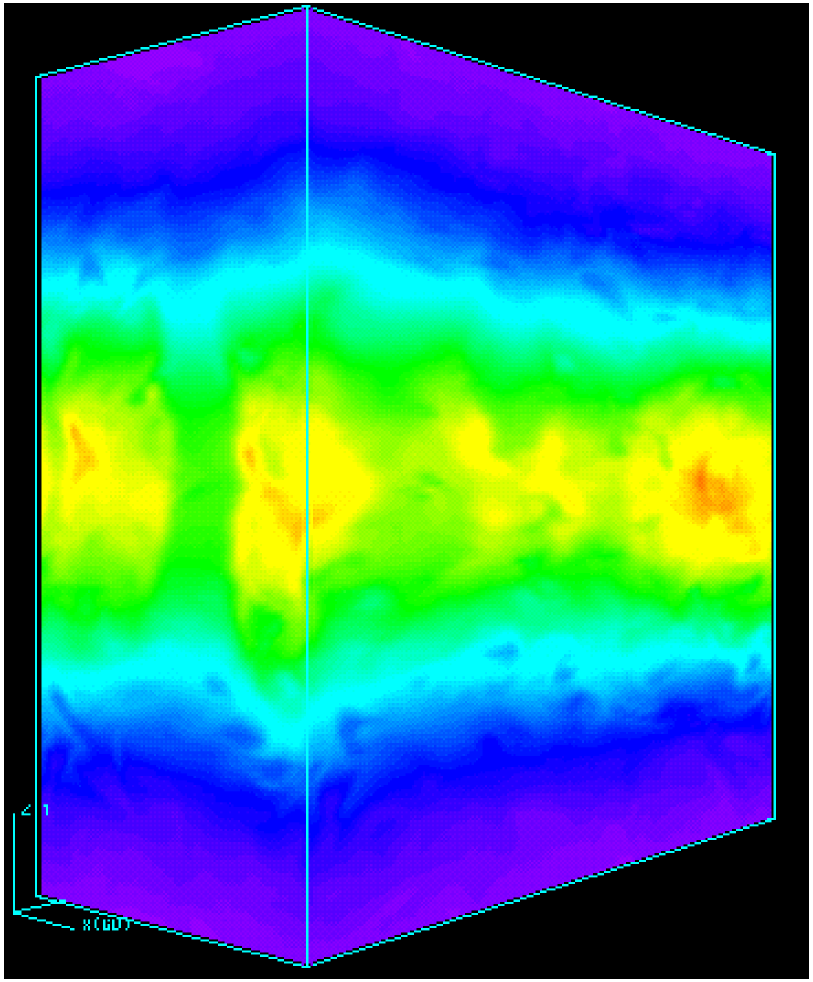


Outer Region: gas pressure dominates.
Opacity: free-free absorption (i.e., inverse Bremsstrahlung)

Middle Region: gas pressure dominates Opacity: electron scattering

Inner Region: radiation pressure dominates.
Opacity: electron scattering

Accretion disc Vertical Structure

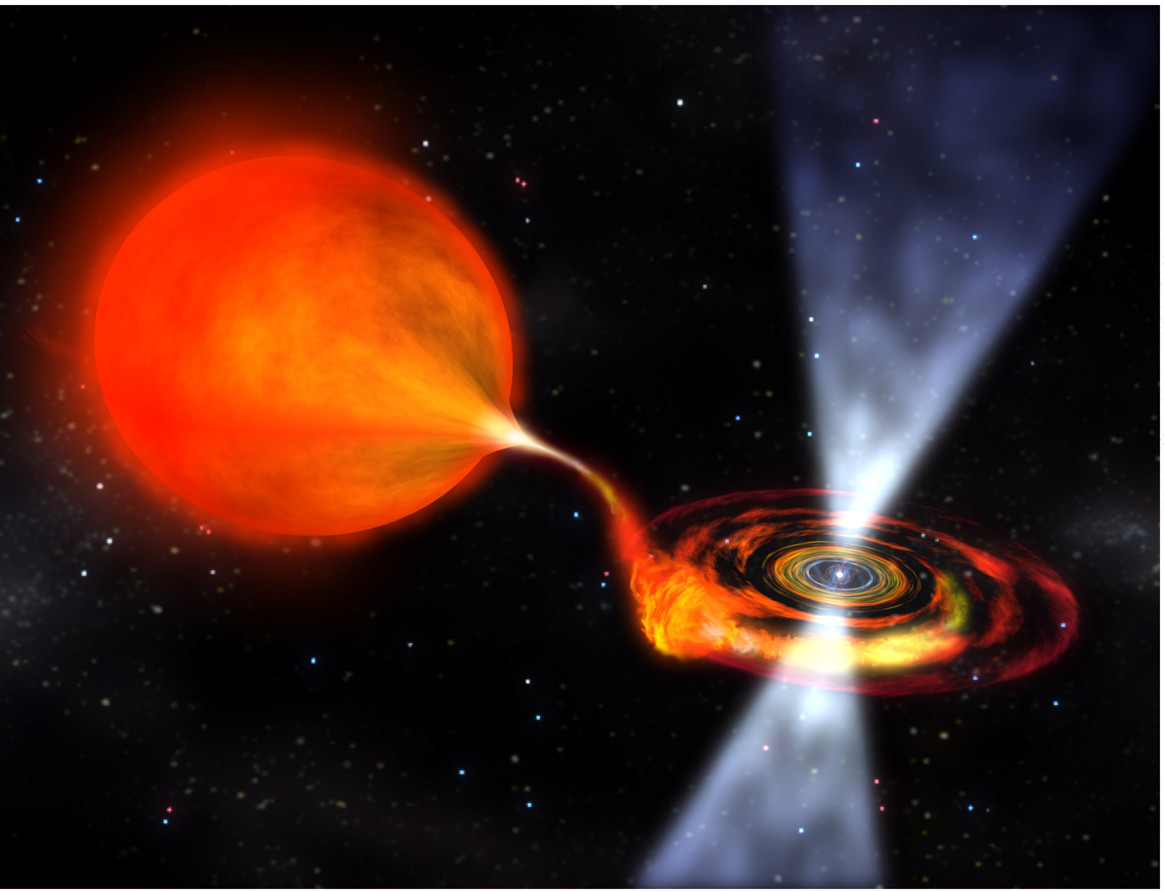


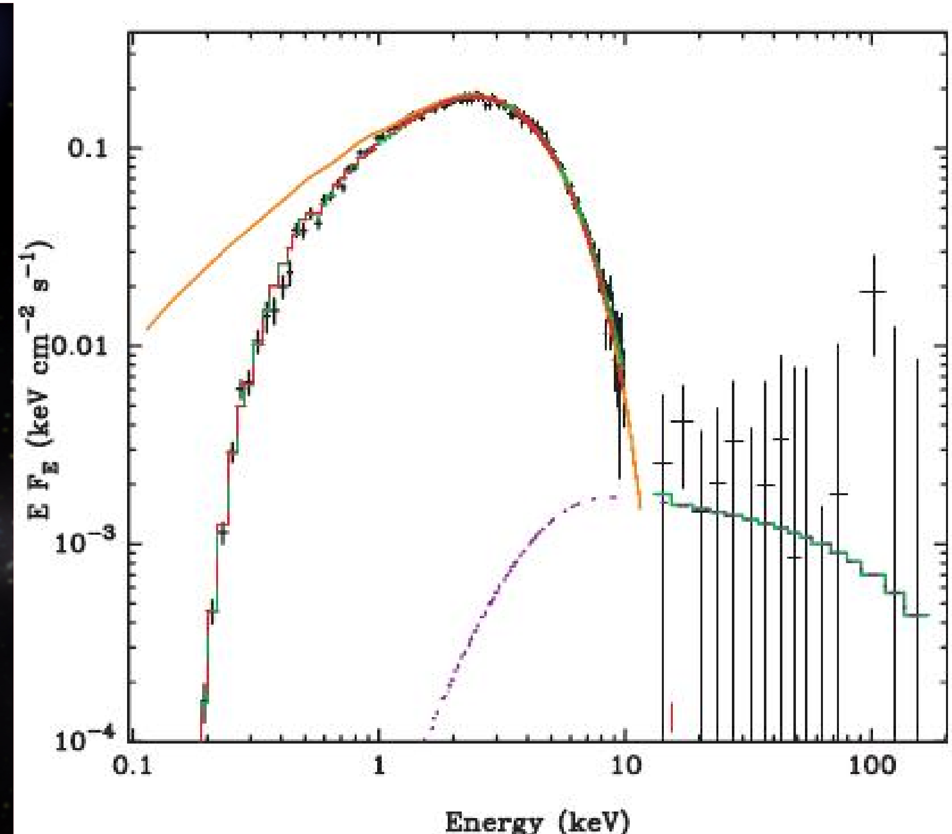
$$\frac{1}{\rho}\frac{dP}{dz} = -\frac{GM}{r^2}\frac{z}{r}$$

$$h = \left(\frac{P}{\rho}\right)^{1/2}\left(\frac{r^3}{GM}\right)^{1/2} \approx \frac{c_s}{\Omega}$$

Vertical disc is in hydrostatic equilibrium

Accretion Disks around Black Holes

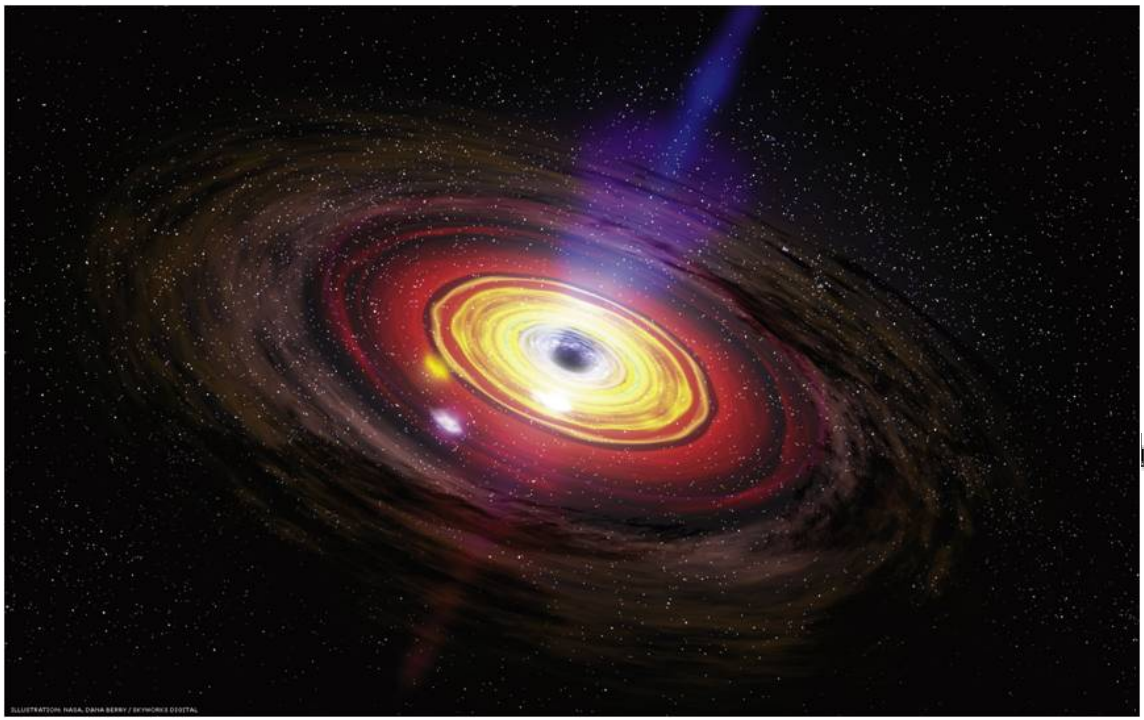


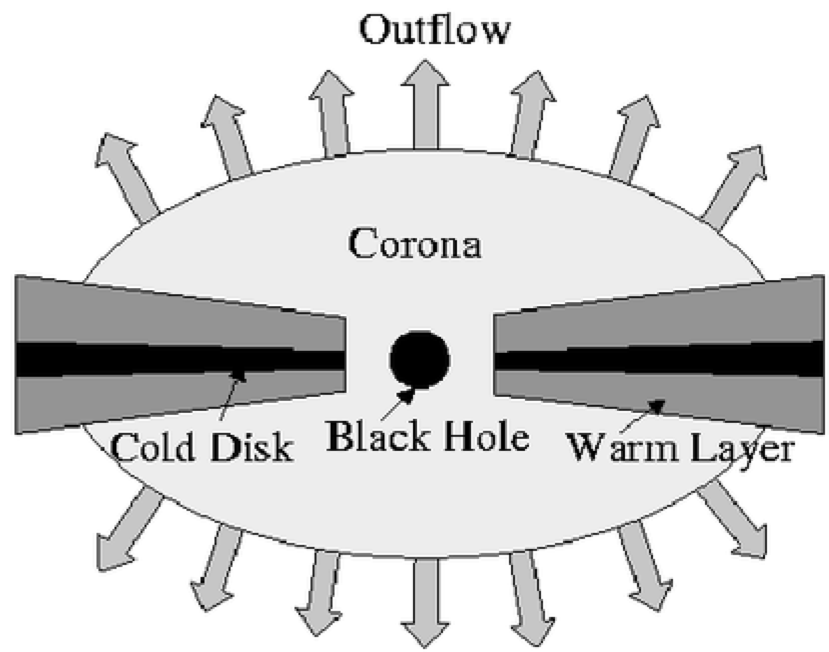


Used to infer the Black Hole spin period (and test General Relativity)

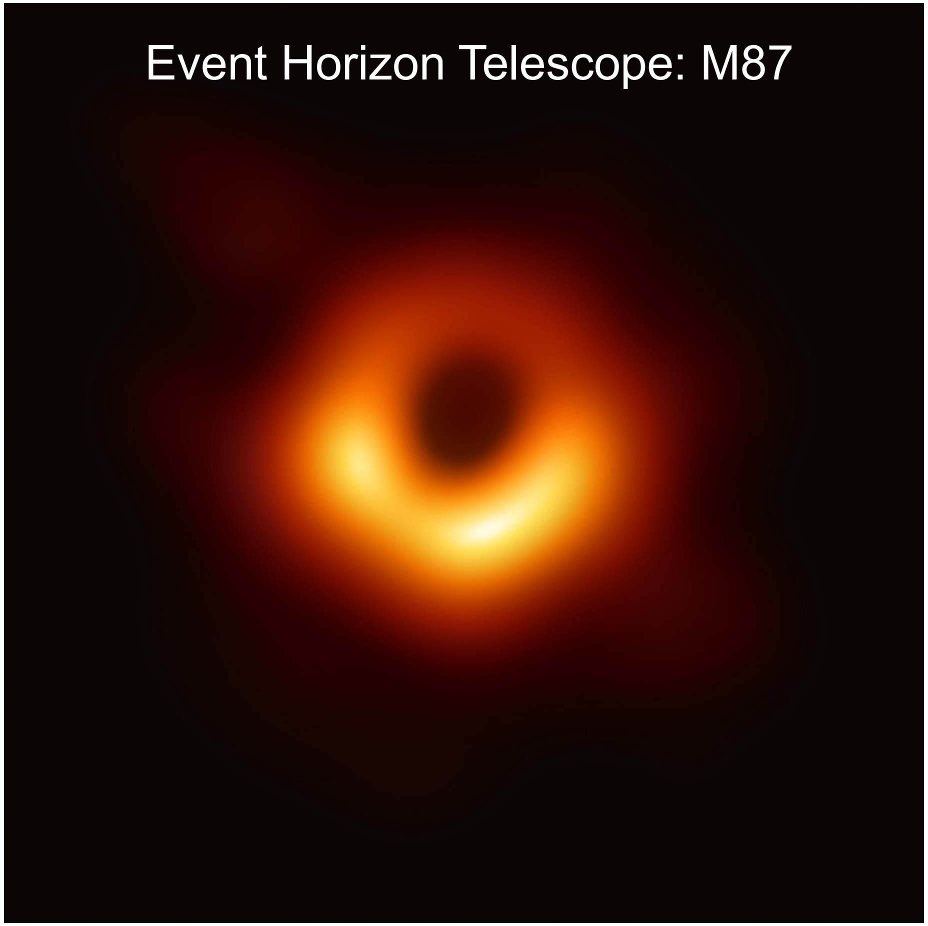
McClintock, Narayan & Steiner (2013)
http://arxiv.org/pdf/1303.1583.pdf

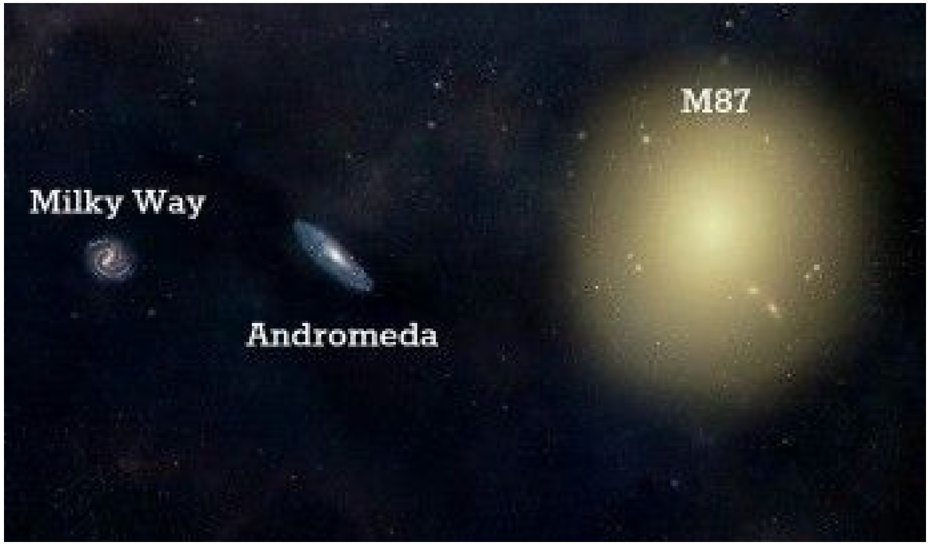
Accretion Disks around Supermassive Black Holes





Accretion Disks around Supermassive Black Holes





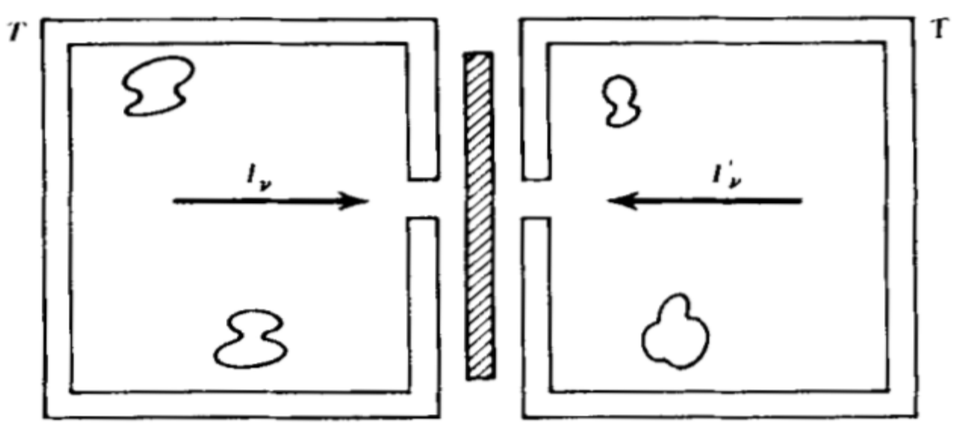
First “direct” image of a black hole obtained in radio wavelengths with the Event Horizon Telescope, an array of telescopes spread across the Earth.

Why do we see blackbodies?

If particles and radiation are in equilibrium it means that there is a steady-state condition: there is no net flow of energy in nor out of a volume element, nor any transfer of energy between matter and radiation.

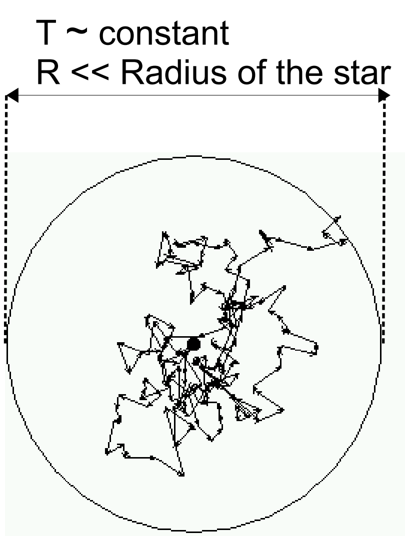
Every process, such as the absorption of a photon, occurs at the same rate as its inverse process, such as the emission of a photon. Such an idealised condition is referred to as thermodynamic equilibrium. A blackbody is by definition in thermodynamic equilibrium

You might now wonder how is possible that a star or an accretion disk, or a neutron star emits like a blackbody. For example in a star there is a net outward flow of energy. The temperature varies from millions of degrees in the core to thousands of degrees in the atmosphere. So we do we see blackbody-like emission ?



Local Thermodynamic Equilibrium

Suppose that the typical distance traveled by particles and photons between collisions—their mean free path—is small compared to the scale over which the temperature changes significantly. This situation applies to most of the stellar interior, where density and temperature are high, so that the mean distance between collisions is small. You can think of this situation as the particles and photons being confined to a limited volume of nearly constant temperature.



In such cases it is possible to derive a simple expression for the energy flux, relating it to the local temperature gradient. This result is called the Rosseland approximation.

Rosseland Approximation

Summary of Radiation Properties

	Thermal	Blackbody	Bremsstrahlung	Synchrotron	Inverse Compton
Optically thick	–	YES			
Maxwellian distribution of velocities	YES	YES			
Relativistic speeds	–	–			
Main Properties	Matter in thermal equilibrium	Matter AND radiation in thermal equilibrium			

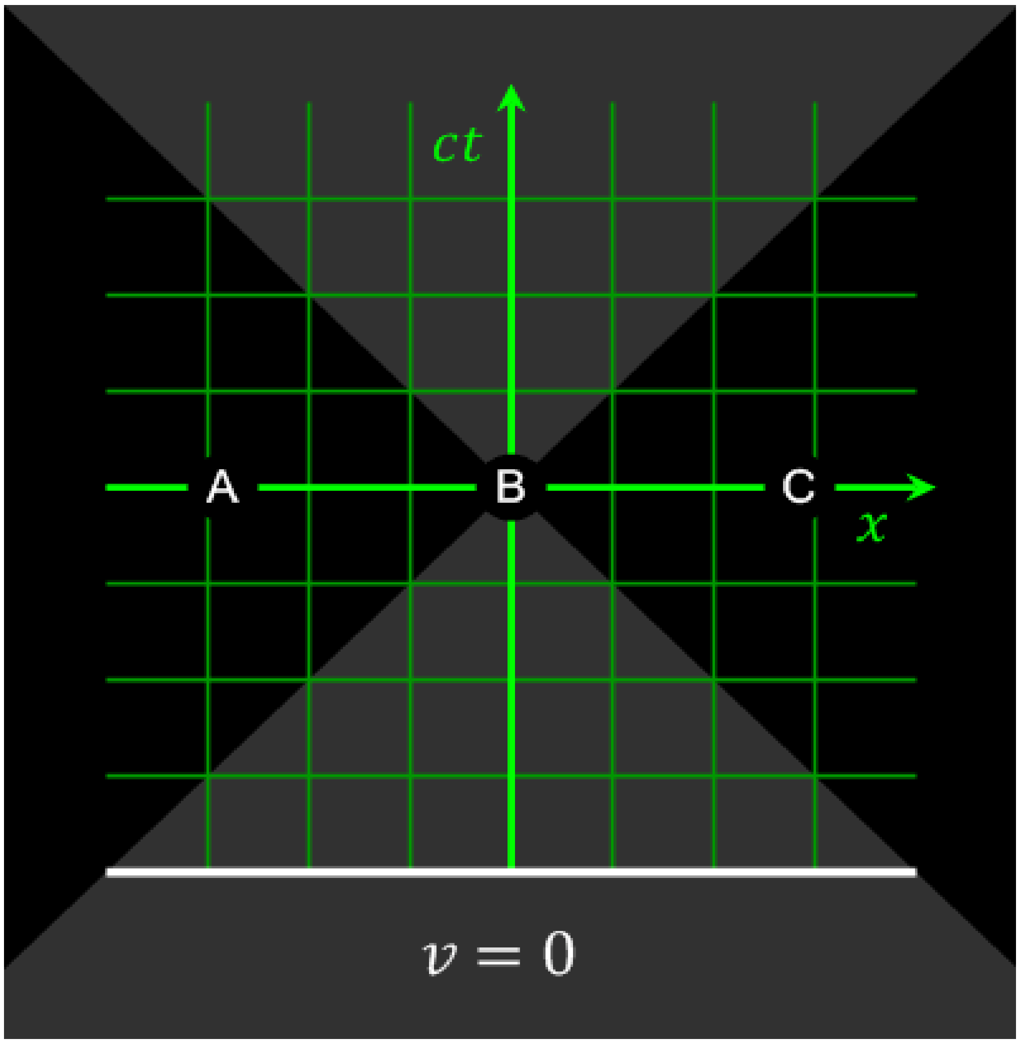
Rule of thumb: Blackbody is always thermal, but thermal radiation is not always blackbody

Blackbody → **thermal** radiation

What We Have Seen So Far

Topic	Desciption
Some definitions (specific brightness, flux, energy density, radiation pressure, etc...).	
Constancy of Specific Intensity in Free Space	Brightness does not depend on distance (in free space)
The Radiative Transfer Equation	How specific intensity varies with absorption, emission and scattering. Optical depth. Some simple analytical solution
Thermal Radiation: blackbody radiation and Kirchhoff's law	Thermal radiation emerges when the matter is in thermal equilibrium. Blackbody radiation emerges when matter AND radiation are in thermal equilibrium.
Stefan-Boltzmann law and entropy of blackbody radiation	Blackbody is the maximum entropy radiation. Stefan-Boltzmann law can be derived from simple thermodynamics
Properties of Blackbody Radiation	Wien law, Rayleigh-Jeans law, perfect emitter
Local Thermodynamic Equilibrium and Rosseland Mean Opacity	Real bodies are never blackbodies, but many systems are very good approximations since there is LTE.

Relativity of Simultaneity



$$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$x = \gamma\left(x' + vt'\right)$$

$$y = y'$$

$$z = z'$$

$$t = \gamma\left(t' + vx'/c^2\right)$$

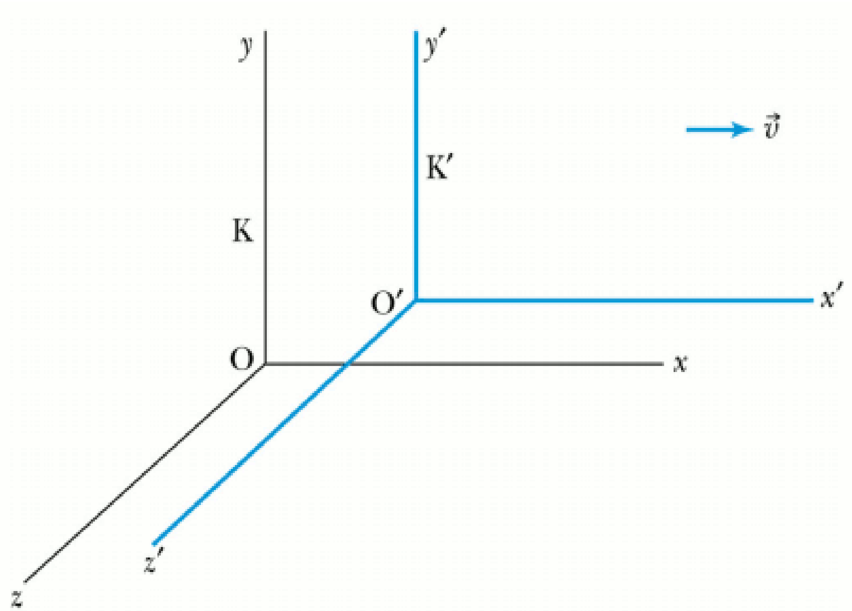
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

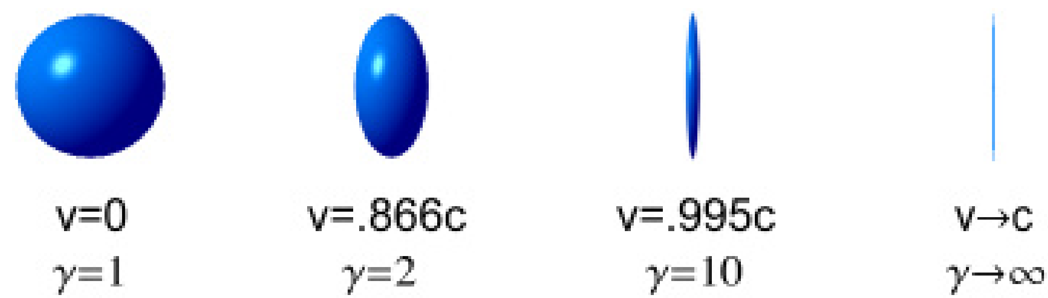
$$x' = \gamma\left(x - vt\right)$$

$$y' = y$$

$$z' = z$$

Ruler:Measuring a bar





Lorentz-Fitzgerald Contraction

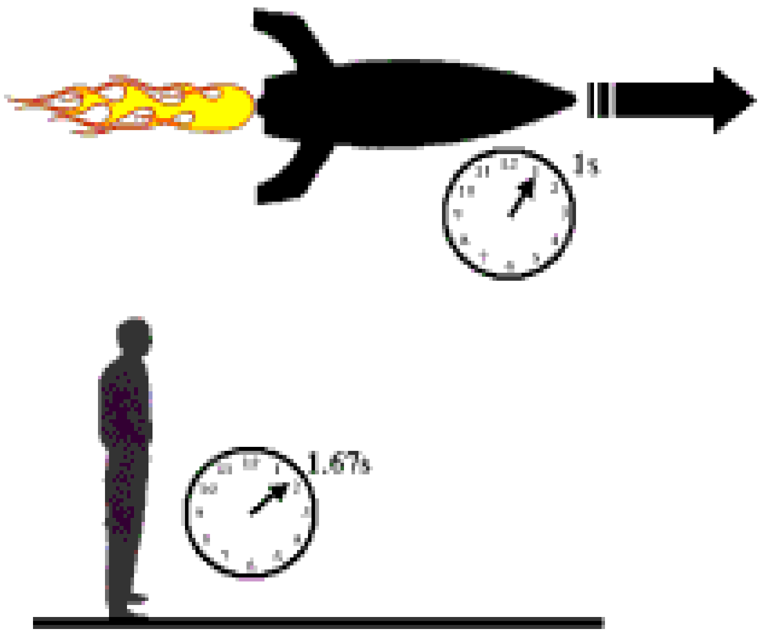
$$L = L'\sqrt{1-\frac{v^2}{c^2}} = \frac{L'}{\Gamma}$$

L’ = length of object in K’
L = length of object in K

Note: the two observers in K and K' would measure the same effect with respect to each other. How is that possible?

Solution: Lorentz transformation of time is **NOT** Lorentz invariant since it depends also on space. Therefore temporal simultaneity is **NOT** Lorentz invariant. Therefore each observer does not see the other carrying the measurement of the two ends of the stick at the *same time*.

Clocks: Time Intervals



Time dilation effect

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}} = \Delta t' \Gamma$$

Time in the lab frame flows faster than in the moving frame.

Same story here: both observers will see the each other's clock slowing down.
Each would object that the clocks used by the other to measure the time interval were not synchronized.

Observability of Lorentz contraction and Time dilation

Question for you: Lorentz contraction and time dilation assume that you are carrying your measurements with rods and clocks, i.e., you can carry the measurement “in place”.

But what happens when you use photons?
This is the situation we encounter in astronomy, basically all information is carried by photons and we make measurements by collecting photons on a detector (either by taking a picture or by recording the photons’ time of arrivals).

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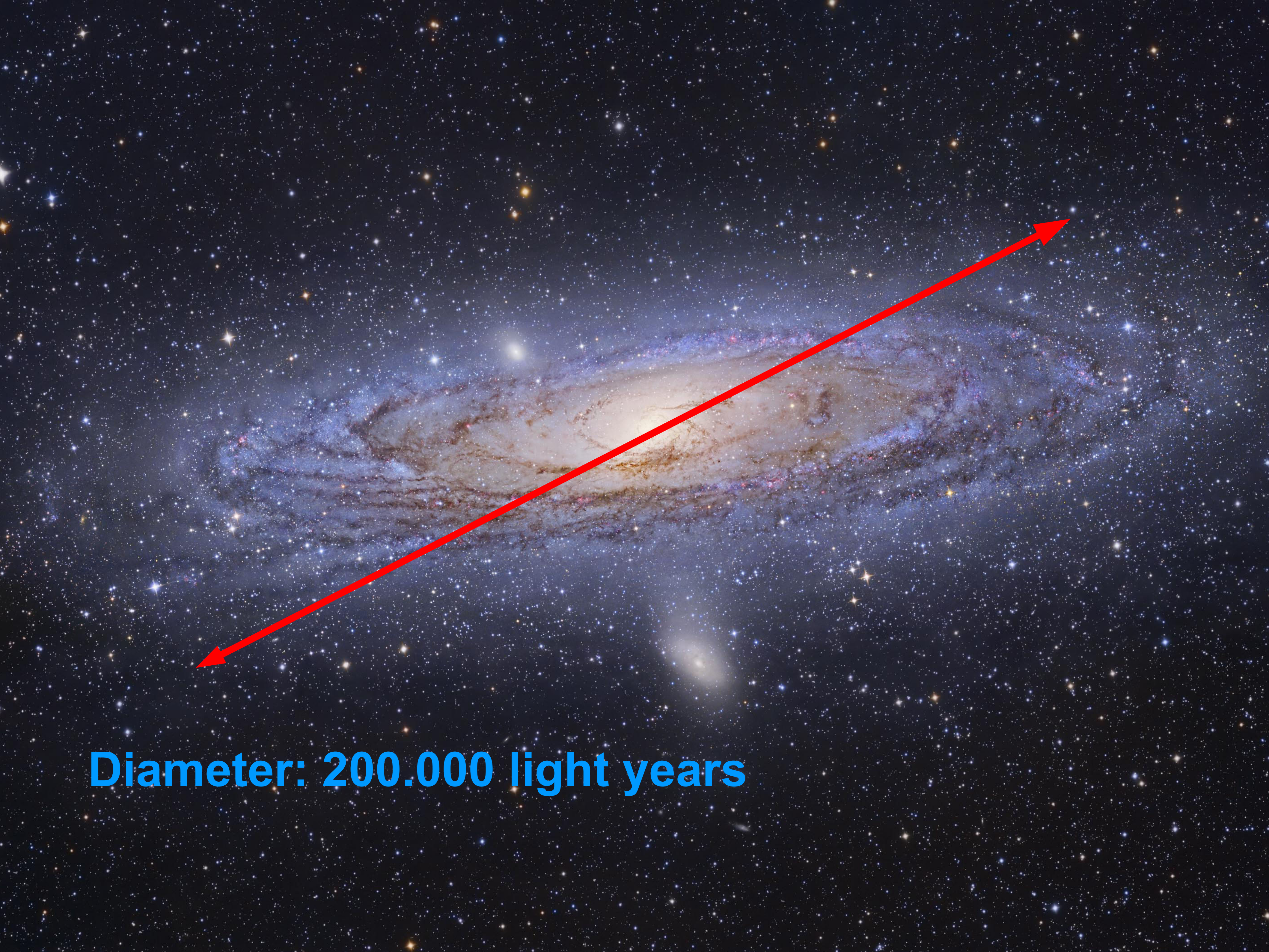
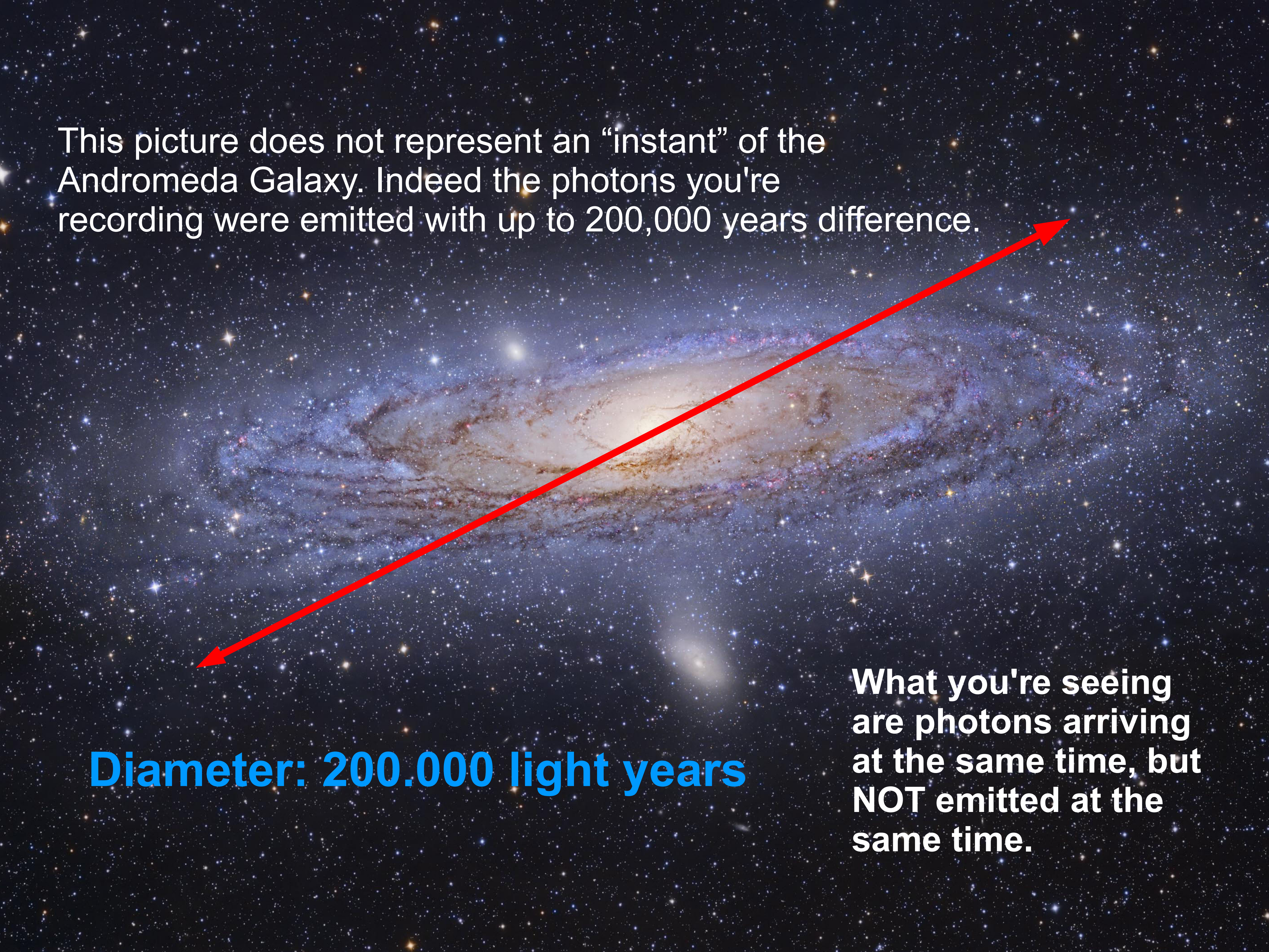
We will see now that the fact that we use photons instead of rulers and clocks “in place” changes completely the effect observed. This doesn’t mean of course that the Lorentz contraction and/or time dilation do not occur. It means simply that the finite propagation speed of light introduces distortions in the effect measured when using photons.


Diameter: 200.000 light years


This picture does not represent an “instant” of the Andromeda Galaxy. Indeed the photons you're recording were emitted with up to 200,000 years difference.
Diameter: 200.000 light years
What you're seeing are photons arriving at the same time, but NOT emitted at the same time.

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Invisibility of the Lorentz Contraction*

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It is shown that, if the apparent directions of objects are plotted as points on a sphere surrounding the observer, the Lorentz transformation corresponds to a conformal transformation on the surface of this sphere. Thus, for sufficiently small subtended solid angle, an object will appear—optically—the same shape to all observers. A sphere will photograph with precisely the same circular outline whether stationary or in motion with respect to the camera. An object of less symmetry than a sphere, such as a meter stick, will appear, when in rapid motion with respect to an observer, to have undergone rotation, not contraction. The extent of this rotation is given by the aberration angle $(\theta-\theta')$, in which θ is the angle at which the object is seen by the observer and θ' is the angle at which the object would be seen by another observer at the same point stationary with respect to the object. Observers photographing the meter stick simultaneously from the same position will obtain precisely the same picture, except for a change in scale given by the Doppler shift ratio, irrespective of their velocity relative to the meter stick. Even if methods of measuring distance, such as stereoscopic photography, are used, the Lorentz contraction will not be visible, although correction for the finite velocity of light will reveal it to be present.

INTRODUCTION

EVER since Einstein presented his special theory of relativity[1] in 1905 there seems to have been a general belief that the Lorentz contraction should be visible to the eye. Indeed, Lorentz stated[2] in 1922 that the contraction could be photographed. Similar statements appear in other references too numerous to be mentioned, and even Einstein's first paper leaves the impression,[3] perhaps unintentionally, that the contraction due to relativistic motion should be visible. The usual statement is that moving objects "appear contracted," which is somewhat ambiguous. The special theory predicts that the contraction can be observed by a suitable experiment, and the words "observe" and "see" seem to be used interchangeably in this connection.

There is, however, a clear distinction between observing and seeing. An observation of the shape of a fast-moving object involves simultaneous measurement of the position of a number of points on the object. If done by means of light, all the quanta should leave the surface simultaneously, as determined in the observer's system, but will arrive at the observer's position at different times. Similar restrictions would apply to the use of radar as an observational method. In such observations the data received must be corrected for the finite velocity of light, using measured distances to various points of the moving object. In seeing the object, on the other hand, or photographing it, all the light quanta arrive simultaneously at the eye (or shutter), having departed from the object at various earlier times. Clearly this should make a difference between the contracted shape which is in principle observable and the actual visual appearance of a fast-moving object.

CONFORMALITY OF ABERRATION

The basic question of the visibility of the Lorentz contraction may be stated as that of the appearance of a rapidly moving object in an instantaneous photograph. The object, of known shape when at rest, is assumed to have a high uniform speed relative to the camera. The camera is assumed to be at rest in a Galilean (unaccelerated) frame of reference. Of course it would make no difference if the camera were, instead, considered to move at high speed past the stationary object, but the photograph produced must be examined at rest, so it is simpler to consider the camera as

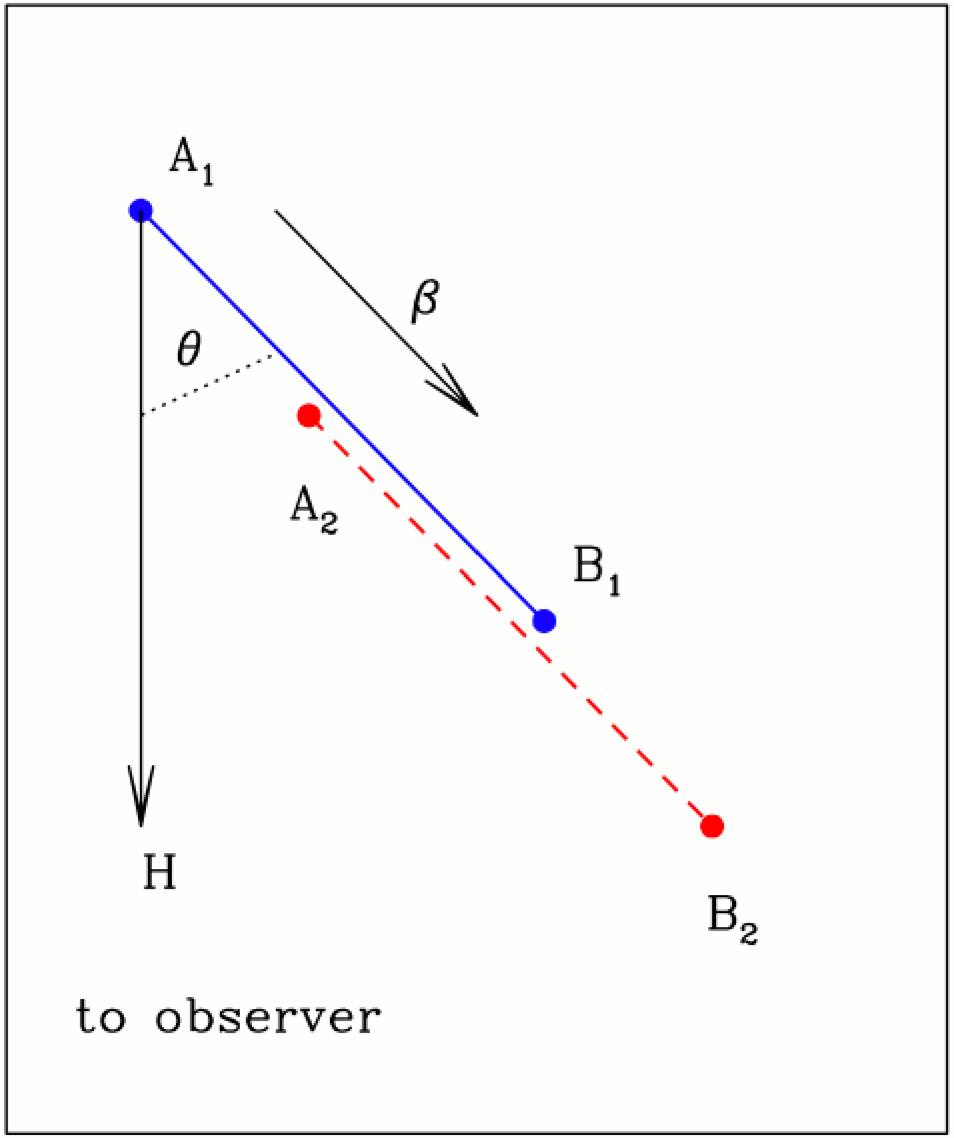
J. Terrell: "*Invisibility of the Lorentz Contraction"*

R. Penrose: "*The Apparent Shape of a Relativistically Moving Sphere*"

These papers were published in 1959. The effects of the finite speed of light seem maybe obvious to you, but that's how long it took to realize this (special relativity was first published in 1905…).

Actually A. Lampa (Austrian) realized this earlier, in 1924. But for some reasons his work was mostly ignored.
A. Lampa: "*"Wie erscheint nach der Relativitätstheorie ein bewegter Stab einem ruhenden Beobachter?"*

Photons: Measuring a bar



$$A_1 B_1 = L$$

If you use a "ruler" you will see $L = \frac{L'}{\Gamma}$

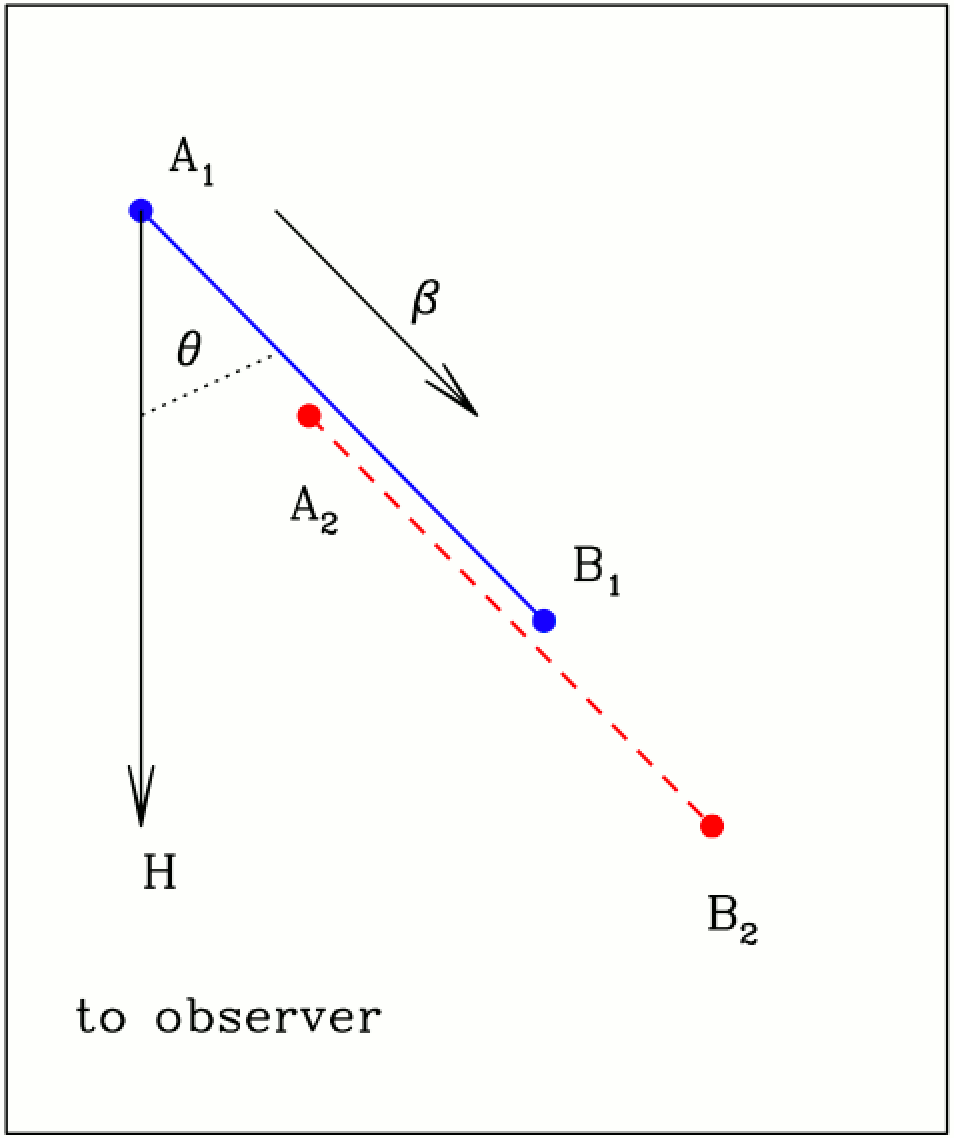
However, if you take a picture (i.e., you use photons) you will see something very different.

Call H the point reached by the photon emitted in A1 after a time Δt , such that the extreme B1 of the rod will be at B2 (i.e., H and B2 have the same distance to the observer).

A photon emitted in H and B2 will thus reach the observer simultaneously (i.e., create the "picture" on the camera). The length A1B1 is thus measured as HB2. Is HB2 = L'/Γ ?

No!

Photons: Measuring a bar



$$A_1 B_1 = L$$

$$B_1 B_2 = \beta c \Delta t$$

$$A_1 H = c \Delta t$$

With a bit of algebra one can find:

$$A_1 H = A_1 B_2 \cos\theta = \frac{L' \cos\theta}{c \Gamma (1 - \beta\cos\theta)}$$

Then:

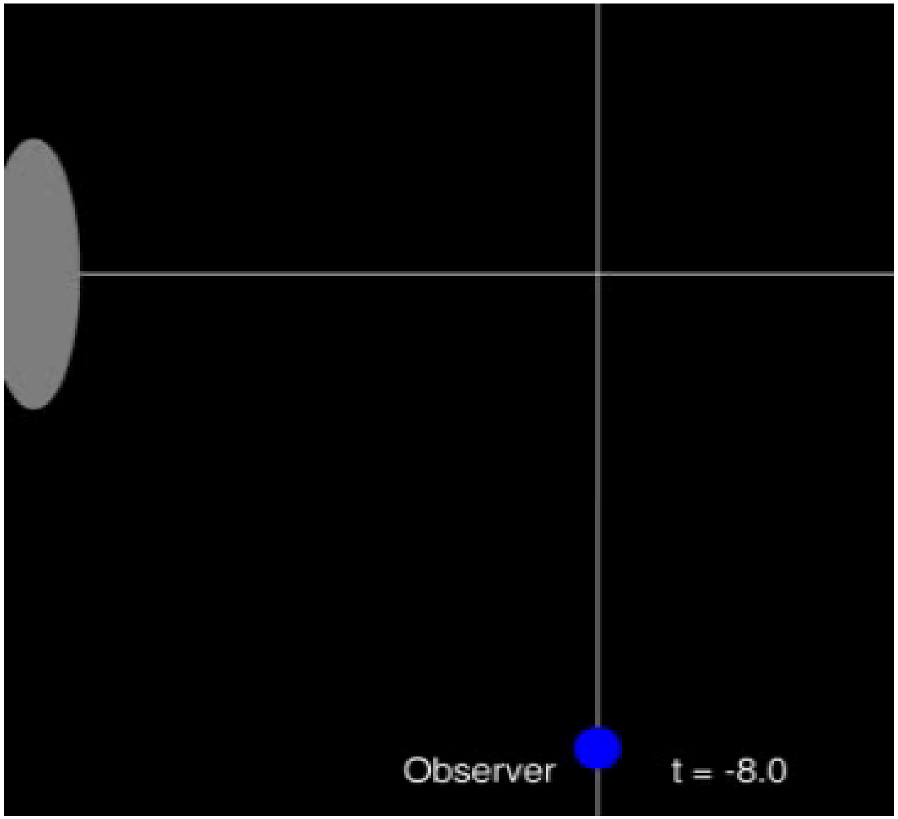
$$A_1 B_2 = \frac{A_1 H}{\cos\theta} = \frac{L'}{\Gamma(1-\beta\cos\theta)} = \delta L'$$

$$\Lambda = H B_2 = A_1 B_2 \sin\theta = L' \delta \sin\theta$$

Note the difference between the ruler and photon case

Measured with a "ruler"

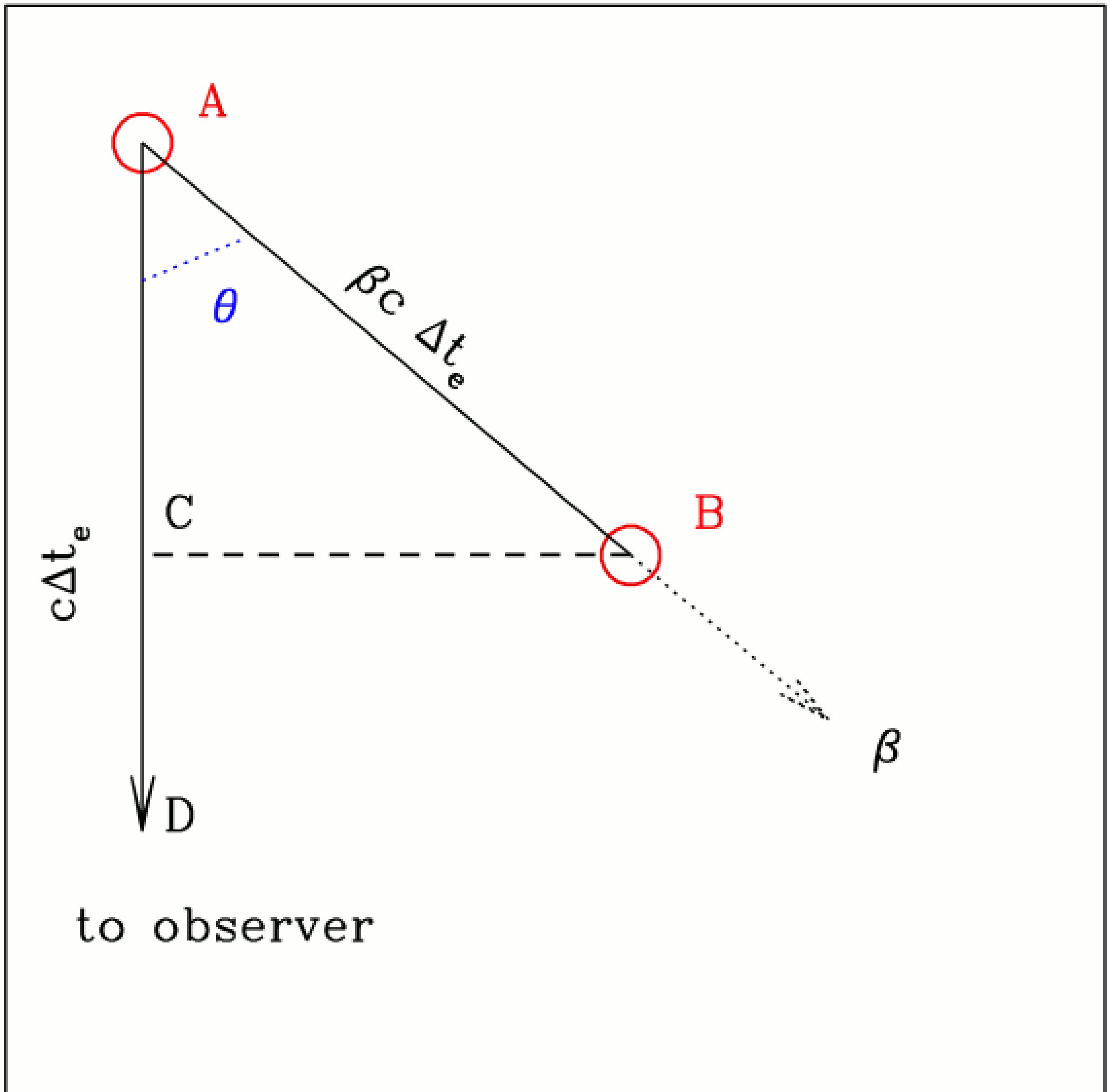
Measured with "photons"



A sphere would be contracted along the direction of motion (Lorentz contraction)

However, due to the finite speed of propagation of light, photons arriving ***simultaneously*** at the observer will still produce a spherical object with no visible Lorentz contraction.

Photons: Time Intervals



We show now that the time dilation effect is completely reversed when you do your measurements with photons instead of using clocks "in place".
From Lorentz transformations we would expect a time dilation.

However, we will show now that, when using photons, we have a **time contraction**:

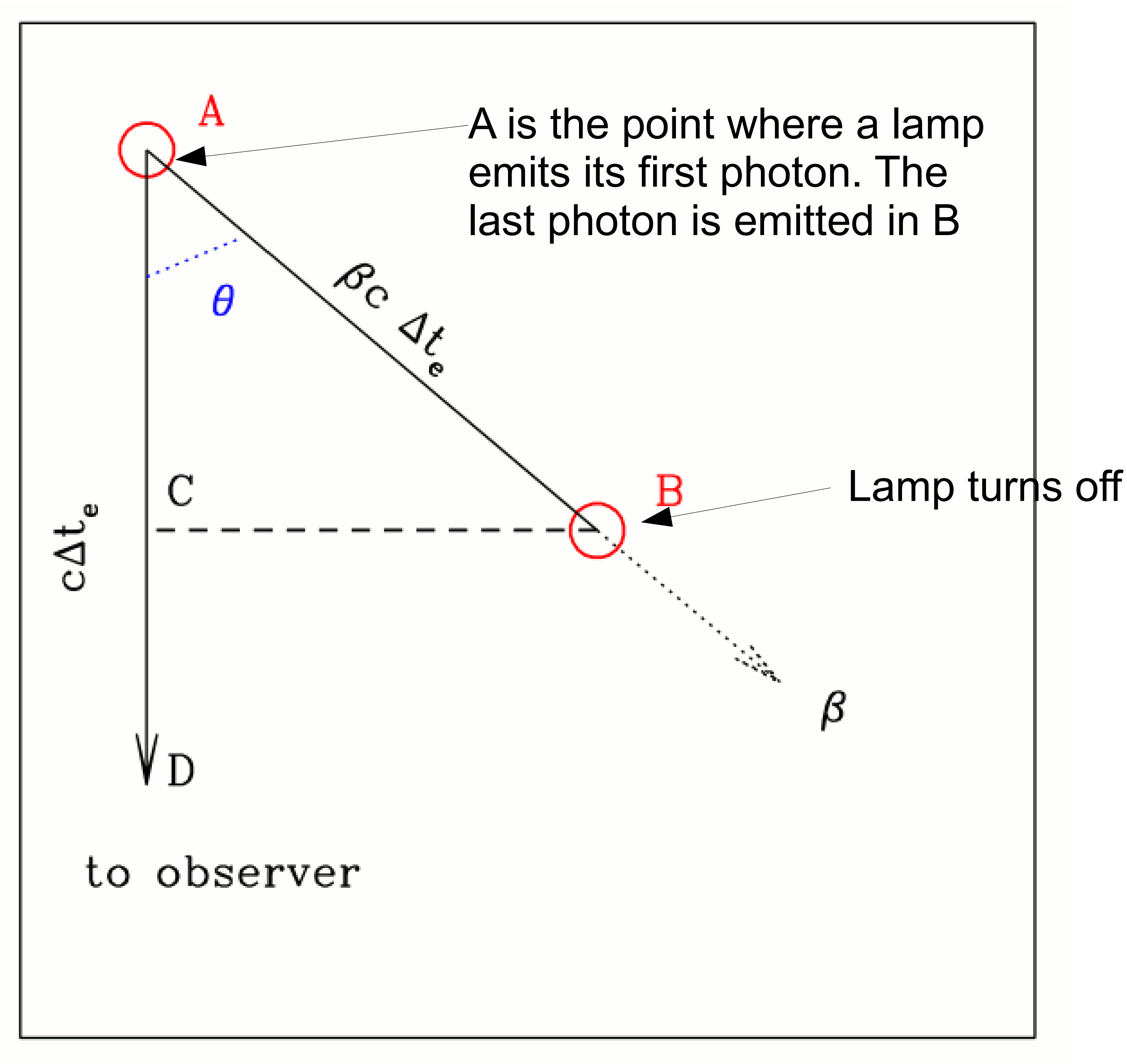
$$\Delta t_a = \frac{\Delta t_e'}{\delta}$$

Here the subscript "a" and "e" refer to the "arrival" and emission time.
Note that:

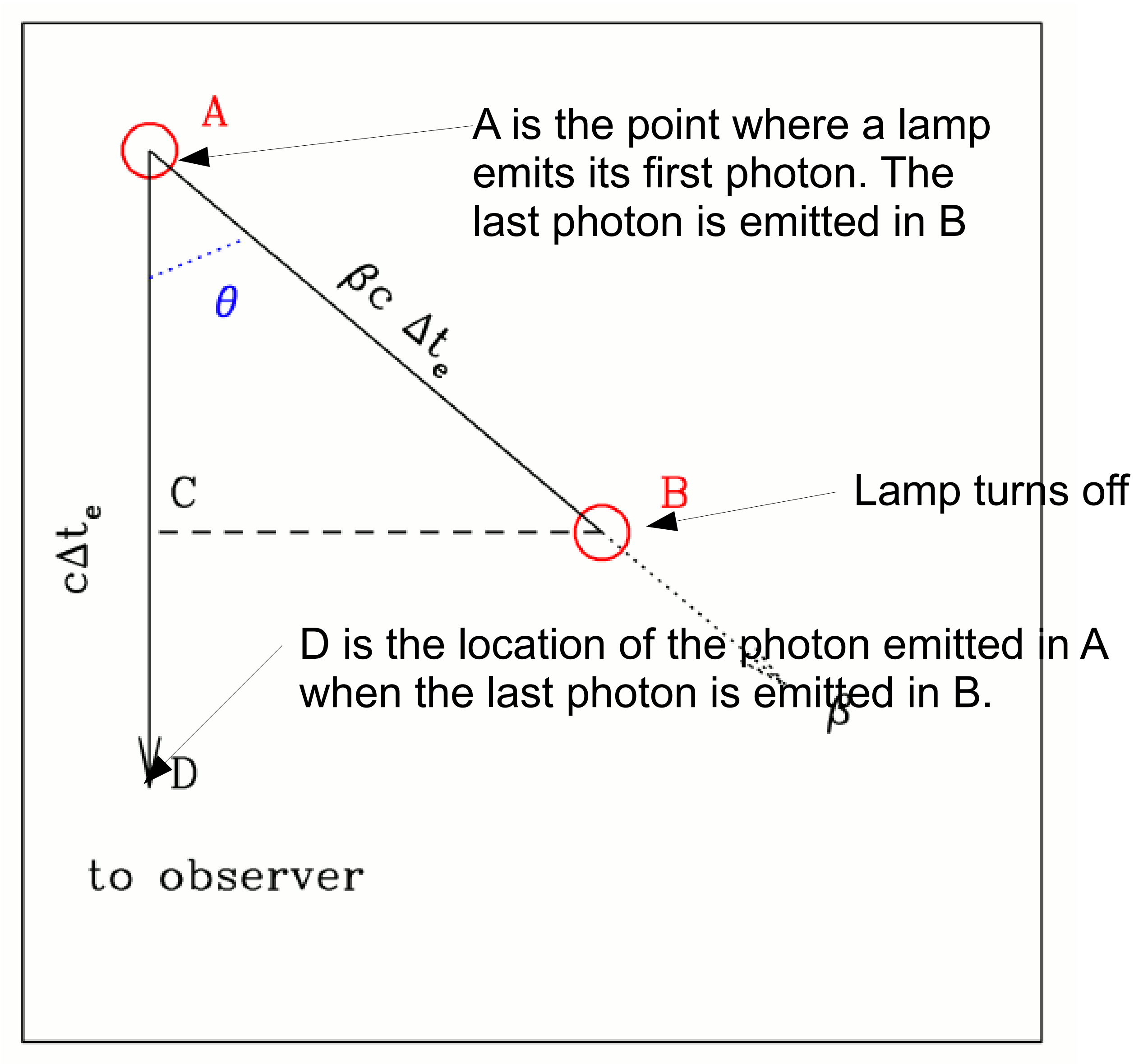
$$\Delta t_e = \Delta t_e' \Gamma$$

is not measurable (with photons) but we measure Δt_a. Again, this is just an effect due to the use of photons and the fact that they propagate with a finite speed.

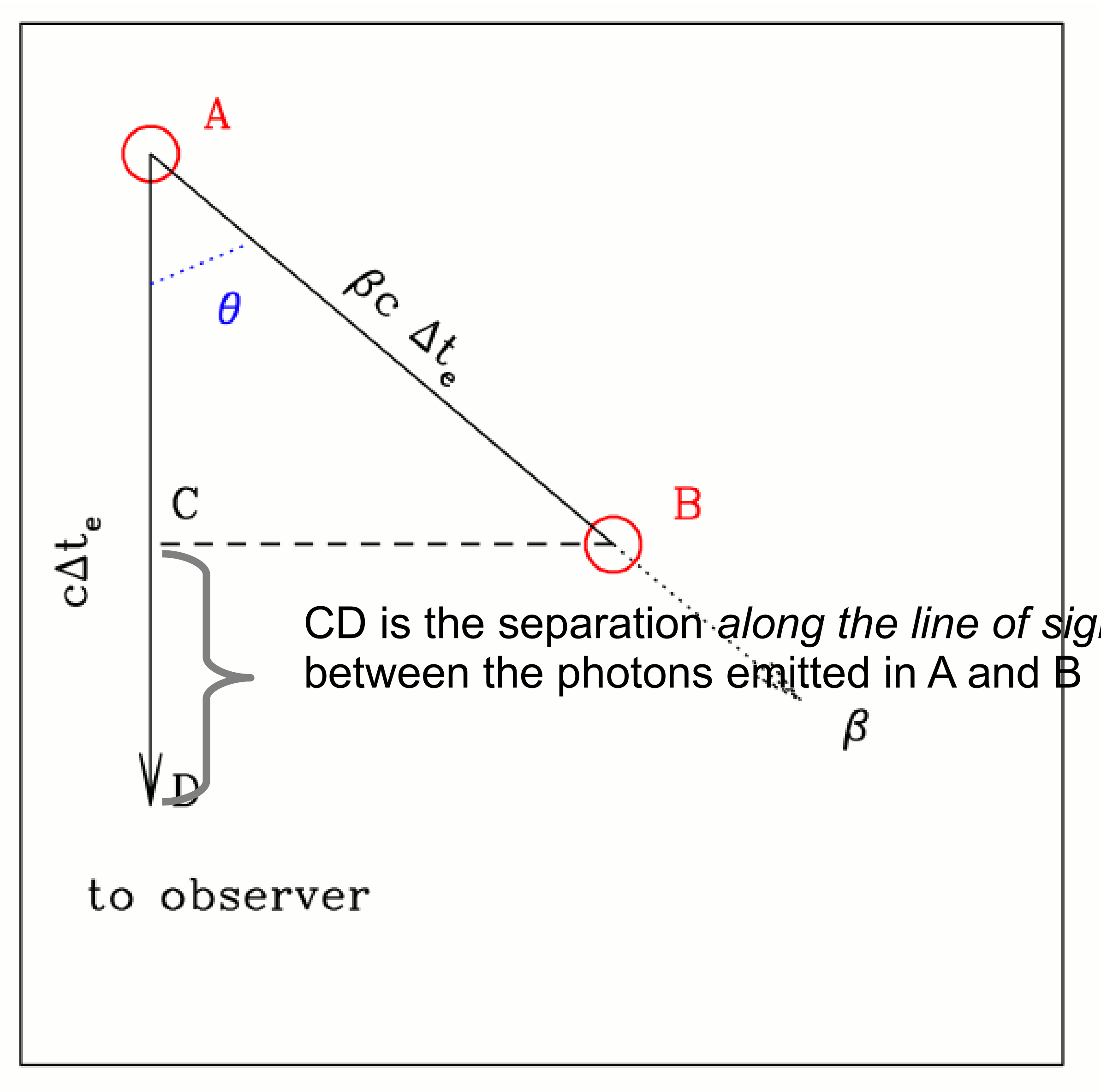
Photons: Time Intervals



Photons: Time Intervals

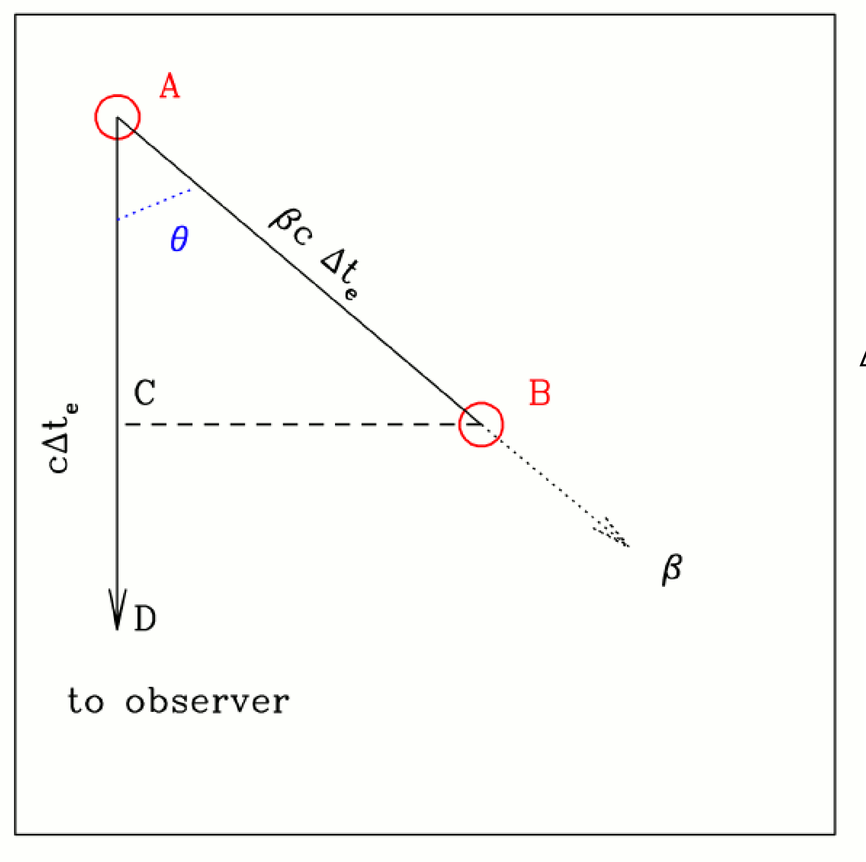


Photons: Time Intervals



CD is the separation *along the line of sight* between the photons emitted in A and B

Photons: Time Intervals



An observer will thus measure the interval CD/c, where c is the speed of light.

$$\Delta t_a = \frac{CD}{c} = \frac{AD - AC}{c} = \Delta t_e - \beta \Delta t_e \cos\theta = \Delta t_e (1 - \beta \cos\theta)$$

$$\Delta t_e (1 - \beta \cos\theta) = \Gamma \Delta t'_e (1 - \beta \cos\theta) = \frac{\Delta t'_e}{\delta}$$

Remember that delta is:

$$\delta = \frac{1}{\Gamma (1 - \beta \cos\theta)}$$