Gamma Ray Burst Spectrum: Slow Cooling

Exercise done during Lecture 8

Exercise

A Gamma-ray burst produces the most relativistic jets we know of, with a bulk Lorentz factor $\Gamma = 100$ (the Lorentz factor corresponding to the overall motion of the jet). As the jet propagates into the interstellar medium, it continuously accelerates electrons to relativistic speeds with a power-law distribution:

$$N(\gamma)d\gamma = N_0\gamma^{-p}d\gamma, \tag{1}$$

where $N(\gamma)d\gamma$ represents the number density of electrons with $p > 2$. When a Gamma Ray Burst explodes at a distance D from us, at a certain time t (observer's frame) we measure a synchrotron spectrum with the following shape (see Figure 1):

$$F_\nu = \begin{cases} \nu^2 & \nu < \nu_t \\ \nu^{1/3} & \nu_t < \nu < \nu_m \\ \nu^{-0.75} & \nu_m < \nu < \nu_c \\ \nu^{-1.25} & \nu > \nu_c \end{cases} \tag{2}$$

Here the subscript t refers to the *transition* frequency (or *absorption* frequency) in the synchrotron spectrum that marks the self-absorbed part of the spectrum. The frequency ν_c is the cooling frequency, above which the electrons have already lost a significant amount of energy. The frequency ν_m instead corresponds to the peak of the spectrum

- (a) What is the value of the electron power-law index p?
- (b) Derive expressions for the break frequencies and explain which parts of the spectrum represent the different power laws.
- (c) Can you explain why below ν_m the power-law has a slope of $1/3$?
- (d) Can you explain why below ν_t the power-law has a slope of 2 instead of $5/2$?

Solution

(a) The value of p will be 5/2[1]. This can be derived from the spectral index between ν_m and ν_c which corresponds to $-(p-1)/2$.

(b) We have that ν_t is the absorption frequency, i.e. below this value the source is optically thick. In the case of ν_m, it corresponds to the peak of the spectrum and it is given

[1] it is just a coincidence that p=5/2, do not confuse this number with the absorption power law coefficient s=5/2 that we have seen in class for the self-absorbed part of a synchrotron spectrum.

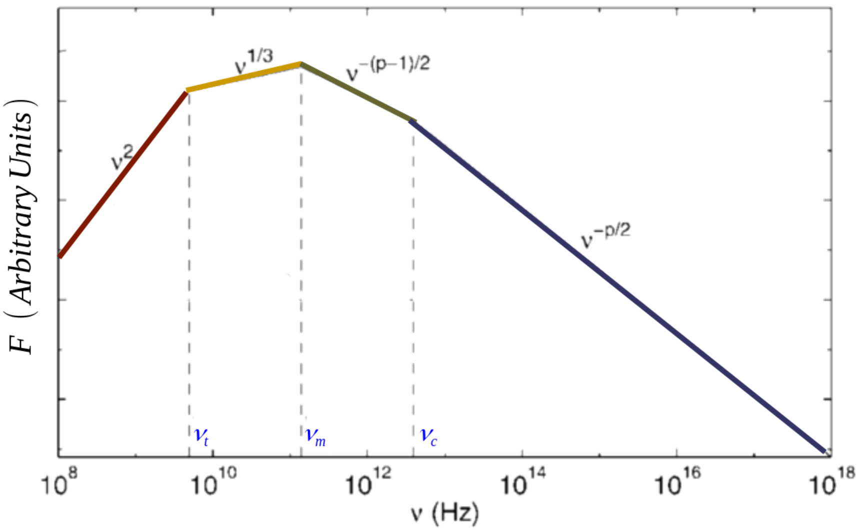

Figure 1. The spectrum of the GRB described in the text.

by $4\gamma^2\nu_L/3$, where ν_L is the Larmor frequency. For ν_c, we have the cooling frequency, corresponding to electrons that have already cooled down by time t. Although we haven't derived this explicitly in class, this can be calculated with the cooling time, i.e., by dividing the electron energy with the synchrotron power $P_{\rm synch}$ (and so have energy/power = energy/energy/time = time), and deriving the γ corresponding to electrons already cooled. This gives us:

$$\gamma_c = \frac{9m^3c^5}{4e^4B^2t} \tag{3}$$

and from this we have $\nu_c = 4\gamma_c^2\nu_L/3$. Note that for frequencies larger than ν_c the power law distribution index changes from p to p + 1 and thus we can see that the spectral index will become −5/4.

(c) Since the electrons have a power-law energy distribution, there will be a minimum and maximum energy (or Lorentz factor γ if you prefer). If the transition frequency ν_t is lager than the minimum frequency ν_m correspondings to the minimum Lorentz factor γ_m then the spectrum will be as described above. However, what happens if $\nu_t < \nu_m$? All frequencies between ν_m and ν_c will have the usual power law slope $s = (p-1)/2$. Below ν_m the spectrum continues with a slope of 1/3 down to ν_t . This comes from the single particle synchrotron spectrum (which has a slope of 1/3 below its peak frequency). Indeed this part of the spectrum will be dominated by the tail of the emission of those electrons with $\gamma = \gamma_m$. This continues until we reach the self-absorbed part at ν_t.

(d) Below ν_t we recover the Rayleigh-Jeans limit ($I_\nu \propto \nu^2$) because the electrons behave as a single-temperature medium. Why? Take the initial power law of electrons and let's associate an effective temperature to each electron via: $\gamma mc^2 = 3k_BT_e$ where m is the mass of the electron, T_e its effective temperature and c and k_B are the speed of light and Boltzmann constant, respectively[2]. Now, this means that each electron in the power law distribution, each with a different γ, can be thought to have a certain effective tempearture T_e. Here "effective" means that the whole ensemble of electrons does not have a single temperature like in a thermal distribution, but the electrons can be thought as each having a certain temperature. This temperature, T_e, is the one they would have in a thermal distribution so that their thermal energy would be equal to their γmc^2. In other words you can decompose the power-law into multiple Maxwellians each with a different temperature $T_{e,1}$, $T_{e,2}$,..., $T_{e,N}$. However, when you reach the minimum energy γ_m, those electrons would all have a single effective temperature $T_{e,min}$ and thus when the radiation becomes self-absorbed you see a blackbody. Note that this will also happen at very low frequencies when $\nu_m < \nu_t$ (i.e., you'll have the usual self-absorbed part with $\nu^{5/2}$ from ν_t down to ν_m and ν^2 below ν_m).

[2]Note that the equipartition theorem for relativistic particles says that the average energy of the particle is $3k_BT$, not $3/2k_BT$ as in the non-relativistic case. This is, however, irrelevant for the problem under consideration.