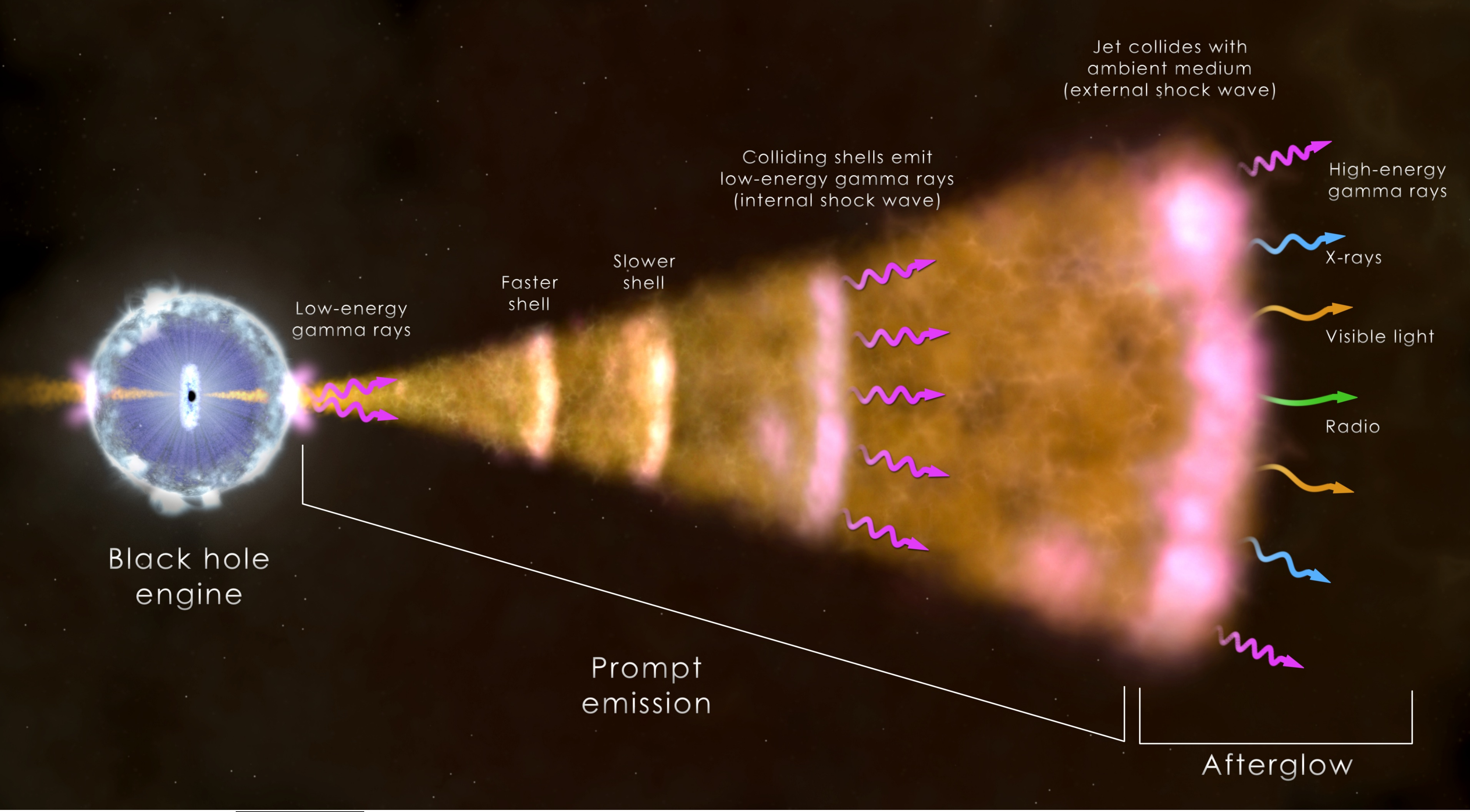
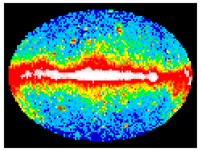
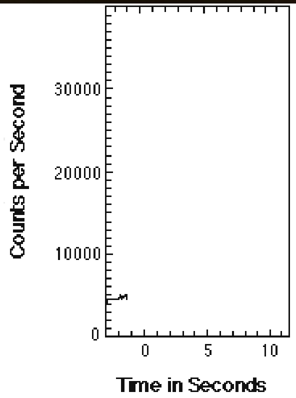




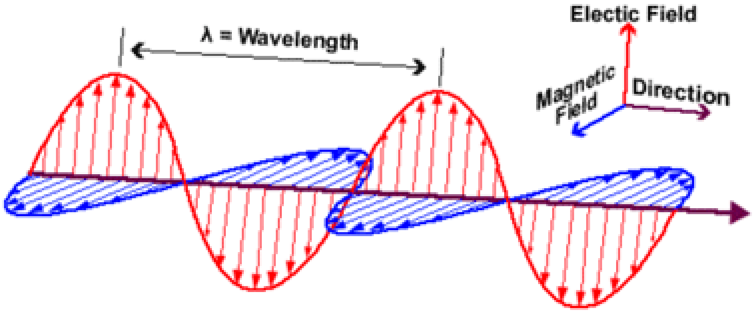

# Compton & Inverse Compton Scattering

# Thomson Scattering

Thomson Scattering is a process by which an electromagnetic wave is scattered in to random directions by a free electron. It is applicable when:

1. $h\nu \ll mc^2$ , with m the electron mass and
2. in the non-relativistic regime (v << c)

First of all, let's consider a linearly polarized e.m. wave incident on a free electron.




*Linearly polarized wave*

*Which forces act on the electron as the e.m. interacts with it?*

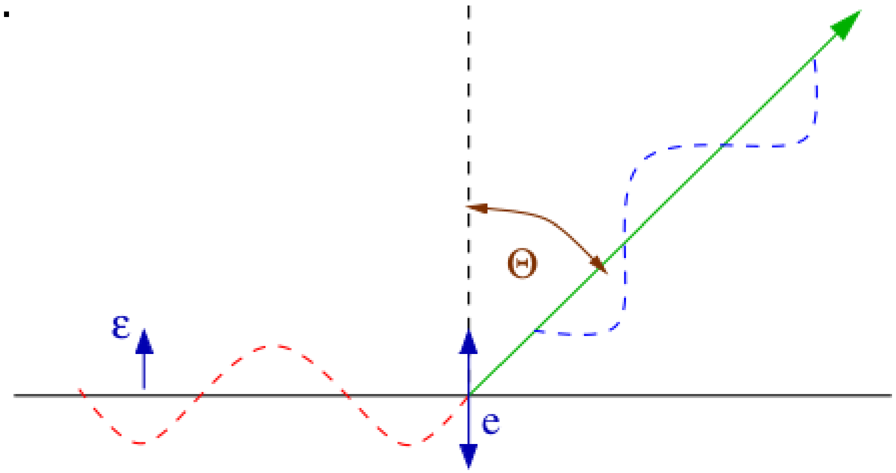
If we are in the non-relativisic limit, then v<<c and thus we can neglect the Lorentz force. (Remember that an e.m. wave has |E| = |B|)

We are left with the electric force:

$$F = e\,\boldsymbol{\epsilon}\,E_0 \sin(\omega_0 t)$$ *force of a linearly polarized wave acting on an electron*

Here $\boldsymbol{\epsilon}$ defines the direction of the **E** field.
We can rewrite this force as:

$$m\ddot{\mathbf{r}} = e\boldsymbol{\epsilon}E_0 \sin\omega_0 t$$




Let's recall the definition of a dipole moment from Lecture 5: $\boldsymbol{d} = e\,\boldsymbol{r}$

Therefore:

$$\ddot{\mathbf{d}} = \frac{e^2 E_0}{m}\boldsymbol{\epsilon}\sin\omega_0 t,$$

And: $$\mathbf{d} = -\left(\frac{e^2 E_0}{m\omega_0^2}\right)\boldsymbol{\epsilon}\sin\omega_0 t$$

Now remember what we said in Lecture 4 about the Dipole Approximation:

# Dipole Approximation

Following the same procedure as for the single particle case, we can find the total power emitted by an ensemble of particles (in the **non-relativistic** limit) in the so-called *dipole approximation:*

$$\frac{dP}{d\Omega} = \frac{\ddot{\mathbf{d}}^2}{4\pi c^3} \sin^2\Theta$$

$$P = \frac{2\ddot{\mathbf{d}}^2}{3c^3}.$$

*We can therefore calculate the time average power from* $\ddot{\mathbf{d}} = \frac{e^2 E_0}{m} \boldsymbol{\epsilon} \sin \omega_0 t,$

*(and remembering that the <sin^2(x)> = 1/2)*

This gives:

$$\begin{cases} \dfrac{dP}{d\Omega} = \dfrac{e^4 E_0^2}{8\pi m^2 c^3} \sin^2\Theta \\ P = \dfrac{e^4 E_0^2}{3 m^2 c^3}, \end{cases}$$

Remembering that the time averaged Poynting flux is defined as: $\langle S \rangle = \dfrac{c}{8\pi} E_0^2$

we can write the power as:

$$\frac{dP}{d\Omega} = \langle S \rangle \frac{d\sigma}{d\Omega} = \frac{cE_0^2}{8\pi} \frac{d\sigma}{d\Omega}$$

where we have defined the differential cross section $d\sigma$ for scattering into $d\Omega$ for a polarized electromagnetic wave:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{polarized}} = \frac{e^4}{m^2 c^4} \sin^2\Theta = r_0^2 \sin^2\Theta$$

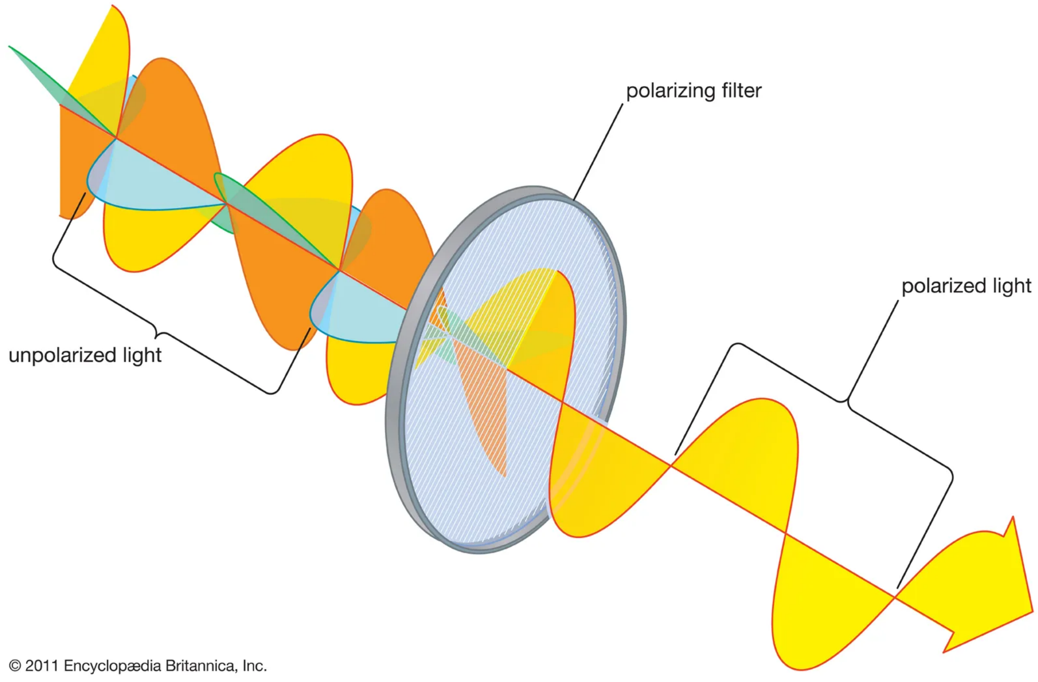
*(classical electron radius)*

To find the total cross section we integrate over the whole solid angle and we get:

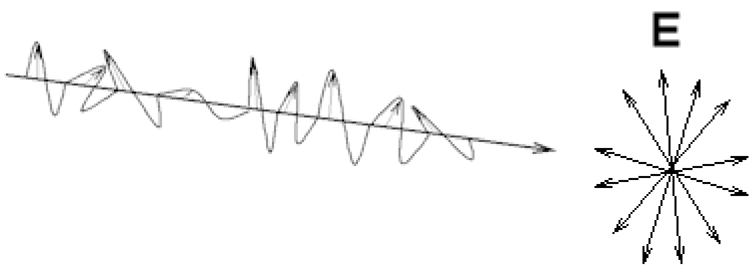
$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = 2\pi r_0^2 \int_{-1}^{1} (1-\mu^2) d\mu. = \frac{8\pi}{3} r_0^2$$

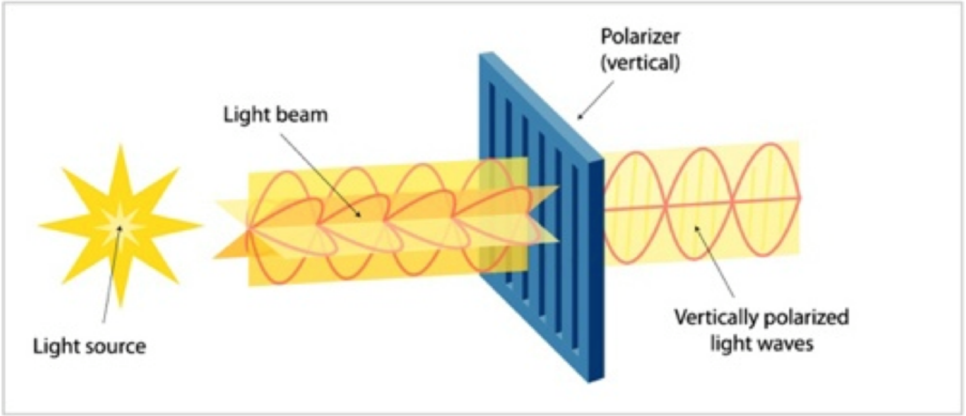
*This one above is the Thomson cross section for an electron and polarized e.m. waves.*
*Note that also the outgoing e.m. wave is polarized (in the plane defined by* $\boldsymbol{\epsilon}$ *and* $\boldsymbol{n}$*)*

What about unpolarized radiation?
The R&L syas that: "unpolarized radiation can be defined as the superposition of two linearly polarized waves *with perpendicular axis".*



*Unpolarized wave*

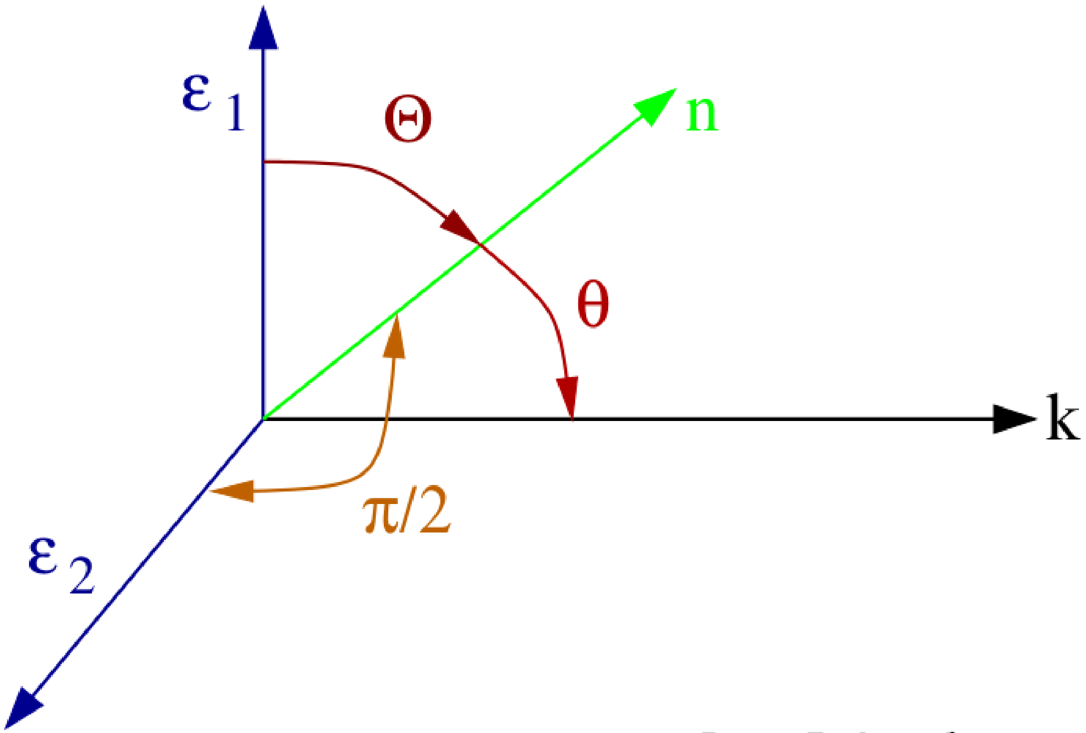



An unpolarized electromagnetic wave traveling in the x-direction is a superposition of many waves. For each of these waves the electric field vector is perpendicular to the x-axis, but the angle it makes with the y-axis is different for different waves. For a polarized electromagnetic wave traveling in the x-direction, the angle the electric field makes with the y-axis is unique. Natural light is, in general, unpolarized. The direction of the electric field changes too quickly to be measured.

In other words: unpolarized radiation is a misnomer, since light in this state is composed by a rapidly varying succession of different polarization states. Perhaps a better name would be *randomly polarized* light.

Mathematically this wave can be represented by two orthogonal linearly polarized waves of equal amplitude varying *incoherently.* The word “incoherent” means that the sinusoidal dependence (coherence) of the electric field is lost.




$$\theta = \pi/2 - \Theta$$

Let's call **k** the direction of propagation before the scattering and **n** the direction after the scattering.
Now choose the two orthogonal waves such that the first wave is along eps1 in the plane **n – k** and the second is along eps2 which is orthogonal to this plane and eps1.

Let $\Theta$ be the angle between $\epsilon_1$ and $\mathbf{n}$

Now the total cross section will be the average of the two polarized states.

*Differential Thomson cross section for unpolarized radiation:*

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{1}{2}\left[\left(\frac{d\sigma(\Theta)}{d\Omega}\right)_{\text{pol}} + \left(\frac{d\sigma(\pi/2)}{d\Omega}\right)_{\text{pol}}\right]$$
$$= \frac{1}{2} r_0^2 (1 + \sin^2\Theta)$$
$$= \frac{1}{2} r_0^2 (1 + \cos^2\theta),$$

What shape is this?

*Remember what we said in lecture 5: the power emitted per unit solid angle by an accelerating charge depends as the square of the sinusoid of the angle between the direction of the acceleration and the direction of propagation of radiation*

*LECTURE 5*

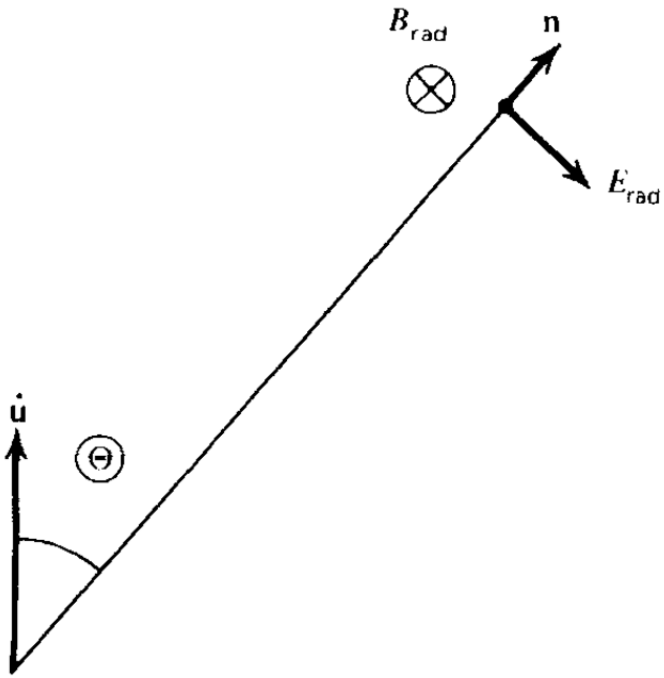
# Larmor’s Formula

What can we say about the radiation field when the velocity is <<c? (non-relativistic case)

In this case beta<<1 and thus we can simplify the electric and magnetic field expressions and obtain:

$$\mathbf{E}_{\mathrm{rad}} = \left[ (q/Rc^2)\mathbf{n} \times (\mathbf{n} \times \dot{\mathbf{u}}) \right]$$

$$\mathbf{B}_{\mathrm{rad}}$$




What is the Poynting vector **S**? (remember that the Poyinting vector defines the direction towards which the energy carried by the em fields is directed. Here **S** is parallel to **n;**
**S** has units of erg/s/cm^2, i.e., energy flux).

Since:

$$|\mathbf{E}_{\mathrm{rad}}| = |\mathbf{B}_{\mathrm{rad}}| = \frac{q\dot{u}}{Rc^2} \sin\Theta$$

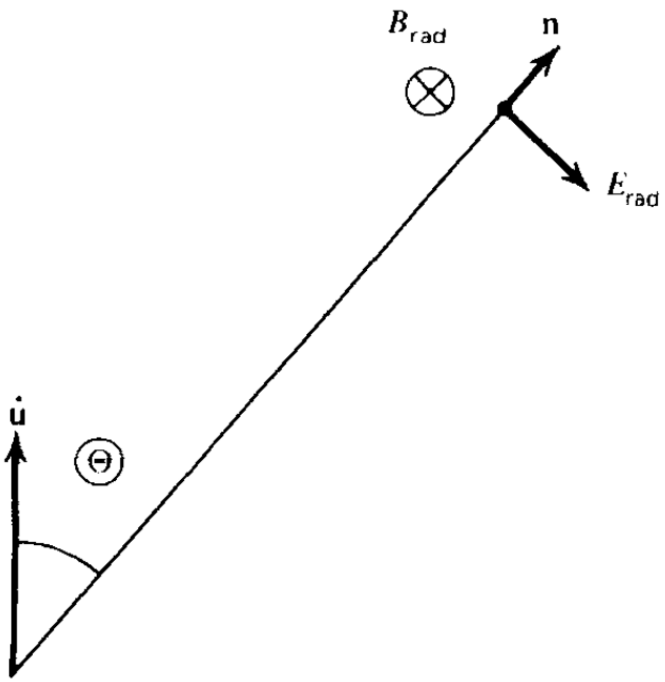
The Poynting vector has magnitude:

$$S = \frac{c}{4\pi} E_{\mathrm{rad}}^2 = \frac{c}{4\pi} \frac{q^2 \dot{u}^2}{R^2 c^4} \sin^2\Theta.$$

**LECTURE 5**

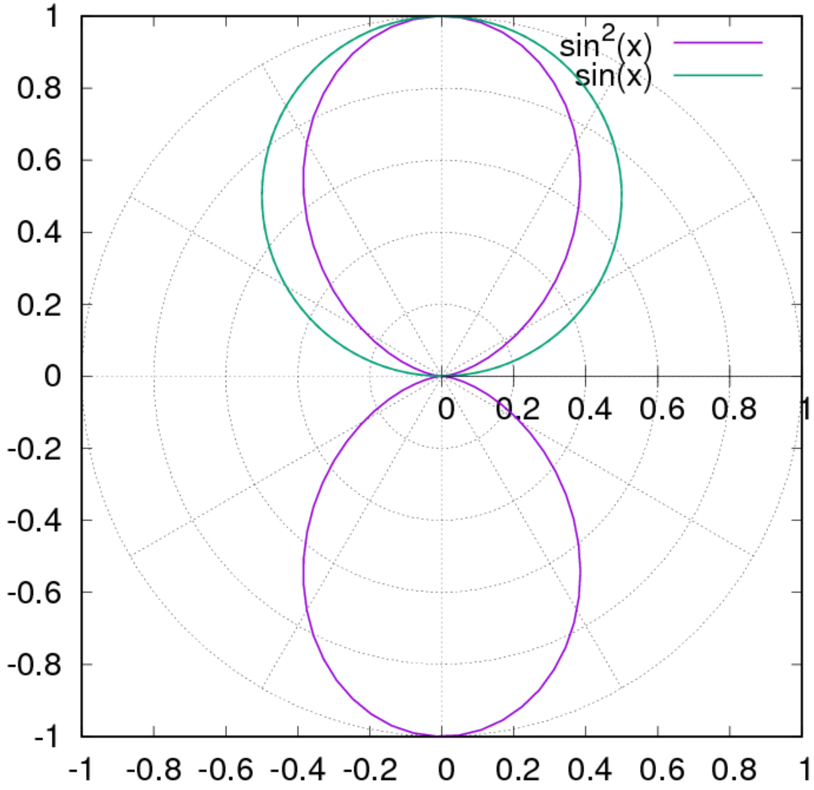
# Larmor's Formula

$$S=\frac{c}{4\pi}E_{\mathrm{rad}}^{2}=\frac{c}{4\pi}\frac{q^{2}\dot{u}^{2}}{R^{2}c^{4}}\sin^{2}\Theta.$$




Note the angle theta!

The energy of the em. field is not isotropic but there is a sin^2 !!

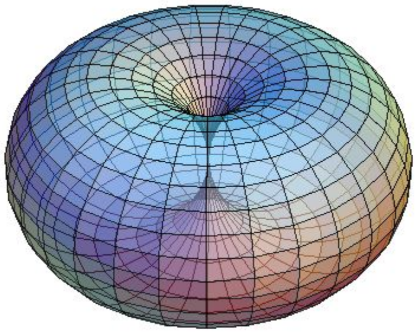


**LECTURE 5**

# Larmor's Formula

Now let's calculate the power in a unit solid angle about **n**. To do this we multiply the Poynting vector (units: erg/s/cm^2) by an area dA (cm^2) to get a power (erg/s). How do we choose dA? We know that the solid angle dOmega = dA/R^2. Therefore:

$$\frac{dW}{dt\, d\Omega} = \frac{q^2 \dot{u}^2}{4\pi c^3} \sin^2\Theta$$



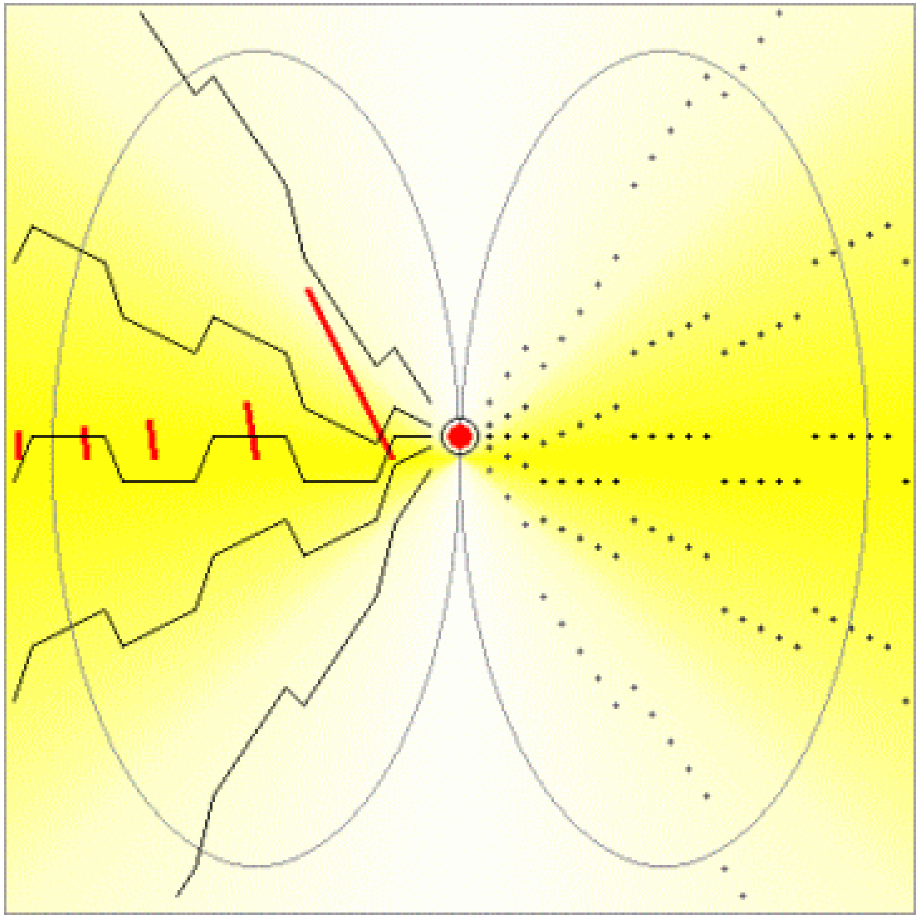
And now we integrate the above expression over the whole solid angle Omega=4*pi and we obtain the total power emitted by an accelerated charge in the non-relativistic approximation:

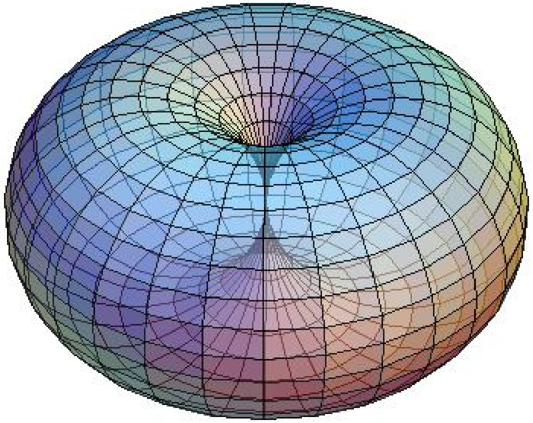
$$P = \frac{2q^2\dot{u}^2}{3c^3}$$

**Larmor's Formula**

**IMPORTANT:** The power emitted is proportional to <u>the square of the charge</u> and <u>the square of the acceleration</u>.

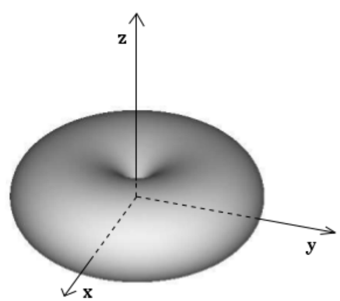
# LECTURE 5

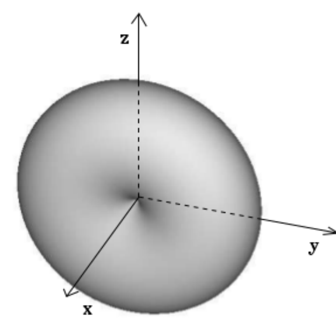




The animation represents a charged particle being switched up and down in a very strong electric field, such that the shape being traced out in time aligns to an approximate square wave. The ovals reference lines drawn to the left and right of the charge correspond to a cross-section through the doughnut toroid, as illustrated in the previous diagram. Based on the criteria of the Larmor formula, when a charge is subject to acceleration, i.e. during the transition positions, it radiates power also subject to the angle θ with respect to the axis of charge motion. As such, the energy density is reflected by the depth of the yellow shading, symmetrical about the axis of motion. However, the intention of left-right sides of the animation is to be somewhat illustrative of wave-particle duality in that the left reflects the electric field lines, while the right reflects the streams of photons being emitted by the charge. The field lines or photon streams are shown at different angles, e.g. 0, 30 and 60 degrees, from the maximum, which is always perpendicular to the axis. Finally, the oscillating red lines on left reflect the total electric field E=E_rad + E_vel as a function of distance. So what you see is the effects of E_vel reducing by 1/R^2, while E_rad only reduces by 1/R and so quickly becomes the dominate field as the radius from the charge increases.

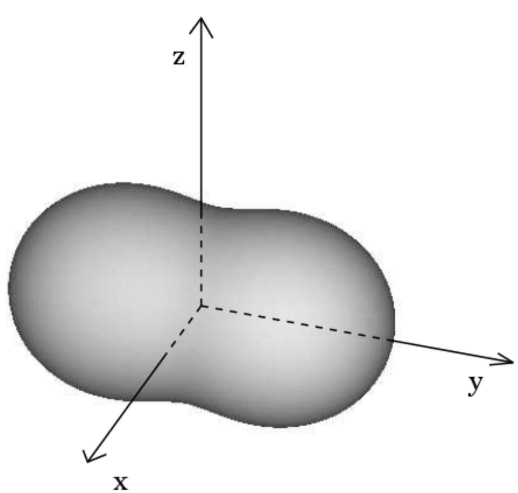
# Pattern of Scattered Radiation (Thomson Electron Scattering)




$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{1}{2}\left[\left(\frac{d\sigma(\Theta)}{d\Omega}\right)_{\text{pol}} + \left(\frac{d\sigma(\pi/2)}{d\Omega}\right)_{\text{pol}}\right]$$
$$= \frac{1}{2} r_0^2 (1 + \sin^2\Theta)$$
$$= \frac{1}{2} r_0^2 (1 + \cos^2\theta),$$

*(Try to plot at home the function sin^2(theta) and 1 + sin^2(theta) in <u>polar</u> coordinates as an exercise).*




Note that after the integration:

$$\sigma_{\text{unpol}} = \sigma_{\text{pol}} = (8\pi/3) r_0^2.$$

Also the scattered radiation will be polarized with a certain degree:

$$\Pi = \frac{1 - \cos^2\theta}{1 + \cos^2\theta}$$

# Electron scattering (Thomson) optical depth

*Remember that:* $\alpha_\nu = n\sigma_\nu$.

*Here sigma is the Thomson scattering cross section (units: cm^2)*
*n is the number density of particles.*

*The Thomson cross section has a value of 6.65e-25 cm^2*

*Let's go back to our problem of the Orion Nebula (Lecture 5).*
*We said n~10,000 and R=10^19 cm.*

*Therefore the optical depth due to electron scattering is:*

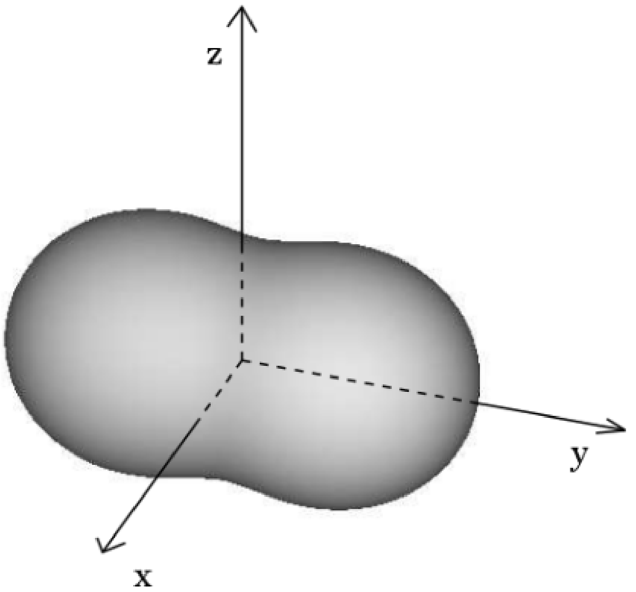
$$\tau = n\sigma_T R \approx 0.07$$

*Thus our HII cloud is optically thin both for electron scattering and for free-free absorption, thus we are really seeing Bremsstrahlung radiation!*

# Recap Thomson Scattering

Recall what we said about Thomson (electron) scattering:

1. It occurs when the photon's energy is << electron rest mass
2. The electrons move non-relativistically: v<<c.

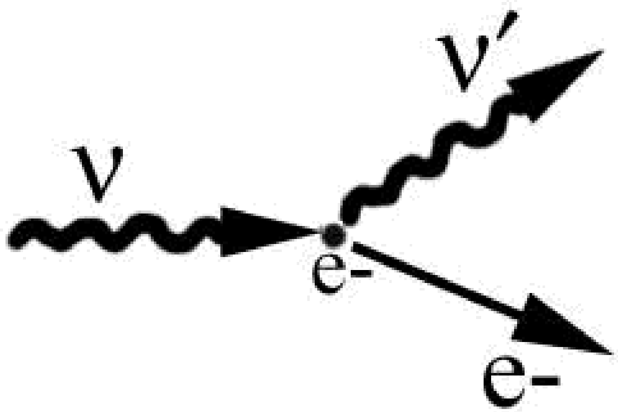



The radiation pattern has a "peanut shape". The incoming and outgoing photon has the same energy and the electron **does not change energy** in the scattering process (elastic or coherent scattering).

# Photon scattering by electrons - Overview

| | **Low energy photons** $\hbar\omega \ll m_e c^2$ | **High energy photons** $\hbar\omega \geq m_e c^2$ |
|---|---|---|
| $v \ll c$ | Thomson scattering<br>Classical treatment<br>frequency unchanged | Compton scattering<br>Quantum treatment incorporating photon momentum<br>frequency decreases |
| $v \sim c$ | $\gamma\hbar\omega \ll m_e c^2$<br>Inverse Compton<br>Photons gain energy from relativistic electrons<br>Approximate with classical treatment in electron rest frame<br>Frequency increases | $\gamma\hbar\omega \geq m_e c^2$<br>Inverse Compton<br>Quantum treatment in electron rest frame<br>Photons gain energy from relativistic electrons |

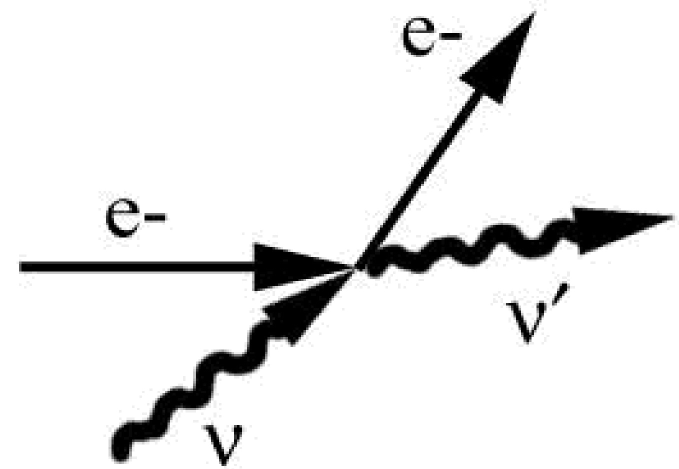
Compton scattering




$\nu' < \nu$

Electron is initially at rest
e- gains energy

Inverse Compton scattering




$\nu' > \nu$

High energy e- initially
e- loses energy

# Direct

**Photon *loses* energy**
**Electron gains energy**

# Inverse

**Photon *gains* energy**
**Electron loses energy**

# Preamble on the notation:

– The prime symbol **'** means that the quantity is calculated in the rest frame K**'** (i.e., the electron's rest frame in this case)

– No prime symbol means the quantity is calculated in the lab frame K (i.e., observer frame).

– The under-script **1** means that the quantity is calculated *after* the scattering has already occurred.

– No under-script means that the quantity is calculated *before* the scattering.

E.g.: E → energy before the scattering in K
E1 → energy after the scattering in K
E' → energy before the scattering in K'
E'1 → energy after the scattering in K'

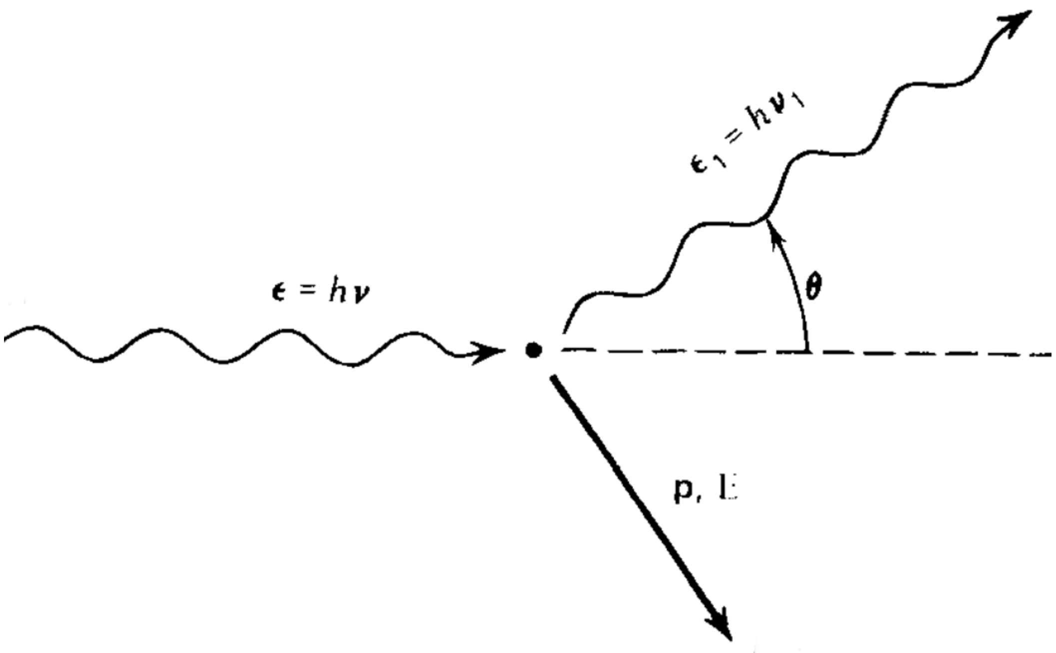
# Direct Compton

Let's start by looking at the momentum and energy of the photon and electrons.
In Thomson scattering the photon has no momentum (classical electrodynamics).
However, from quantum mechanics we do know that a photon has a momentum.
This means that any scattering process cannot be purely elastic since the electron will recoil due to the momentum of the photon.

The photon has initial energy $\epsilon$ and final energy $\epsilon_1$.

The photon has initial momentum $\epsilon/c$ and final momentum $\epsilon_1/c$.

The electron has initial energy mc^2 and final energy E/c.
The electron has initial momentum 0
and final momentum **p**.



# Direct Compton

Using the conservation of energy and momentum it's easy to show that the final and initial photon energies are related in the following way:

$$\epsilon_1 = \frac{\epsilon}{1 + \frac{\epsilon}{mc^2}(1 - \cos\theta)}$$

*Direct Compton Scattering*

This can be rewritten in terms of wavelengths as:

$$\lambda_1 - \lambda = \lambda_c(1 - \cos\theta)$$

where the subscript "c" refers to the Compton wavelength:

$$\lambda_c \equiv \frac{h}{mc}$$

$$= 0.02426\ \text{Å for electrons.}$$

# Direct Compton: elastic vs. inelastic

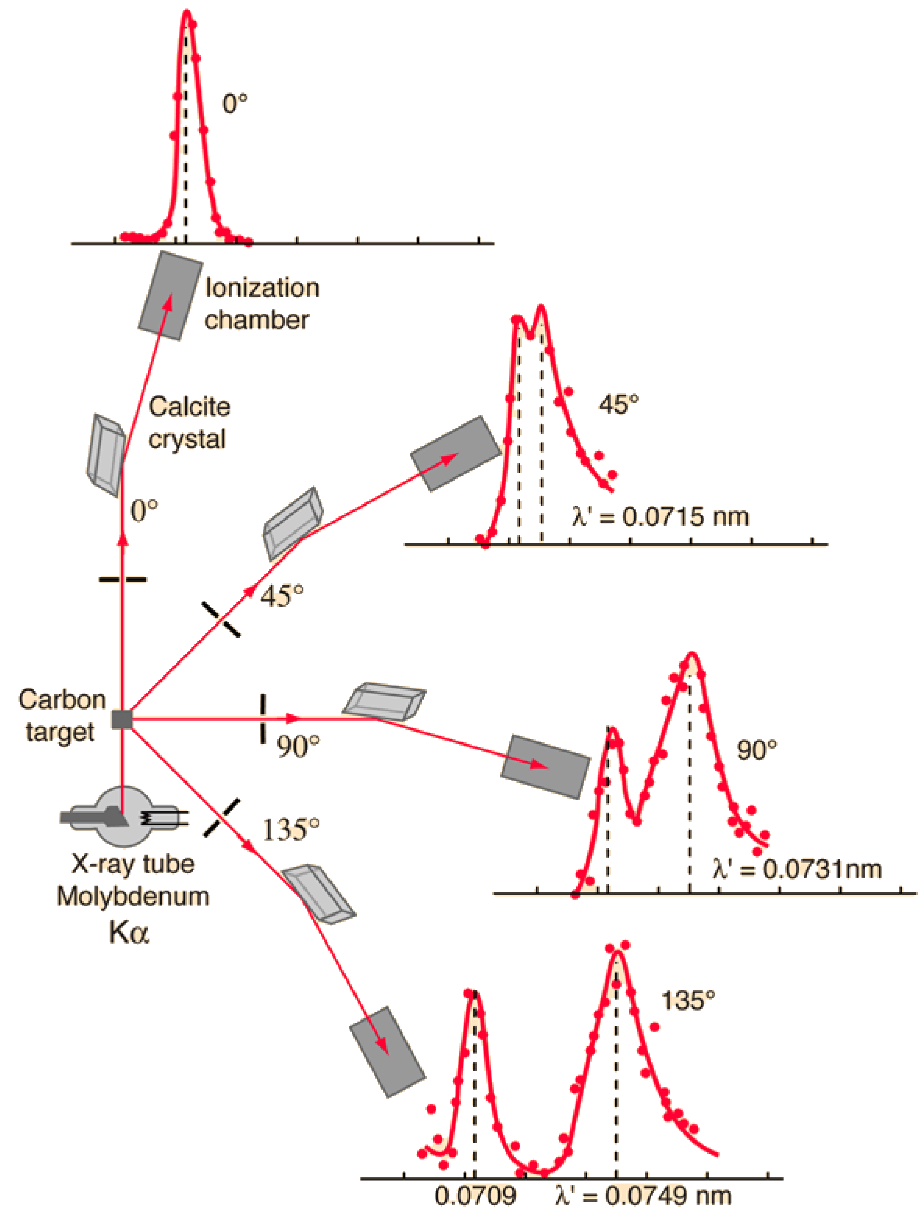
When the wavelength of the *incoming* photon is smaller than the Compton wavelength then the Compton scattering is important. The net effect is to *decrease* the energy of the photon. When the wavelength is larger than the Compton wavelength then elastic scattering (i.e., Thomson scattering) is a good approximation and the photon does not change wavelength (or energy).

**Direct Compton**

$$\lambda_1 - \lambda = \lambda_c(1 - \cos\theta)$$

**Thomson Scattering**

$$\lambda_1 - \lambda \approx 0$$



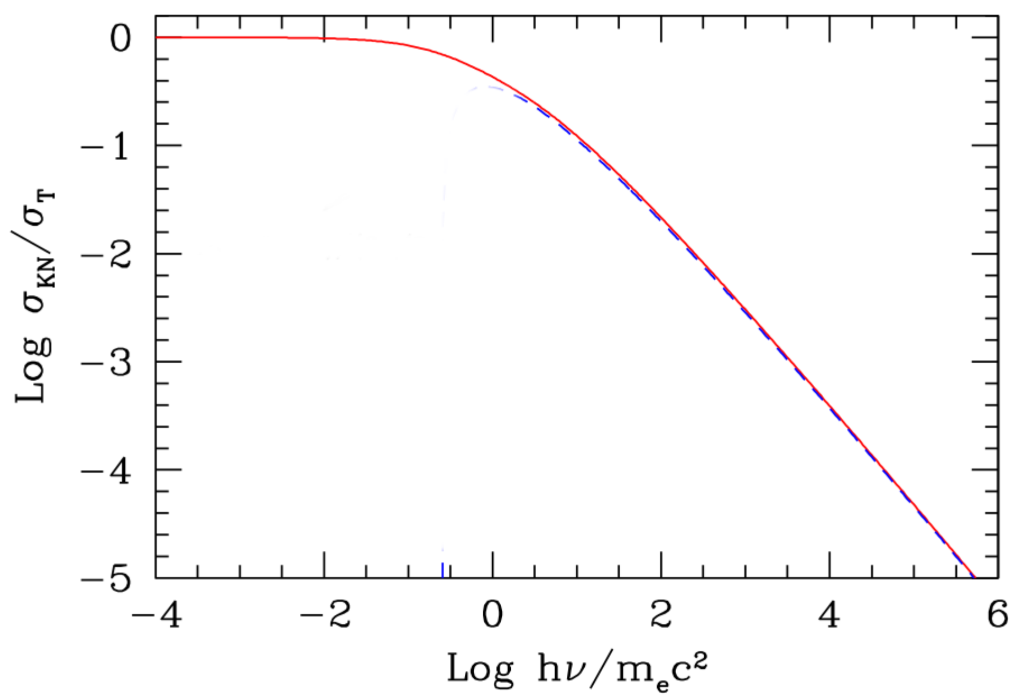
# Klein-Nishina Cross Section

Quantum effects enter in the Compton scattering not only because of the photon momentum. They also change the cross section of the electron.

When the energy of the photon approaches the rest mass energy of the electron, then the Thomson cross section changes and becomes the so-called Klein-Nishina cross section:

$$\sigma = \sigma_T \cdot \frac{3}{4}\left[ \frac{1+x}{x^3}\left\{ \frac{2x(1+x)}{1+2x} - \ln(1+2x) \right\} + \frac{1}{2x}\ln(1+2x) - \frac{1+3x}{(1+2x)^2} \right]$$

The “take home” message is that the KN cross section is smaller than the Thomson one



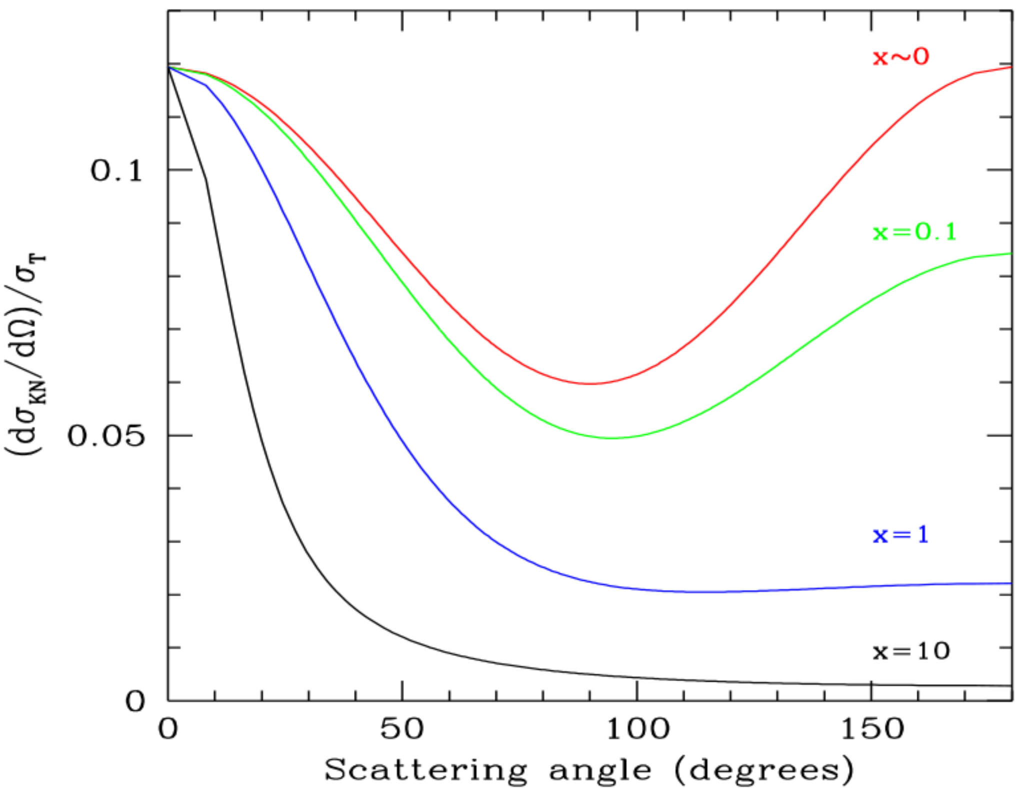
# Klein-Nishina Cross Section

It's important to have a look at the differential KN cross section per unit solid angle, since this is how we understand the radiation pattern of the scattering process:

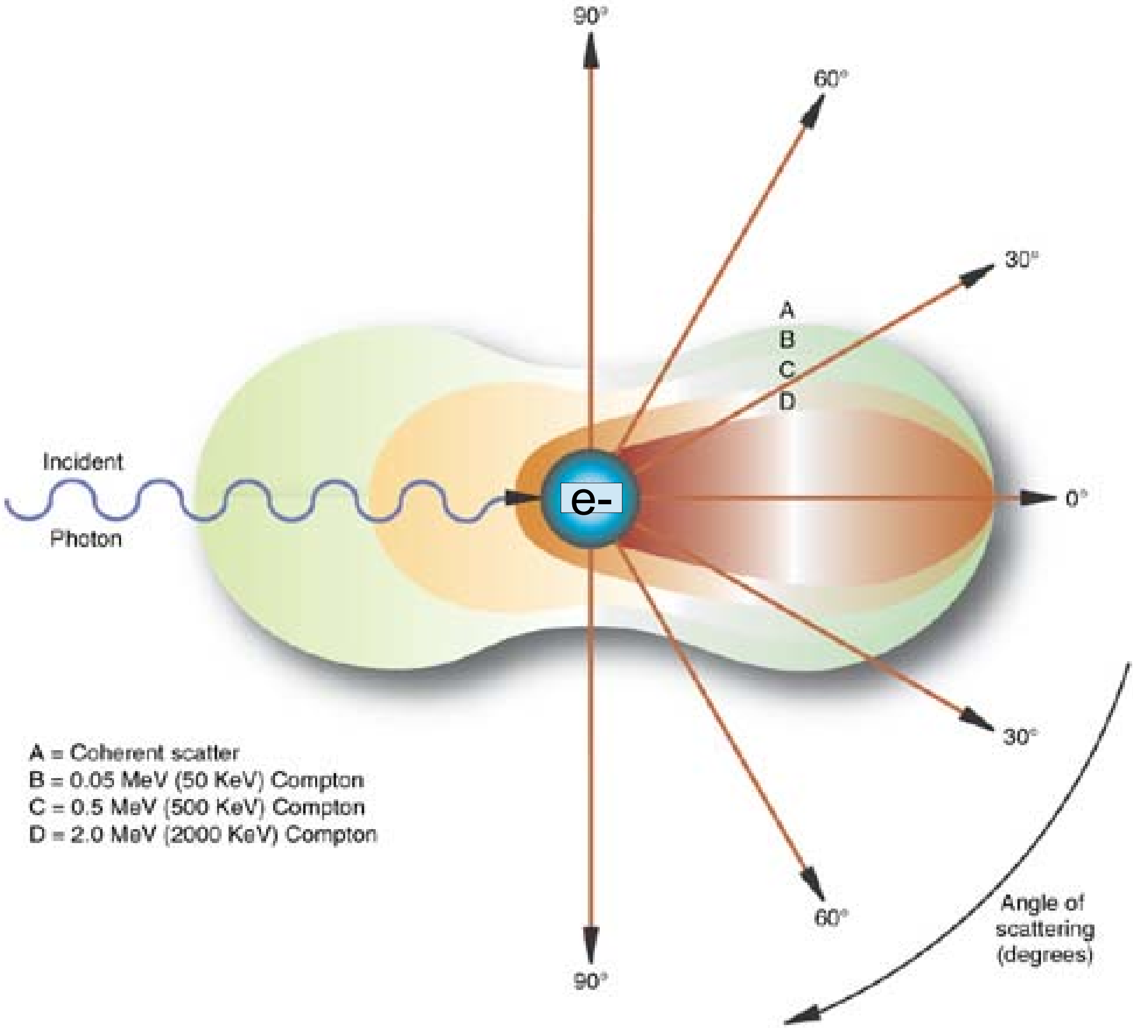
$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}\frac{\epsilon_1^2}{\epsilon^2}\left(\frac{\epsilon}{\epsilon_1} + \frac{\epsilon_1}{\epsilon} - \sin^2\theta\right)$$

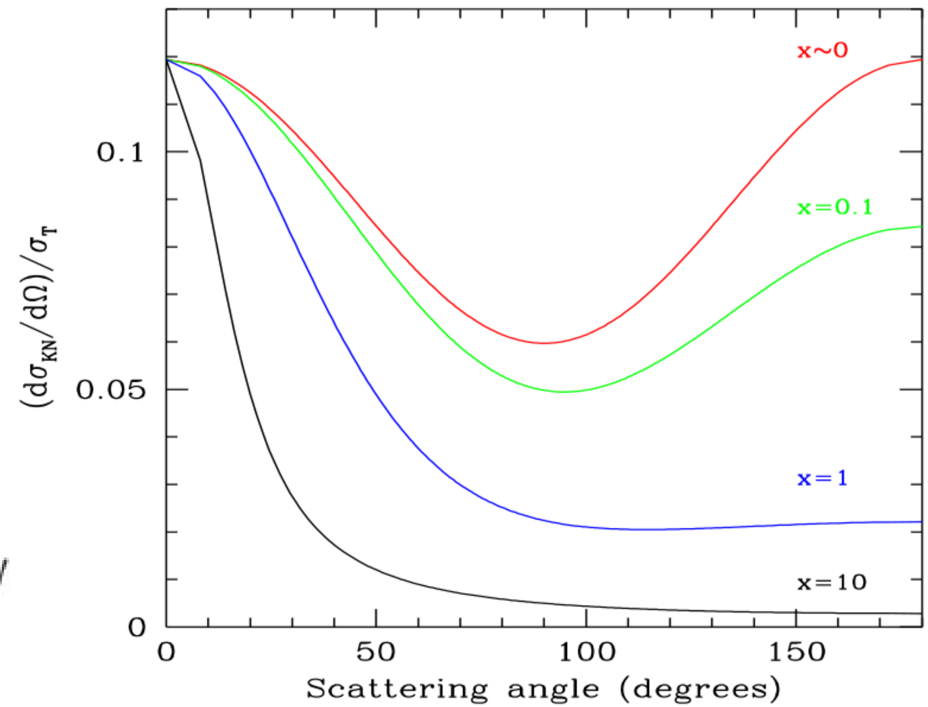
If we plot this function, we see that the scattering is not isotropic but becomes preferentially "forward" as the energy of the photon increases

Note that the x~0 corresponds to the Thomson scattering and the familiar "peanut shape".



# Radiation Pattern



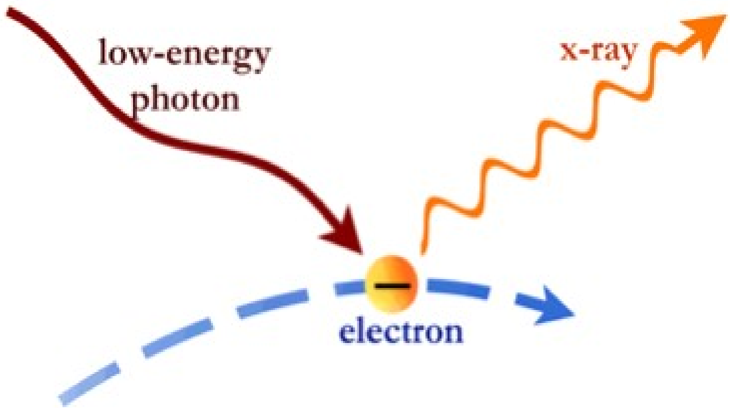




The green “peanut shape” pattern is the Thomson scattering (x~0, coherent scattering)
As the energy is increased the peanut shape disappears and the scattering becomes elongated in the “forward” direction.

# Inverse Compton Scattering

The direct Compton (or simply Compton) scattering is not a very common process in astrophysics, but its inverse process is very widespread: inverse Compton.

This is a *relativistic* and quantum phenomenon at the same time (whereas direct Compton is a purely quantum phenomenon).



# The first question we would like to answer now is:

How does the initial photon energy change after a collision with the relativistic electron?
(Inverse Compton)

# Remember the notation:

– The prime symbol **'** means that the quantity is calculated in the rest frame K**'** (i.e., the electron's rest frame in this case)

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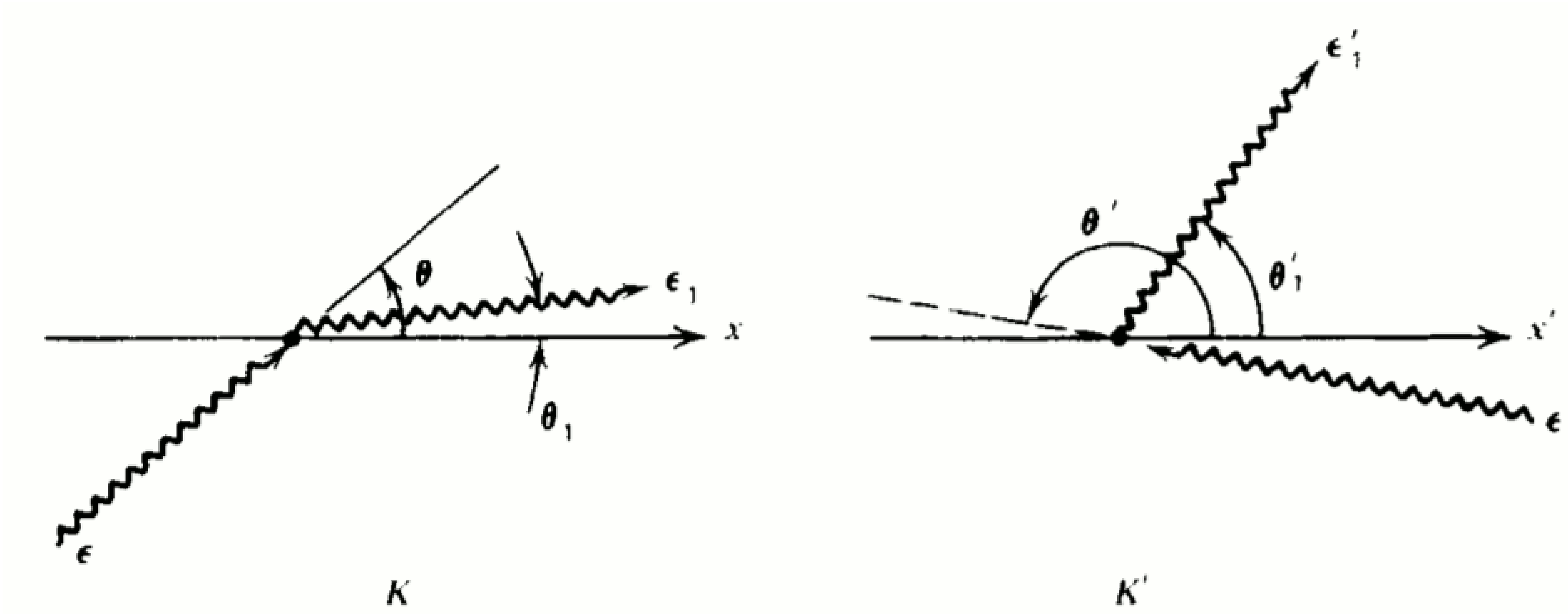
E' → energy before the scattering in K'
E'1 → energy after the scattering in K'

# Inverse Compton

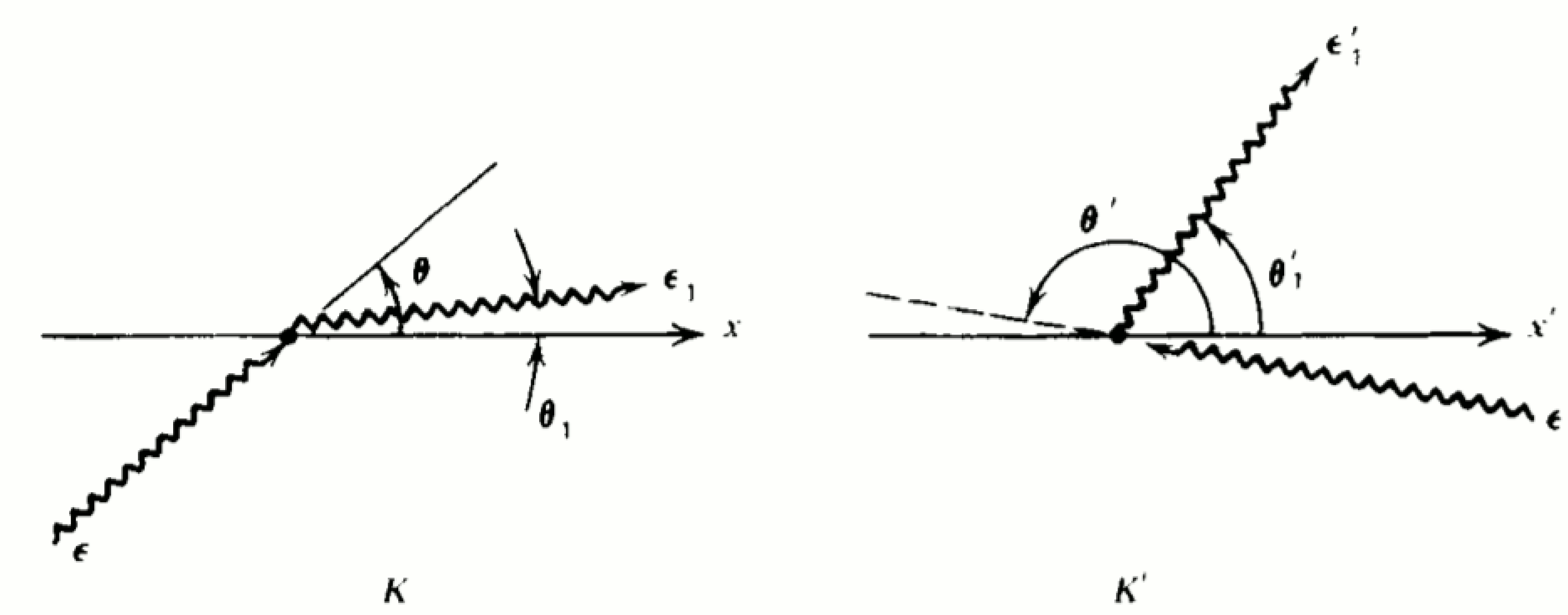



**Step 1**: Photon and e- in lab frame → e- rest frame

**Step 2**: Photon and e- interact in e- rest-frame

**Step 3**: Back to the lab frame

# Inverse Compton



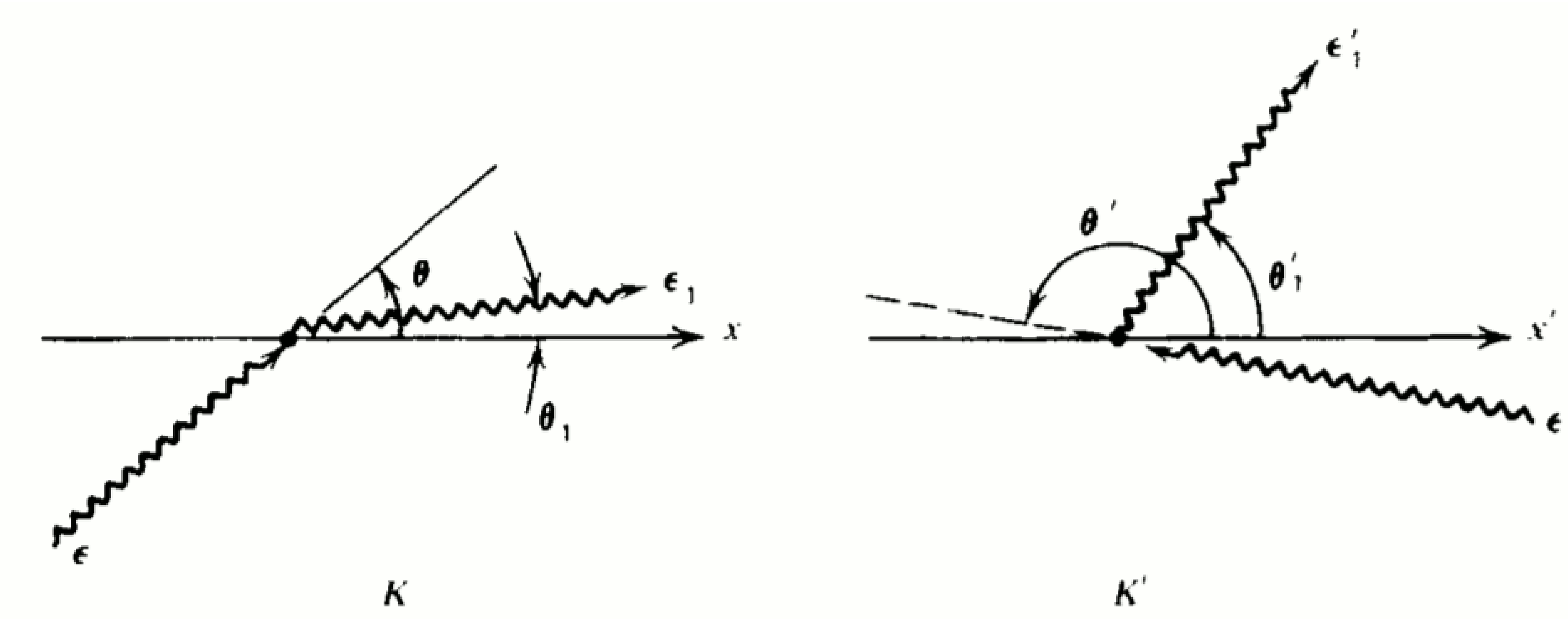

**Step 1**: lab frame → e- rest frame

$$\epsilon' = \epsilon \gamma (1 - \beta \cos\theta)$$

**epsilon** is the energy of the photon in K
**epsilon'** is the energy of the photon in K'

**BEFORE THE SCATTERING (see Eq. 4.12 on Rel. Doppler)**
**Note: The photon energy has *increased* in the e- rest frame (due to the relativistic Doppler boost)**

# Inverse Compton

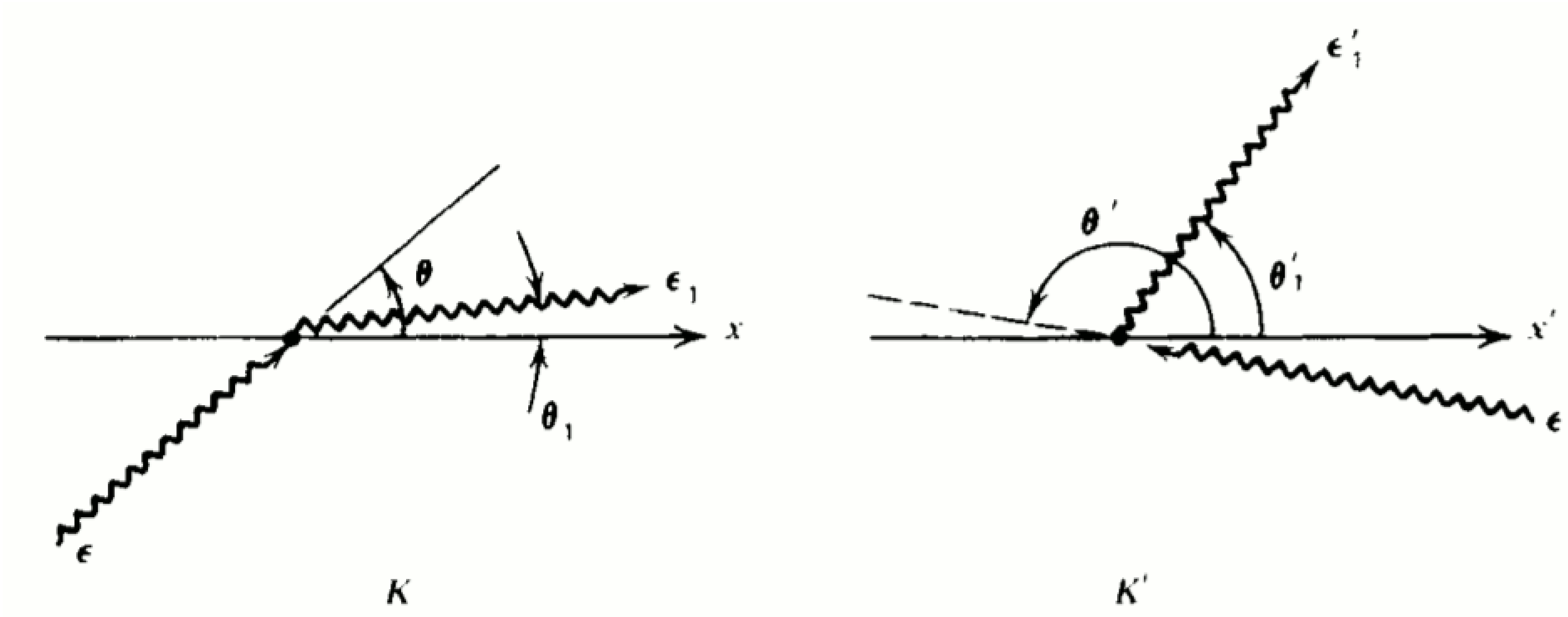



**Step 2**: Photon and e- interact in e- rest-frame

$$\epsilon_1{}' = \frac{\epsilon\,'}{1+\dfrac{\epsilon\,'(1-\cos\Theta)}{mc^2}} \approx \epsilon\,'\left[1-\frac{\epsilon\,'}{mc^2}(1-\cos\Theta)\right]$$

**We are now in the e- rest frame, so we can use the normal Compton formula (Eq. 7.2)**

# Inverse Compton



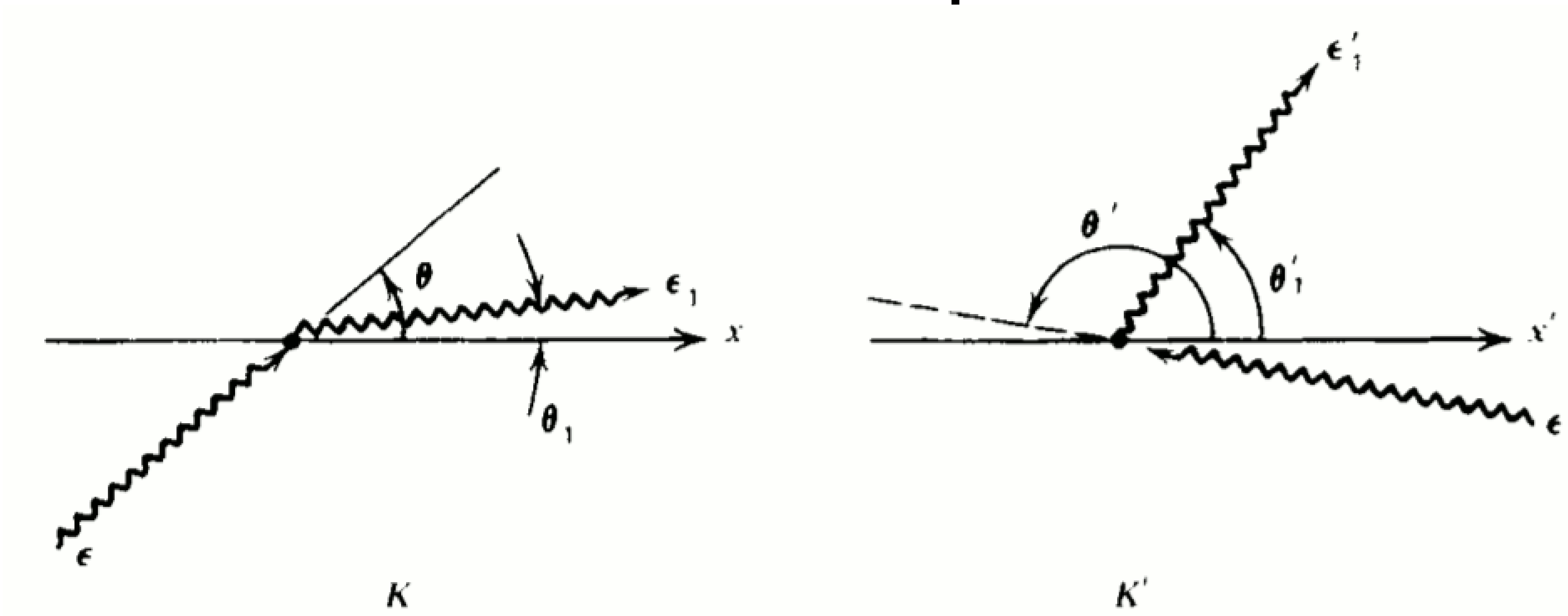

**Step 2**: Photon and e- interact in e- rest-frame

$$\epsilon_1{}' = \frac{\epsilon'}{1 + \frac{\epsilon'(1-\cos\Theta)}{mc^2}} \approx \epsilon'\left[1 - \frac{\epsilon'}{mc^2}(1-\cos\Theta)\right]$$

**Note:** $\cos\Theta = \cos\theta_1{}'\cos\theta' + \sin\theta'\sin\theta_1{}'\cos(\phi' - \phi_1{}')$

Phi is now the azimuthal angle between scattered photon and incident photon in the e- rest frame

# Inverse Compton

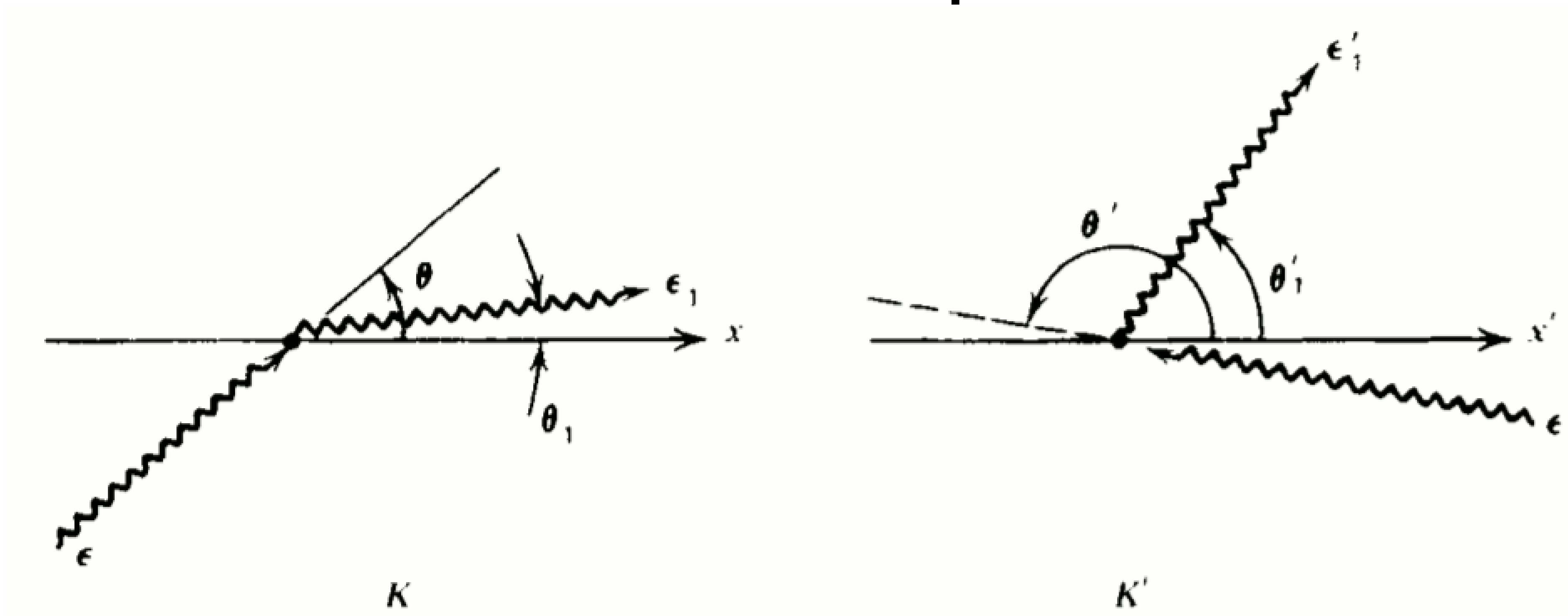



**Step 2**: Photon and e- interact in e- rest-frame

$$\epsilon_1' = \frac{\epsilon'}{1+\frac{\epsilon'(1-\cos\Theta)}{mc^2}} \approx \epsilon'\left[1-\frac{\epsilon'}{mc^2}(1-\cos\Theta)\right]$$

**Note 2: The photon energy has *decreased* in this process since some energy was given away to the electron**

# Inverse Compton

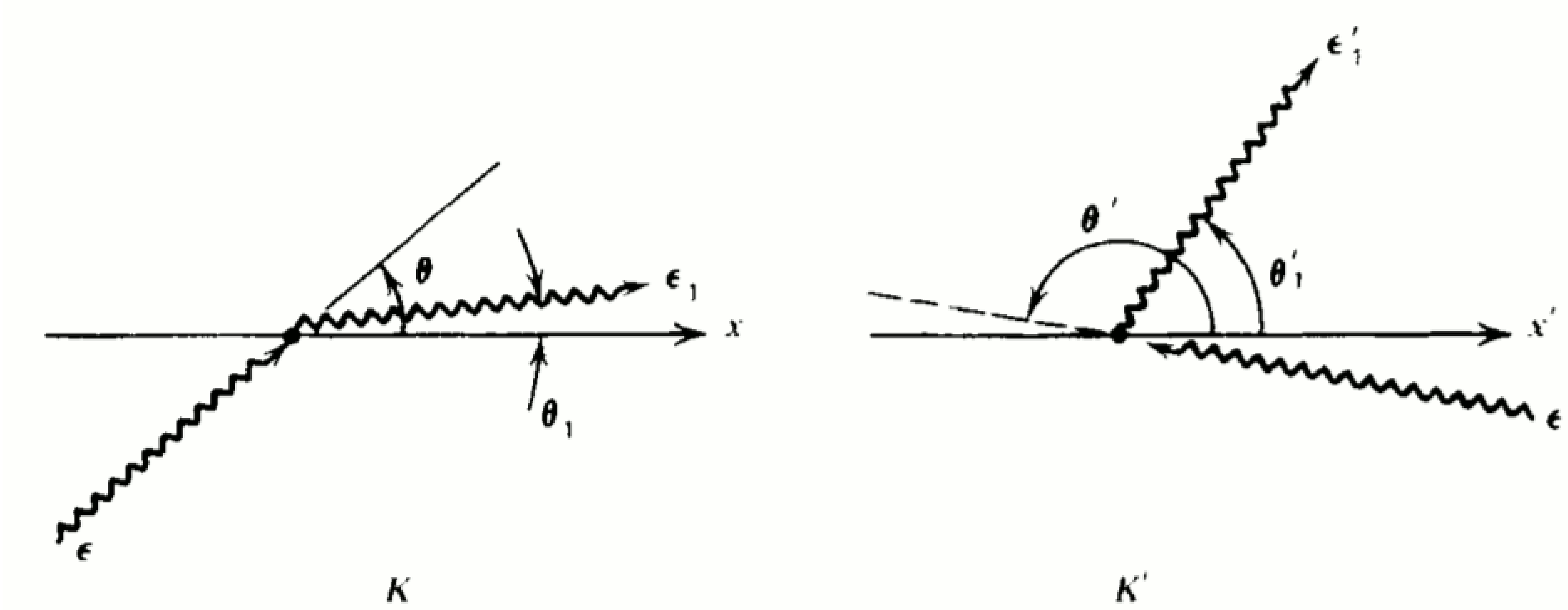



**Step 2**: Photon and e- interact in e- rest-frame

$$\epsilon_1{}' = \frac{\epsilon'}{1 + \frac{\epsilon'(1-\cos\Theta)}{mc^2}} \approx \epsilon'\left[1 - \frac{\epsilon'}{mc^2}(1-\cos\Theta)\right] \approx \epsilon'$$

**Note 3: If the photon energy is still << mc^2 then we are still in the Thomson regime**

# Inverse Compton




**Step 3**: Back to the lab frame

$$\epsilon_1 = \epsilon_1{}' \gamma \left(1 + \beta \cos \theta_1{}'\right)$$

**Note: We have a second Doppler boost because now we go back to the lab frame and use again Eq. 4.2. The photon energy has *increased* again (second relativistic Doppler boost)**

## Some comments:

**i.** In our step 2 we have used the Thomson scattering, i.e., ***we have assumed*** that the photon energy is << mc^2. In other words in the electron rest frame the scattered photon will have the same energy as before the scattering (E'1 = E').

**ii.** The photon *gains* an energy in step 1 and 3 by a factor gamma (so in total it gains a factor gamma^2 because of the double Relativistic Doppler boost).

**Iii.** The photon *loses* energy in step 2 (but this loss is basically ~0 if we're in the Thomson regime).

**Summary:**

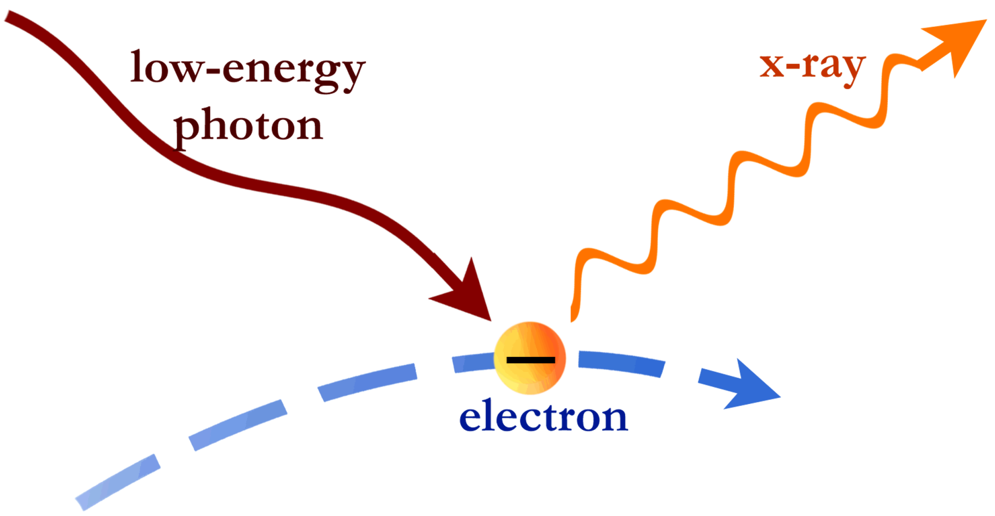
**Step 1. K → K' (1st Rel. Doppler boost)**
**Step 2. Compton Scattering (photon loses energy to the electron)**
**Step 3. K' → (2nd Rel. Doppler boost)**

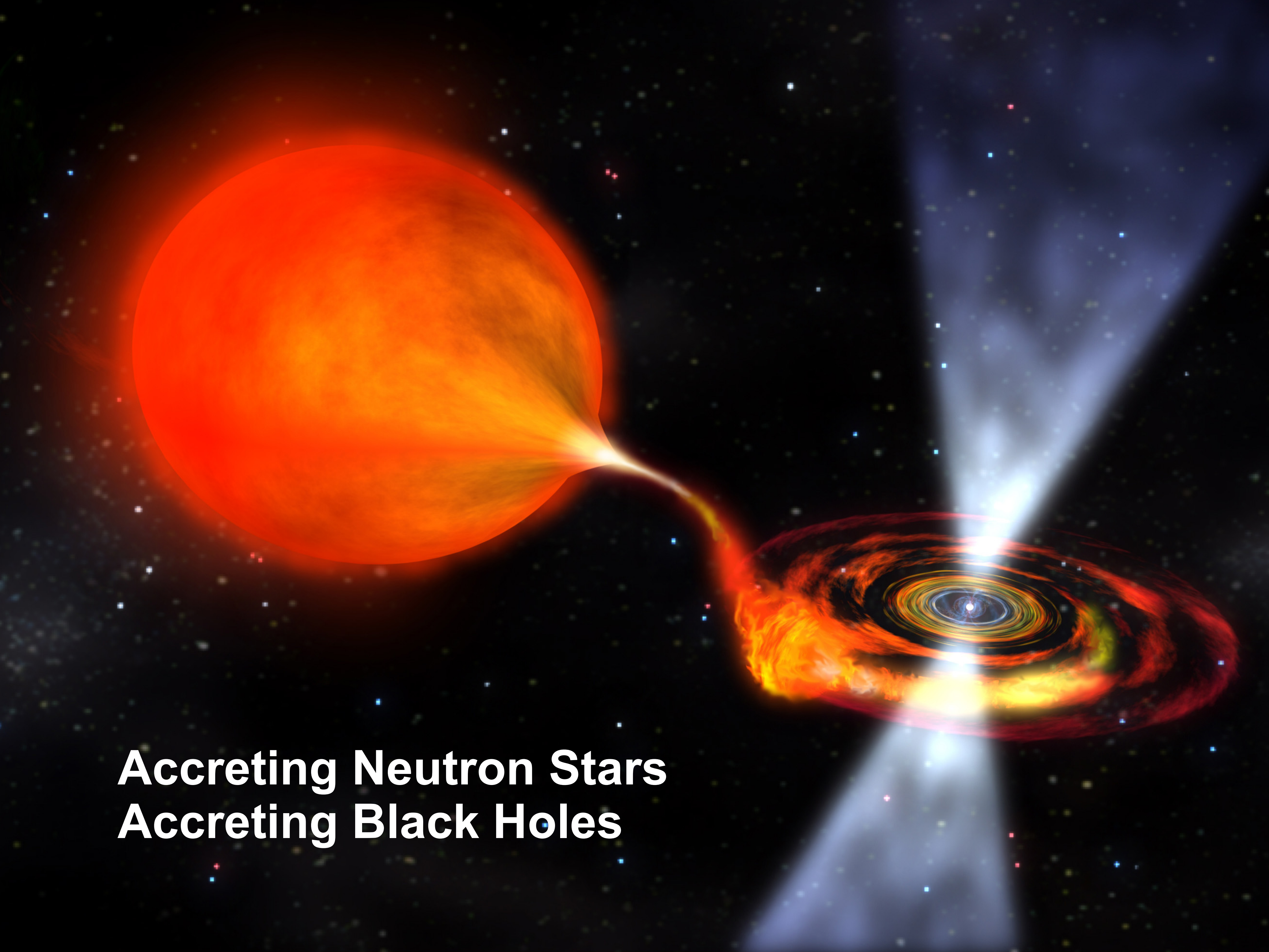
Example:

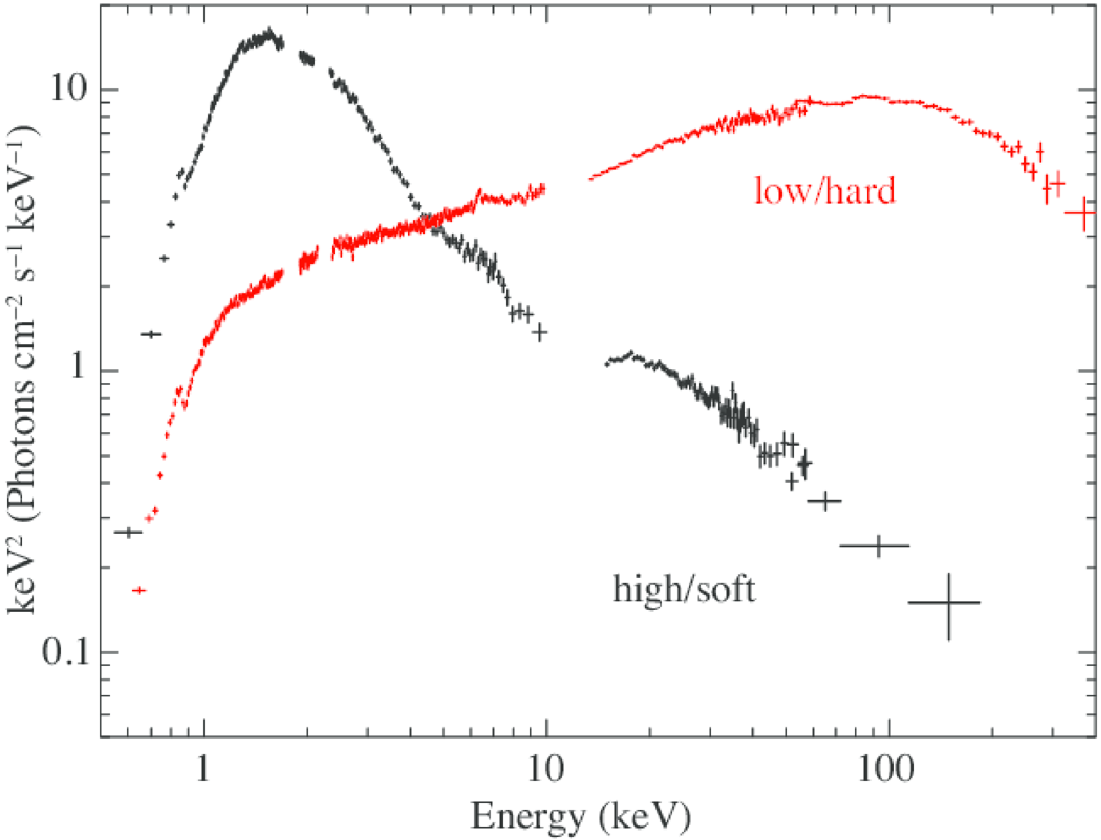
The rest mass energy of an electron is 511 keV.

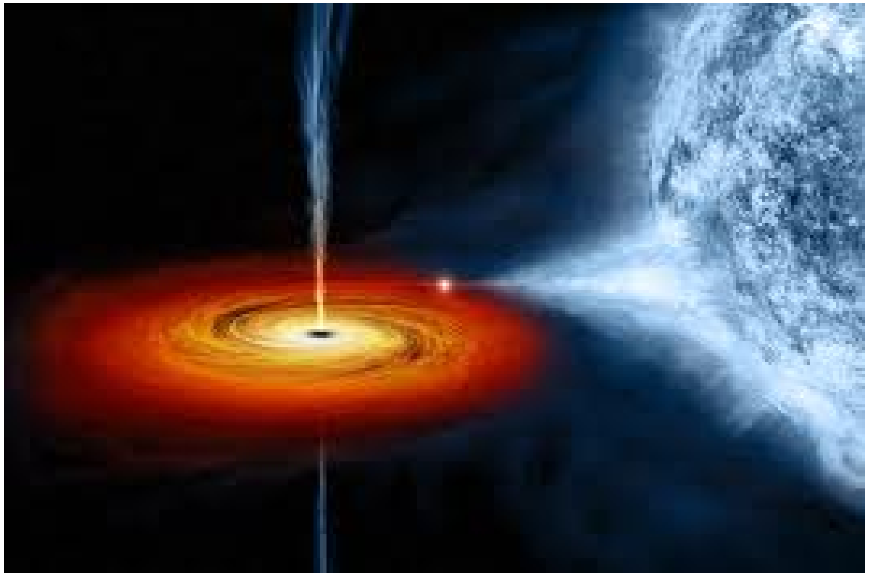
If the photon **in step 2** is still in the Thomson limit (after gaining a factor gamma in energy in **step 1**), then it will gain another factor gamma in energy in **step 3**.

For example, say that gamma = 10 (i.e., electron velocity is ~0.95 c).
Initial photon energy is 0.003 keV (visible light)
After one scattering (**step 1**) the photon has energy of the order 0.03 keV.
Since 0.03 keV << 511 keV we are still in the Thomson limit (**step 2**). We boost the photon again (**step 3**) and the final energy will then be 0.3 keV (soft X-rays)

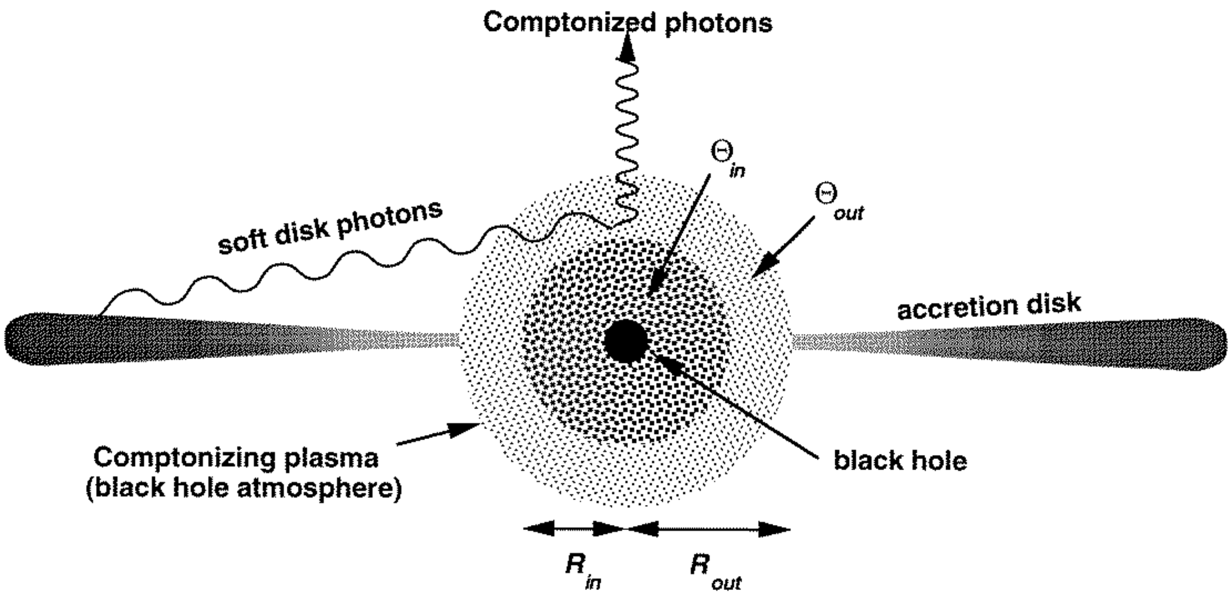



Accreting Neutron Stars
Accreting Black Holes



# Cygnus X-1




15 Msun black hole around a blue supergiant star.

Easily visible in optical with a small telescope (v ~ 9 mag)

# A final step to Inverse Compton

The last equation we derived $\epsilon_1 = \epsilon_1{}' \gamma \left(1 + \beta \cos \theta_1{}'\right)$ might bother you because not all quantities are calculated in the lab frame K.

We can add a final step by transforming the angles from K' to K by using the formula for the aberration of angles (see Lecture 3).

In this way after some algebra we arrive at the expression:

$$\boldsymbol{\epsilon_1 = \epsilon \frac{1 - \beta \cos \theta}{1 - \beta \cos \theta_1}}$$

# Maximum and Minimum Energy

We can now see that wen theta = pi and theta1 = 0, then the photon is scattered along the electron velocity vector (head-on) and epsilon1 is maximized.

$$\epsilon_1 = \epsilon \frac{1+\beta}{1-\beta}$$

When theta = 0 and theta1 = pi, then the photon scatters “from behind” the electron (tail-on) and epsilon1 is minimized

$$\epsilon_1 = \epsilon \frac{1-\beta}{1+\beta}$$

So, in fact, the photon is *gaining energy* in the former case and *losing energy* in the latter.

Question: what happens if we have many photons coming from all directions? Will inverse Compton produce a higher energy photon distribution or not?

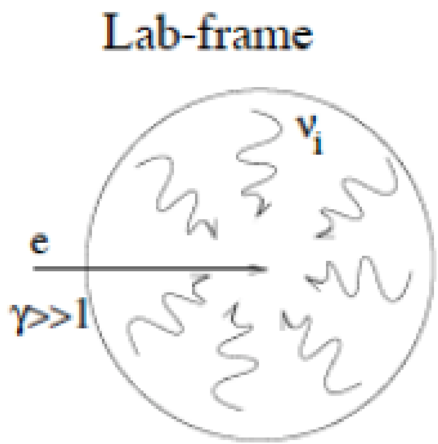
# Answer

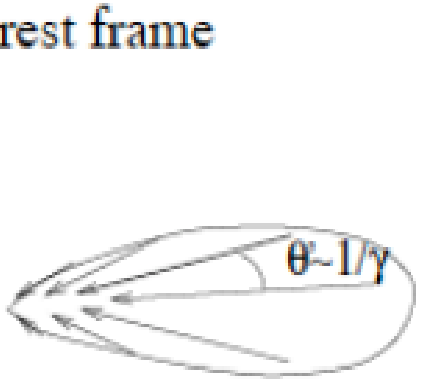
If we have many photons, collisions in which the photon overtakes the electron (in K) will result in a reduced photon energy, but, on average, the energy gain will be positive because there are relatively fewer overtaking than head-on collisions. Remember the example of the blob surrounded by broad line regions (Lecture 4).

# Answer

If we have many photons, collisions in which the photon overtakes the electron (in K) will result in a reduced photon energy, but, on average, the energy gain will be positive because there are relatively fewer overtaking than head-on collisions. Remember the example of the blob surrounded by broad line regions (Lecture 4).

Why?




We'll show later that the average photon energy after one IC scattering is:

$$\langle \epsilon_1 \rangle = \frac{4}{3} \gamma^2 \epsilon$$

# Inverse Compton: isotropic photon distribution

What happens for an isotropic distribution of photons? (But still one single electron) **Let's calculate the total emitted power in this case.**

Let's start by defining the differential photon number density *dn.*

$$dn = dN / dV = \left( dN / dX \right) dx_0$$

*dX* and *dV* are the 4D and 3D space volumes and are related in the following way:

$$dX = dx_0\, dx_1\, dx_2\, dx_3 = dV\ dx_0$$

It can be shown that *dX* is a Lorentz invariant (see Section 4.9 of R&L). Therefore since *dN* is invariant (it's a number), we have that *dn* transforms as time (dx0).

We also know that time and energy (Lorentz) transform in the exact same way, therefore we can create a new (Lorentz) invariant:

$$\frac{dn}{\epsilon} = Lorentz\ Invariant$$

With this invariant we can now easily calculate the number of Compton scatterings per unit time per electron in the general case

The expression for the total scattering rate is most readily written down in terms of the rate in K'. In K' immediately before a scattering the electron is at rest; time intervals are related by dt = gamma*dt', and the scattering rate is:

$$\frac{dN}{dt} = \gamma^{-1} \frac{dN'}{dt'} = \gamma^{-1} c \int \sigma\, dn'$$

Here sigma is the total Compton cross section.

Now we use the invariant $dn/\epsilon = dn'/\epsilon'$ plus the expression we found before for the energy of the photon K' before the scattering occurs:

$$\epsilon' = \epsilon\gamma(1-\beta\cos\theta)$$

In this way we obtain the following expression:

$$\frac{dN}{dt} = \gamma^{-1}\frac{dN'}{dt'} = \gamma^{-1} c \int \sigma\, dn' = c\int \sigma(1-\beta\cos\theta)\, dn$$

What is the meaning of $c(1-\beta\cos\theta)$?

It is just the relative velocity of the photon and electron along the direction of the electron's motion.

We know also that power (=energy/time) is a Lorentz invariant, therefore we can write:

$$\frac{dE_1}{dt} = \frac{dE_1'}{dt'}$$

In the Thomson limit we have $\epsilon_1' \approx \epsilon'$ and thus we can write for the total power emitted:

$$\frac{dE_1}{dt} = \frac{dE_1'}{dt'} = \epsilon_1 \frac{dN}{dt} = \int \sigma_T c \epsilon' dn' = \sigma_T c \gamma^2 \int (1 - \beta \cos\theta)^2 \epsilon \, dn$$

This expression refers now solely to quantities in K.

For an isotropic distribution of photon we have:

$$\frac{dE_1}{dt} = c \sigma_T \gamma^2 \left(1 + \frac{1}{3}\beta^2\right) U_{ph}$$

Where Uph is the total photon energy density

# IMPORTANT

This expression refers only to the power in the scattered radiation.
It is NOT the power in the radiation *before the scattering.*

*Therefore the **net power** emitted by the electron is:*

[Total energy loss of the electron] =
[Power in radiation after scattering] – [Power in radiation before scattering]

$$\frac{dE_1}{dt} = c\,\sigma_T\,\gamma^2\left(1+\frac{1}{3}\beta^2\right)U_{ph} \qquad\qquad \frac{dE_1}{dt} = -\sigma_T\,c\,U_{ph}$$

$$P_{compt} = \frac{dE_{rad}}{dt} = c\,\sigma_T\,\gamma^2\left(1+\frac{1}{3}\beta^2\right)U_{ph} - \sigma_T\,c\,U_{ph} = \frac{4}{3}\sigma_T\,c\,\gamma^2\beta^2\,U_{ph}$$

# Synchrotron vs. Compton Power

$$P_{compt} = \frac{4}{3} \sigma_T c \gamma^2 \beta^2 U_{ph}$$

$$P_{synch} = \frac{4}{3} \sigma_T c \gamma^2 \beta^2 U_B$$

Why are these two powers so similar?
After all we have a very different physical mechanism operating.

# Synchrotron vs. Compton Power

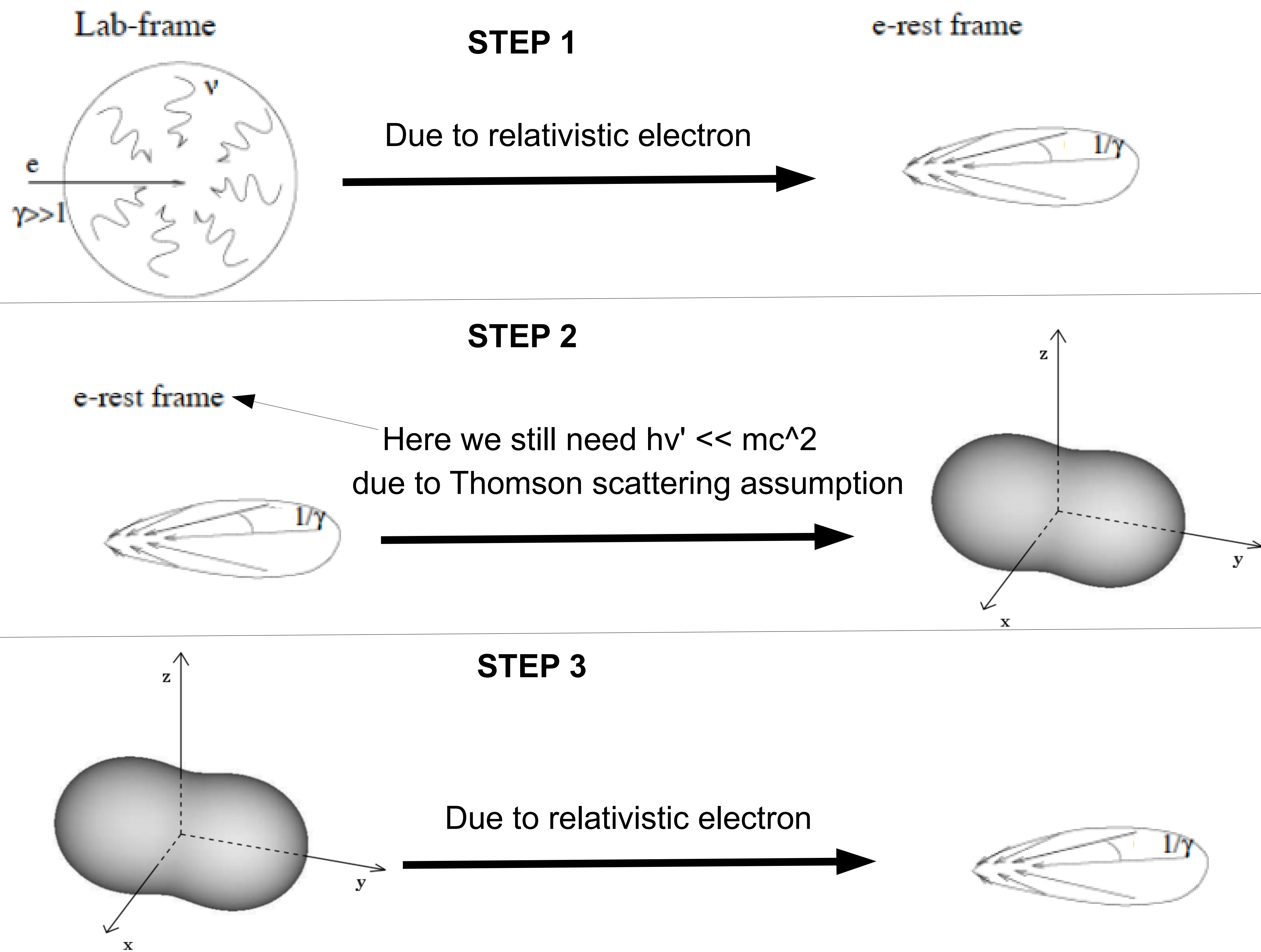
$$P_{compt} = \frac{4}{3} \sigma_T c \gamma^2 \beta^2 U_{ph}$$

$$P_{synch} = \frac{4}{3} \sigma_T c \gamma^2 \beta^2 U_B$$

Why are these two powers so similar?
After all we have a very different physical mechanism operating.

The reason for this is that the energy loss rate depends upon the electric field which accelerates the electron in its rest frame and it does not matter what the origin of that field is.

In the case of synchrotron radiation, the electric field is the (v × B) field due to motion of the electron through the magnetic field whereas, in the case of inverse Compton scattering, it is the sum of the electric fields of the electromagnetic waves incident upon the electron.



Lab-frame
STEP 1
e-rest frame
ν
e
γ>>1
Due to relativistic electron
1/γ
STEP 2
e-rest frame
Here we still need hν' << mc^2
due to Thomson scattering assumption
1/γ
z
y
x
STEP 3
z
y
x
Due to relativistic electron
1/γ

# Single Particle Spectrum

As we did for the Synchrotron radiation, we now want to check what is the spectrum of the (isotropic) scattered radiation emerging after a collision with a single electron

The derivation is lengthy and we'll skip it (See 7.3 in R&L if interested in the mathematical details).

Here we give the general properties.

First we've seen that: $\epsilon_1 = \epsilon \frac{1 - \beta \cos\theta}{1 - \beta \cos\theta_1}$

We've seen that the maximum of this function is: $\epsilon_1 = \epsilon \frac{1+\beta}{1-\beta}$

Which can be rewritten as $\epsilon_1 = \epsilon \gamma^2 (1+\beta)^2 \approx 4 \gamma^2 \epsilon$

Therefore the IC spectrum (for single scattering) must fall off at $4\gamma^2\epsilon$

Second: from the power emitted we know that $P_{compt}=\frac{4}{3}\sigma_T c\gamma^2\beta^2 U_{ph}$

The average number of photons scattered per unit time is:

$$\frac{dN}{dt}=\frac{\sigma_T c U_{ph}}{\epsilon}$$

Therefore the average photon energy after a scattering is:

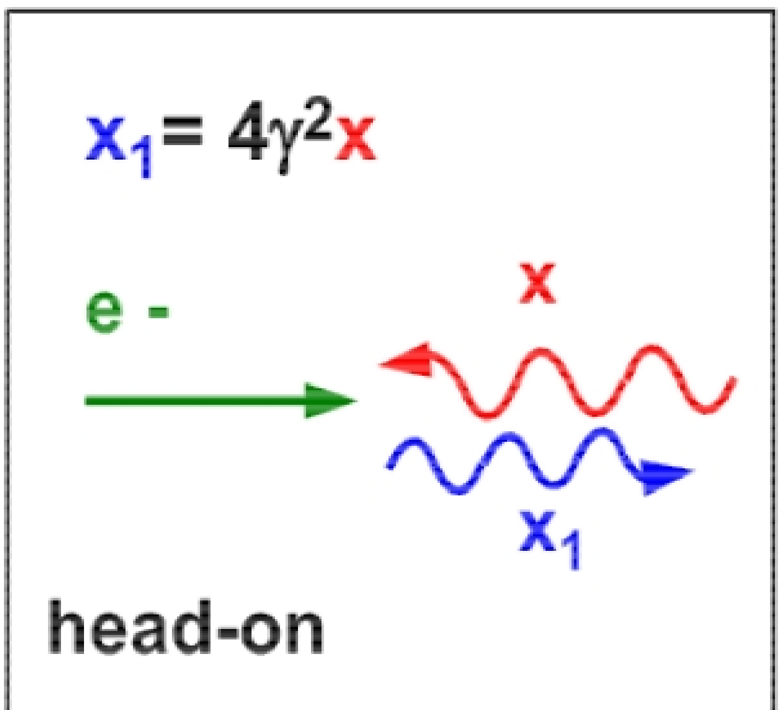
$$\langle\epsilon_1\rangle=\frac{4}{3}\gamma^2\epsilon$$

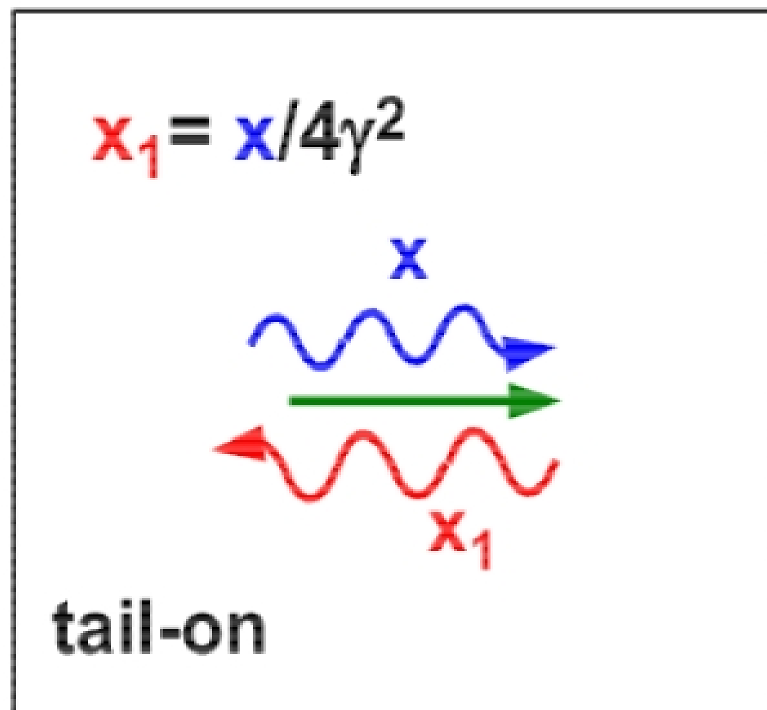
This is a consequence of the fact that:

$$P_{compt}=\left(\frac{\#\text{ of collisions}}{\text{sec}}\right)\text{(average phot. energy after scatt.)}$$

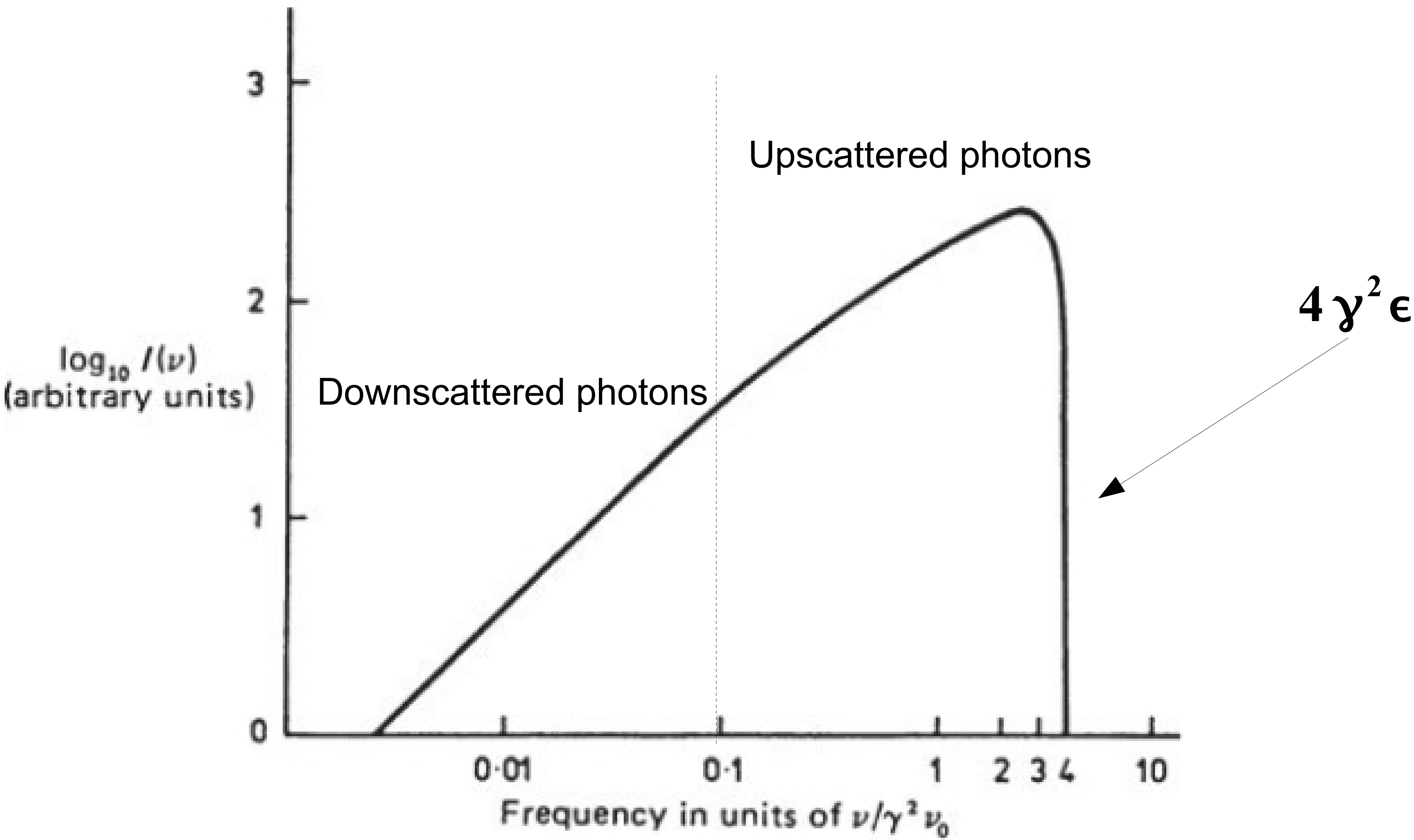
Some photons (tail-on) will be *downscattered*
Some (most) photons (head-on) will be *upscattered*

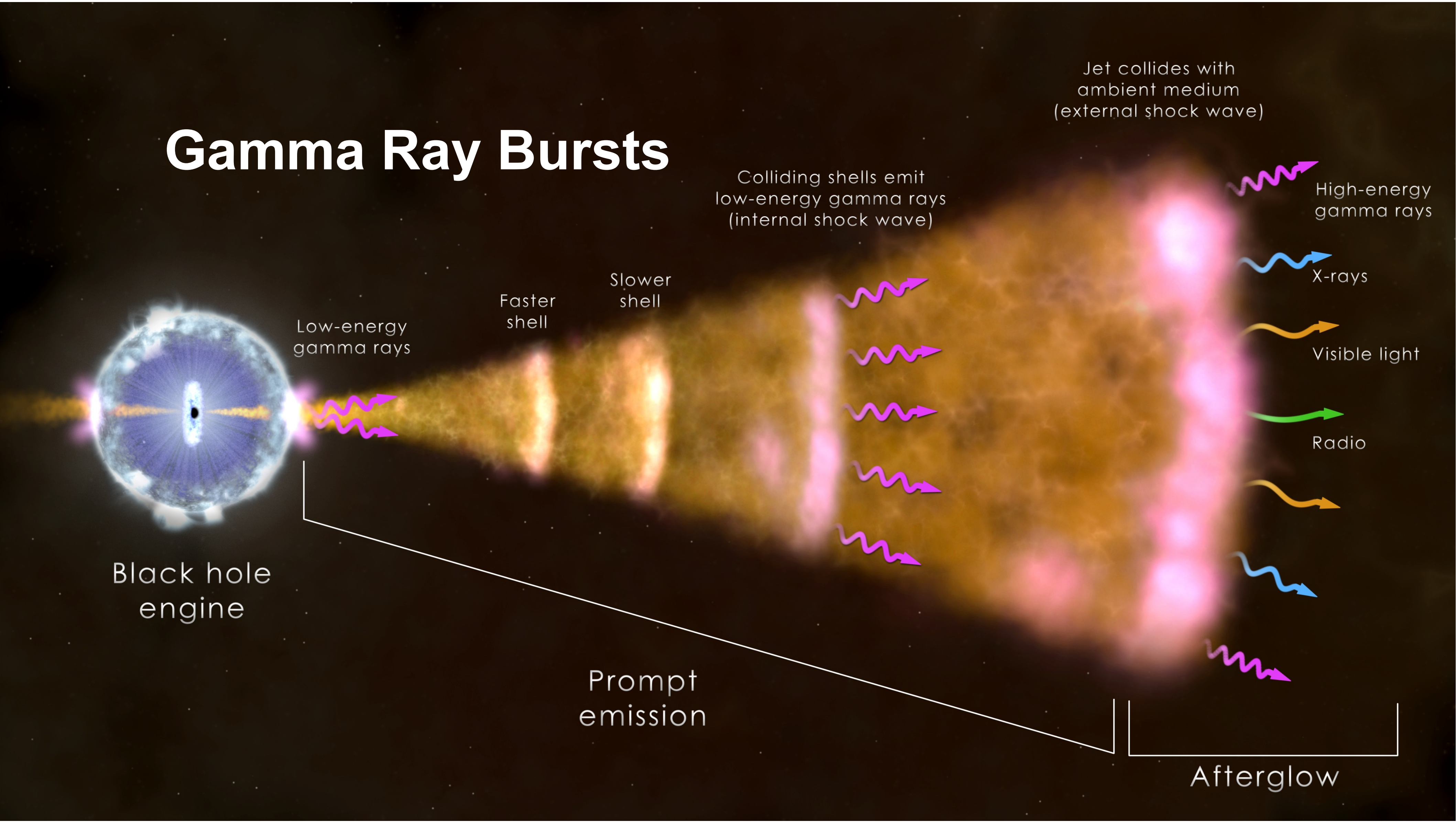
Therefore the low frequency part of the spectrum will be filled by downscattered photons. The high frequency part by upscattered photons.



# Single Particle Spectrum




Gamma Ray Bursts
Jet collides with
ambient medium
(external shock wave)
Colliding shells emit
low-energy gamma rays
(internal shock wave)
High-energy
gamma rays
X-rays
Visible light
Radio
Slower
shell
Faster
shell
Low-energy
gamma rays
Black hole
engine
Prompt
emission
Afterglow

# Sunyaev Zel'dovich Effect