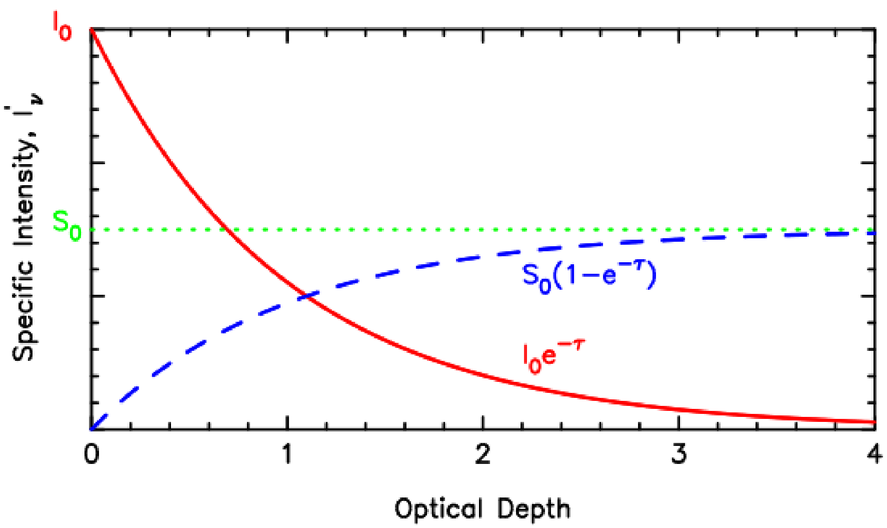
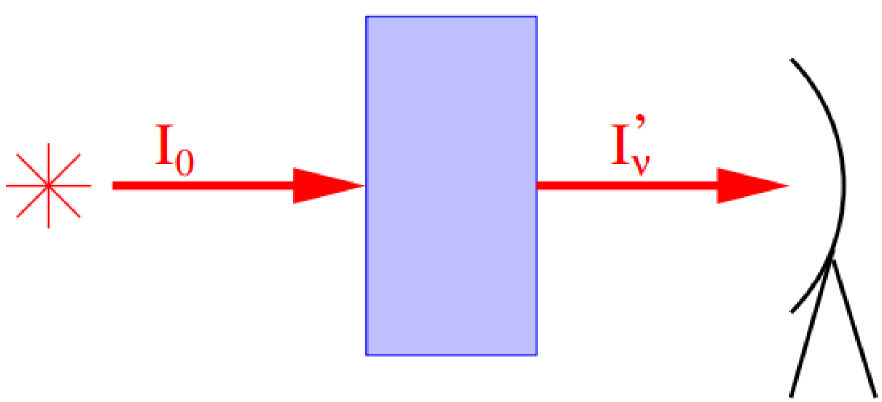
The meaning of the Source Function



The *Source Function* is the value approached by the specific brightness given sufficient optical depth.



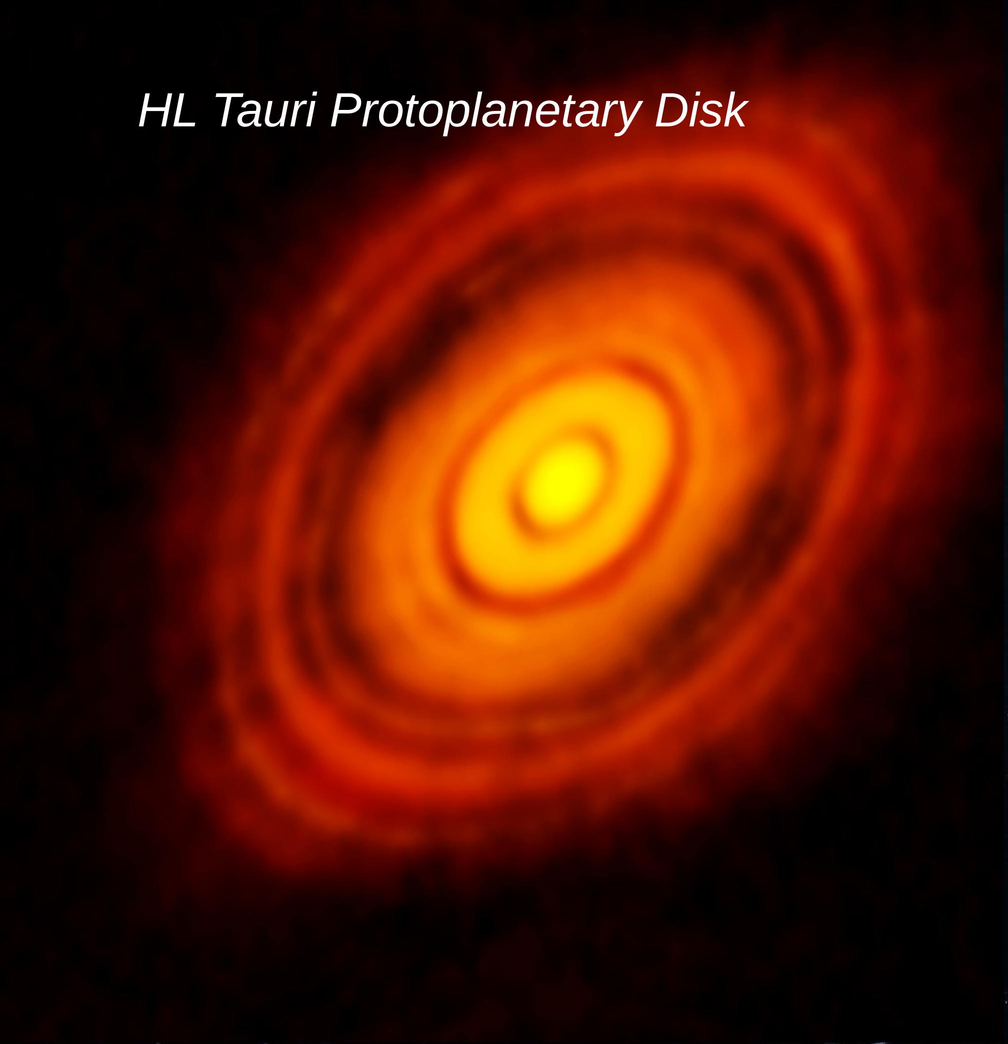
The emergent radiation is the sum of the incident intensity attenuated by the total optical depth plus the sum of each section of cloud emission attenuated by the optical depth from that point to the receiver.

$$I'_\nu = I_0\, e^{-\tau} + S_0(1-e^{-\tau})$$

Example: ALMA

Atacama Large Millimeter Array is an astronomical interferometer that uses microwaves and has provided some of the most spectacular results of the last few years.

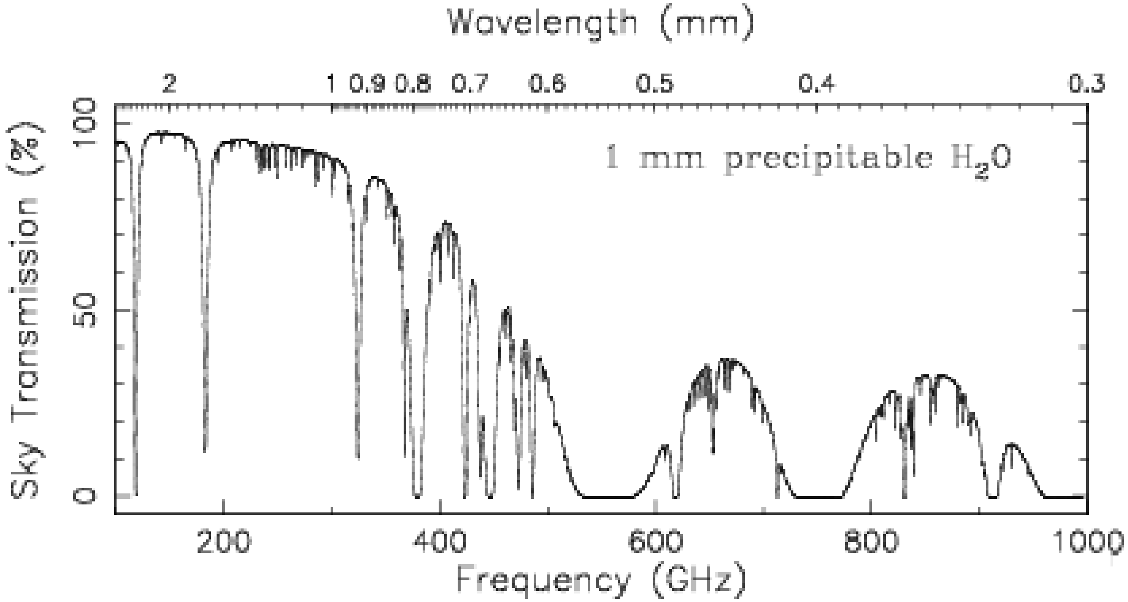
It operates at wavelengths of 0.3-10 mm (30-1000 GHz).


HL Tauri Protoplanetary Disk

Antennae Galaxies

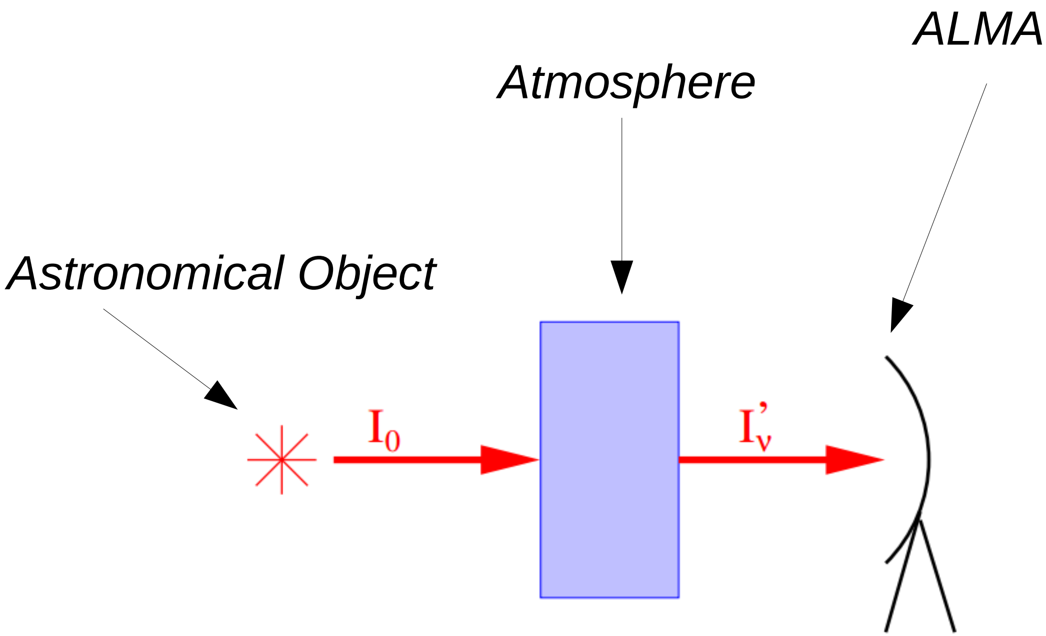
ALMA/Hubble Ultra-Deep Field

Let's go back to our example.
Suppose ALMA is observing at a certain frequency, let's pick the minimum frequency of 30 GHz (10 mm).
The atmosphere has a certain small but non-zero optical depth, due to the absorption of water vapor in that band. A typical value for tau is 0.02. at 30 GHz.



This plot refers to 1 mm precipitable H2O in the atmosphere and refers to a different location than ALMA, but it's a good illustrative example.

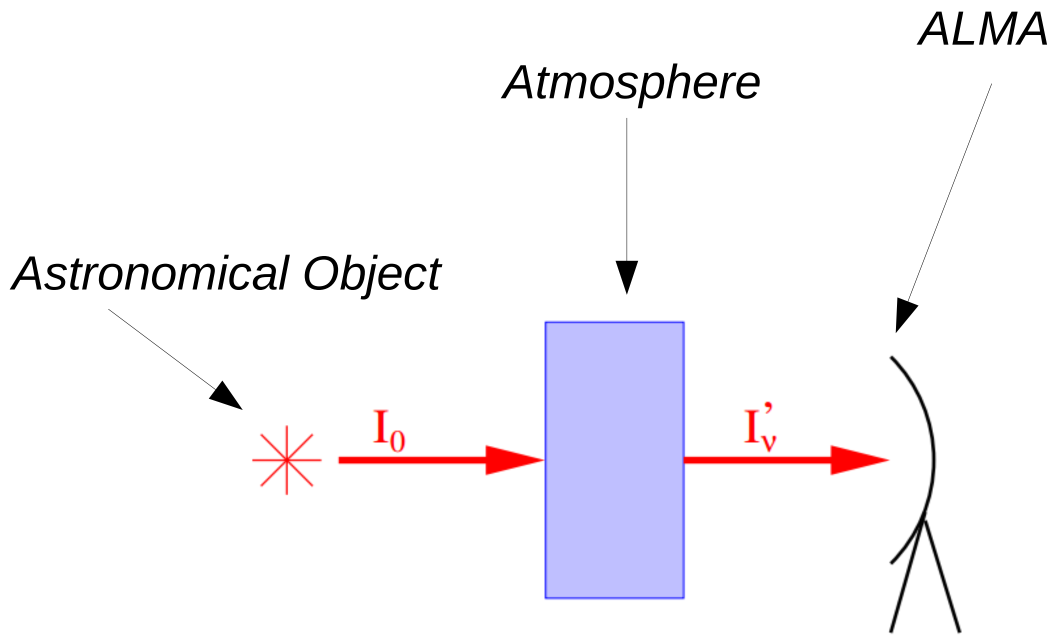
Assume now that the atmosphere is in thermal equilibrium at a temperature of 300 K (this is really not the case, the atmosphere is in LTE and it has a temperature gradient, but let's make things simple)



$$I'_\nu = I_0\, e^{-\tau} + S_0(1 - e^{-\tau})$$

1. The atmosphere is in thermal equilibrium, so we can use Kirchhoff's law:

$$S_\nu = B_\nu(300\,K)$$



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1. The atmosphere is in thermal equilibrium, so we can use Kirchhoff's law:

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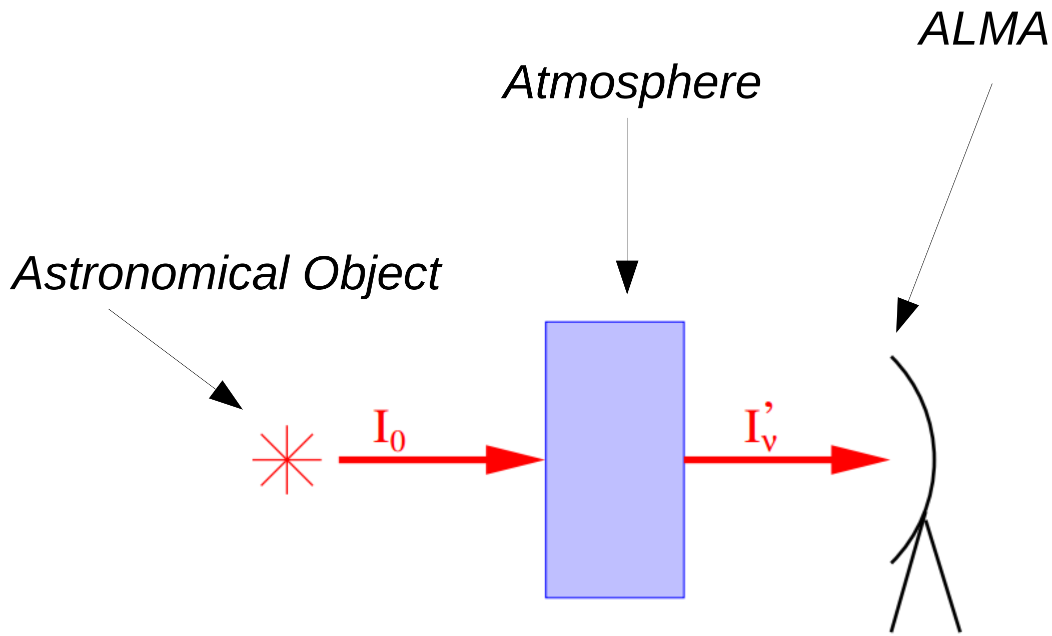
2. We can use the Rayleigh-Jeans approximation if $h\nu \ll kT$

$$h\nu = 2\times 10^{-16}\,erg$$

$$kT = 4\times 10^{-14}\,erg$$

Therefore we can use the RJ approximation: $B_\nu \approx \frac{2\nu^2}{c^2} kT$



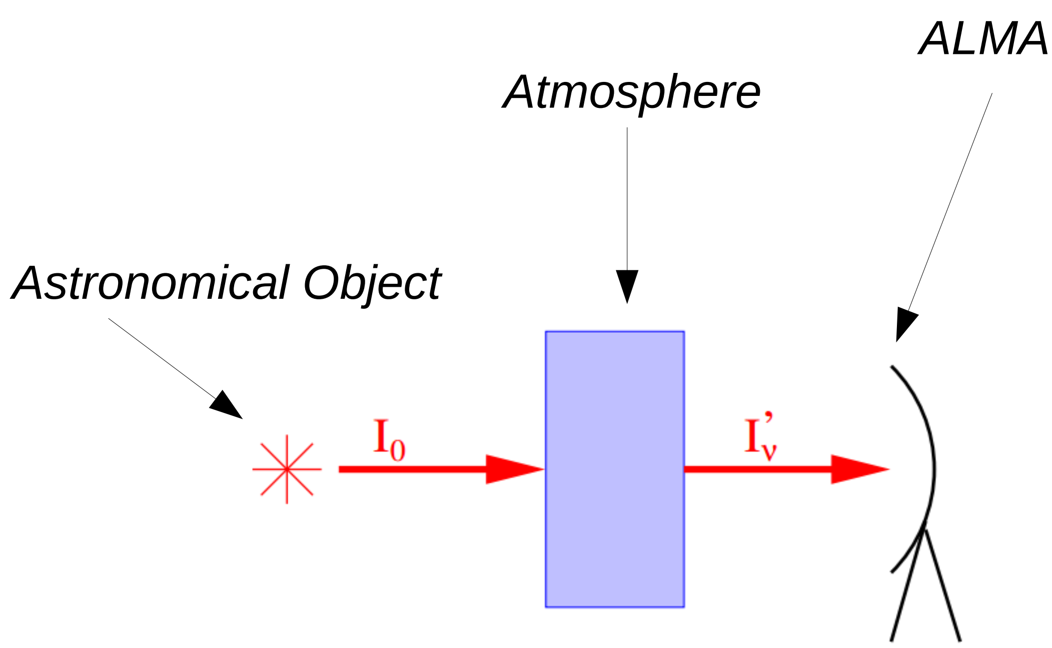
$$I'_\nu = I_0\ e^{-\tau} + S_0(1-e^{-\tau})$$

From the Equation of Transport we know that when the optical depth <<1 as is the case here, we can rewrite to first order the above equation as:

$$I'_\nu \approx I_{\nu,0}(1-\tau_\nu)+\tau_\nu B_\nu(300\,K)$$

Therefore: $\tau_\nu B_\nu(300\,K) \approx \tau_\nu \frac{2\nu^2}{c^2} kT \approx 1.7\times10^{-7}\,erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,ster^{-1}$

This is the brightness of the sky at 30 GHz.



$$I'_\nu = I_0\, e^{-\tau} + S_0(1-e^{-\tau})$$

Suppose we are observing a source with a brightness (at 30 GHz) of:

$$I_{\nu,0} \approx 10^{-6}\, erg\, s^{-1}\, cm^{-2}\, Hz^{-1}\, ster^{-1}$$

We would see:

$$I'_\nu \approx (1-0.02)\times 10^{-6} + 1.7\times 10^{-7}\, erg\, s^{-1}\, cm^{-2}\, Hz^{-1}\, ster^{-1} = 1.15\times 10^{-6}\, erg\, s^{-1}\, cm^{-2}\, Hz^{-1}\, ster^{-1}$$

So the astronomical object appears at 98% of its brightness at 30 GHz due to absorption plus we see some contribution of the sky.